MID TERM EXAMINATION, SUMMER, 1430-1431
Department of Mathematics
King Saud University
MATH: 107
Time: 2 H Full Marks: 30

Question # 1. Marks: 6

Solve the following homogeneous system:

$$\begin{cases} 2x+2y+4z &=0 \\ w-y-3z &=0 \\ 2w+3x+y+z &=0 \\ -2w+x+3y-2z &=0. \end{cases}$$

Question # 2. Marks: 1+5+3=9

Consider the following system of linear equations:

$$\begin{cases} x_1+3x_2+x_3 &=4 \\ 2x_1+2x_2+x_3 &=-1 \\ 2x_1+3x_2+x_3 &=3. \end{cases}$$

(i) Write the system in the matrix form.
(ii) Use the elementary matrix method to find A^{-1}, where A denotes the coefficient matrix.
(iii) Use A^{-1} to find the solution of the above system.

Question # 3. Marks: 3+3+3=9

(a) Evaluate determinant of the matrix A by reducing to reduced-row echelon form, where

$$A=\begin{pmatrix} 1 & 3 & 5 \\ 4 & 14 & 12 \\ -2 & -3 & -20 \end{pmatrix}.$$

(b) If A is a 4×4 matrix, and $\det(A)=8$, and $\det(B)=5$, then evaluate $\det(3A^{-1}B^T)$.
(c) For what values of λ, the matrix A is not invertible

$$A=\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 0 & \lambda \end{pmatrix}?$$

(d) Solve the system for z only by using Cramer Rule:

$$\begin{cases} x+y-z & =2 \\ 3x-y+z & =5 \\ 3x+2y+4z & =0. \end{cases}$$

Question # 4. Marks:2+2+2=6

(a) A force is given by a vector $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and moves a particle from the point $P(2,1,0)$ to the point $Q(4,6,2)$. Find the work done.

(b) If $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$. Find $\text{Comp}_{\mathbf{b}}\mathbf{a}$ and $\text{Proj}_{\mathbf{a}}\mathbf{b}$.

(c) Find the area of the parallelogram with vertices $P(1,3,-2)$, $Q(2,1,4)$ and $R(-3,1,6)$.