TIME: 90 min **KING SAUD UNIVERSITY** **FULL MARKS:50**

M - 107 **DEPARTMENT OF MATHEMATICS**

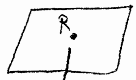
II MID TERM EXAM (SEM II) 1433-1434

Question: 1. **(a) Find the parametric equation of the line L orthogonal to the plane containing the points A(2, 1, 3), B (1, 2, 4) and C(1, 2, 3).**

[6+6+6]

(b) Find the volume of the parallelepiped having adjacent sides AB, AC and AD, where $A(0\ ,\ 0,\ 0)$, $B(\ -2,\ 2,-3\ )$, $C(-1,-1,1)$ **and** $D(\ 4,-1,-1)$.

(c) Find the equation of the plane P_2 **that passes through the point R (3 , 1, -2) and parallel to plane** $P_1:\ x-5y+3z=5$. **Also find the distance between these planes.**



Question: 2 . (a) If the acceleration of a moving particle is given by $a(t)=i+2j+6tk$,

[6+6]

find the object's velocity and position given that the initial velocity is $v(0)=j-k$ **and the initial position is** $r(0)=i-2j+3k$.

(b) Write the surface $r^2+4z^2=16$ **in the Cartesian coordinates form. Identify the surface. Find its traces on the coordinate planes and sketch the surface.**

Question: 3. (a)If the position vector of an object is $r(t)=\left(\frac{1}{2}t^2+1\right)i+\left(t^2-2\right)j+\left(t^3+3\right)k$,

[8+6+6]

find the general formula for the tangential and normal components of acceleration and for the curvature of the curve C .

(b) Let C be the curve determined by the vector valued function

$$r(t)=(t\cos t)i+(t\sin t)j+3tk\ ,$$

Find the parametric equation of tangent line to curve C at the point $P\left(0,\frac{\pi}{2},\frac{3\pi}{2}\right)$.

$3y^{-2}$

$-3y^{-1}$ $3y^{-2}$ $-6y^{-3}$

(c) Find the second partial derivatives of

$$f(x,y)=x^3e^{2y}+\sin^2 y\cos 2x+3\sqrt{x}-\frac{3}{y}.$$

$\frac{1}{2}x^{\frac{1}{2}}$ $(x)^{\frac{1}{2}-1}$

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Q 1 (a)

(6)

$\overrightarrow{AB} = \langle -1, 1, 1 \rangle$, $\overrightarrow{AC} = \langle -1, 1, 0 \rangle$

vector orthogonal to the plane containg A, B and C

$$N = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix} = \langle -1, -1, 0 \rangle$$

Equation of line parallel to N and passing through point A (2, 1, 3) is

$x = 2 - t$, $y = 1 - t$, $z = 3$, $t \in \mathbb{R}$.

(b)

(6)

$\overrightarrow{AB} = \langle -2, 2, -3 \rangle$, $\overrightarrow{AC} = \langle -1, -1, 1 \rangle$, $\overrightarrow{AD} = \langle 4, -1, -1 \rangle$

Volume of parallelepiped $= |\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD})| = \begin{vmatrix} -2 & 2 & -3 \\ -1 & -1 & 1 \\ 4 & -1 & -1 \end{vmatrix}$

$= -2(1+1) - 2(1-4) - 3(1+4)$

$= -4 + 6 - 15 = |-13| = 13$ unit^3

(c) (i) vector normal to plane P_2 is $N = \langle 1, -5, 3 \rangle$

(6)

Equation of plane P_2 with normal vector N and passing through point R (3, 1, −2) is

$1(x-3) - 5(y-1) + 3(z+2) = 0$

$x - 5y + 3z + 8 = 0$

(ii) Distance between two plane is distance of R from P_1

$$d = \frac{|3 - 5 - 6 - 5|}{\sqrt{1 + 25 + 9}} = \frac{12}{\sqrt{35}}$$

Q.2 (a)

(6)

$V(t) = \int (i + 2j + 6t k)\,dt = ti + 2tj + 3t^2 k + c_1 i + c_2 j + c_3 k$

$V(0) = j - k = c_1 i + c_2 j + c_3 k \Rightarrow c_1 = 0$, $c_2 = 1$, $c_3 = -1$

$V(t) = ti + (2t+1)j + (3t^2 - 1)k$

$r(t) = \int [ti + (2t+1)j + (3t^2 - 1)k]\,dt$

$= \frac{t^2}{2} i + (t^2 + t)j + (t^3 - t)k + c_1 i + c_2 j + c_3 k$

$r(0) = i - 2j + 3k = c_1 i + c_2 j + c_3 k \Rightarrow c_1 = 1$, $c_2 = -2$, $c_3 = 3$

(2)

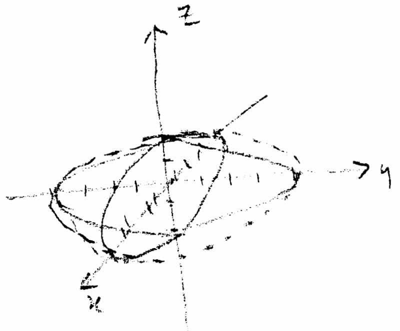
(b) $r^2 = x^2 + y^2$

(6) $r^2 + 4z^2 = 16$

① $\Rightarrow x^2 + y^2 + 4z^2 = 16$

② It is ellipsoid.

④



③

$x=0$	$y^2+4z^2=16$	Ellipse
$y=0$	$x^2+4z^2=16$	Ellipse
$z=0$	$x^2+y^2=16$	circle (Ellipse

Q.3 (a)

(8)

$r(t) = \left(\frac{t^2}{2} + 1\right)i + (t^2-2)j + (t^3+3)k$

$r'(t) = ti + 2tj + 3t^2k$

$r''(t) = i + 2j + 6tk$

$r' \cdot r'' = t + 4t + 18t^3 = 5t + 18t^3$

$r' \times r'' = \begin{vmatrix} i & j & k \\ t & +2t & 3t^2 \\ 1 & +2 & 6t \end{vmatrix} = (12t^2 - 6t^2)i - (6t^2 - 3t^2)j + (+2t - 2t)k$

$= 6t^2 i - 3t^2 j$

$\|r'(t)\| = \sqrt{t^2 + 4t^2 + 9t^4} = \sqrt{5t^2 + 9t^4}$

$\|r' \times r''\| = \sqrt{36t^4 + 9t^4} = \sqrt{45t^4} = 3\sqrt{5}\,t^2$

(i) $a_T = \frac{r' \cdot r''}{\|r'\|} = \frac{5t + 18t^3}{\sqrt{5t^2 + 9t^4}} = \frac{5 + 18t^2}{\sqrt{5 + 9t^2}}$

(ii) $a_N = \frac{\|r' \times r''\|}{\|r'\|} = \frac{3\sqrt{5}\,t^2}{\sqrt{5t^2 + 9t^4}}$

(iii) $K = \frac{\|r' \times r''\|}{\|r'\|^3} = \frac{3\sqrt{5}\,t^2}{[5t^2 + 9t^4]^{3/2}}$

(b)

(6)

$r(t) = \langle \cos t - t\sin t,\ \sin t + t\cos t,\ 3\rangle \qquad t = \frac{\pi}{2}$

$r'(\frac{\pi}{2}) = \langle -\frac{\pi}{2}, 1, 3\rangle$, Point $(0, \frac{\pi}{2}, \frac{3\pi}{2})$

$x = 0 - \frac{\pi}{2}t, \quad y = \frac{\pi}{2} + t, \quad z = \frac{3\pi}{2} + t, \quad t \in \mathbb{R}$

(c)

(6)

$f_x = 3x^2e^{2y} - 2\sin^2 y \sin 2x + \frac{3}{2\sqrt{x}}$

$f_{xx} = 6xe^{2y} - 4\sin^2 y \cos 2x - \frac{3}{4(x)^{3/2}}$

$f_{xy} = 6x^2e^{2y} - 4\sin y \cos y \cos 2x$

$f_y = 2x^3e^{2y} + 2\sin y \cos y \cos 2x + \frac{3}{y^2}$

$f_{yy} = 4x^3e^{2y} - 2\sin^2 y \cos 2x + 2\cos^2 y \cos 2x - \frac{6}{y^3}$

$f_{yx} = 6x^2e^{2y} - 4\sin y \cos y \sin 2x$