

Second Mid Term Exam., Summer, 1434
M 107 Full Marks: 25 Time: 90 Min.

Question 1. [Marks: 2+2+2=6]

(a) A particle moves along the curve

$$r(t) = (t^3 + 1)i + 2tj + t^2k,$$

where t is time. Find velocity \mathbf{v} and acceleration \mathbf{a} at $t = 1$. Also, find the component of velocity $Com_{\mathbf{b}}\mathbf{v}$, and component of acceleration $Com_{\mathbf{b}}\mathbf{a}$ in the direction of the vector $\mathbf{b} = i + j + 2k$. $\mathbf{t} = (1)$

(b) Find all values of c such that \mathbf{a} and \mathbf{b} are orthogonal, where $\mathbf{a} = \langle 4, 2, c \rangle$ and $\mathbf{b} = \langle 1, -2, 3c \rangle$.

Question 2. [Marks: 2+2+4=8]

(a) Determine whether the two lines l_1 and l_2 intersect, and if so, find the point of intersection, and also, find the angle between l_1 and l_2 :

$$l_1 : x = 1 + 2t, y = 1 - 4t, z = 5 - t$$

$$l_2 : x = 4 - v, y = -1 + 6v, z = 4 + v.$$

(b) Find the volume of the box having adjacent sides AB , AC and AD where $A(2, 1, -1)$, $B(3, 0, 2)$, $C(4, -2, 1)$ and $D(5, -3, 0)$.

Question 3. [Marks: 2+1=3]

Sketch the graph of the equation $16x^2 - 4y^2 - z^2 + 1 = 0$ in an xyz -coordinate system, and identify the surface.

Question 4. [Marks: 4]

Find the position vector $r(t)$ if the acceleration and initial conditions are given by

$$a(t) = i + 2tj + t^2k, \quad r(0) = 2k, \quad v(0) = i + j.$$

Question 5. [Marks: 2+2=4]

Find the normal component of acceleration and curvature of the curve $r(t) = \cos t i + \sin t j + k$, at $t = \frac{\pi}{2}$.