TIME: 90 min | KING SAUD UNIVERSITY | FULL MARKS:50

M - 107

DEPARTMENT OF MATHEMATICS

II MID TERM EXAM (SEM II) 1434-1435

Question: 1. (a) Determine the angle between vectors $\vec{a} = <2,-2,1>$ and $\vec{b} = <3,0,4>$.

[5+6+6] (b) Find the parametric equation of the line passing through point A and perpendicular to the plane containing points A(0, 1, 1), B(1,0,1) and C(1,1,0).

(c) Find the distance from point $D(-1,-1,-1)$ to the plane passing through points A(0, 1, 1), B(1,0,1) and C(1,1,0).

Question: 2. (a) Determine whether the vectors $\vec{a} = <1,4,-7>$, $\vec{b} = <2,-1,4>$

[5+5+5] and $\vec{c} = <0,-9,18>$ lie in the same plane.

(b) Identify the surface $x^2 + y^2 - 4z^2 = 16$. Find its traces on the coordinate planes and sketch the surface.

(c) Find domain of the vector valued function

$$r(t) = \ln(t)i + \left(\sqrt{1-t^2}\right)j + \left(\frac{1}{t-1}\right)k$$

Question: 3. (a) If the acceleration of a moving particle is given by $a(t) = 6ti + (2t+2)j + (\cos t)k$,

[6+8+4] find its velocity and position given that the initial velocity is $v(0) = i - k$, and the initial position is $r(0) = 2j + 3k$.

(b) If the position vector of an object is $r(t) = (\sin 2t)i + (\cos 2t + t)j + tk$, find the general formula for the tangential and normal components of acceleration and for the curvature of the curve C.

(c) Let C be the curve determined by the vector valued function

$$r(t) = \left(\tfrac{1}{2}t^2 + 1\right)i + \left(t^2 - 2\right)j + \left(t^3 + 3\right)k,$$

Find the parametric equations of tangent line to curve C at the point t = 1.

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Solution

Question: 1. (a) Determine the angle between vectors $\vec{a} = <2,-2,1>$ and $\vec{b} = <3,0,4>$.

[5+6+6] (b) Find the parametric equation of the line passing through point A and perpendicular to the plane containing points A(0, 1, 1), B(1,0,1) and C(1,1,0).

(c) Find the distance from point $D(-1,-1,-1)$ to the plane passing through points A(0, 1, 1), B(1,0,1) and C(1,1,0).

Solution (a) $a = \langle 2,-2,1\rangle$, $b = \langle 3,0,4\rangle$

$a \cdot b = 6+0+4 = 10$

$\|a\| = \sqrt{4+4+1} = \sqrt{9} = 3$ $\|b\| = \sqrt{9+0+16} = \sqrt{25} = 5$

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$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = \frac{10}{(3)(5)} = \frac{2}{3}$ $\Rightarrow$ $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

(b) To write parametric equations of Line, we need point and vector perpendicular to the plane

Point $A(0,1,1)$, $\overrightarrow{AB} = \langle 1,-1,0\rangle$, $\overrightarrow{AC} = \langle 1,0,-1\rangle$

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$$N = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \langle 1,1,1\rangle$$

Parametric Equation of line are

$x = 0+t$, $y = 1+t$, $z = 1+t$, $t \in \mathbb{R}$.

(c) Equation of plane

Normal vector $N = \overrightarrow{AB} \times \overrightarrow{AC} = \langle 1,1,1\rangle$

Point $A(0,1,1)$

$1(x-0) + 1(y-1) + 1(z-1) = 0$

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$x+y+z-2 = 0$

Distance from point $D(-1,-1,-1)$ to the plane $x+y+z-2=0$

is $h = \frac{|-1-1-1-2|}{\sqrt{1+1+1}} = \frac{5}{\sqrt{3}}$

Question: 2 . (a) Determine whether the vectors $\vec{a} = <1,4,-7>$, $\vec{b} = <2,-1,4>$ and $\vec{c} = <0,-9,18>$ **lie in the same plane.**

[5+5+5]

(b)Identify the surface $x^2 + y^2 - 4z^2 = 16$. Find its traces on the coordinate planes and sketch the surface.

(c) Find domain of the vector valued function

$$r(t) = \ln(t)i + \left(\sqrt{1-t^2}\right)j + \left(\frac{1}{t-1}\right)k$$

Solution

(a) If vectors lie in same plane, then $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} = 18 - 144 + 126 = 144 - 144 = 0$$

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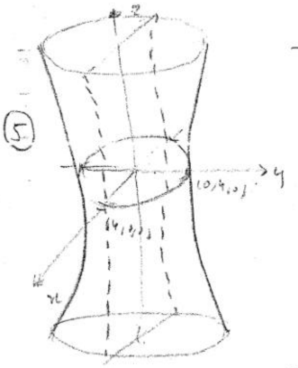
(b) $x^2 + y^2 - 4z^2 = 16$ is hyperboloid of one sheet



5

plane	Equation of Trace	Name
$x=0$	$y^2 - 4z^2 = 16$	hyperbola
$y=0$	$x^2 - 4z^2 = 16$	hyperbola
$z=0$	$x^2 + y^2 = 16$	Ellipse (circle)

(c) Domain of $r(t) = \ln(t)\,i + \sqrt{1-t^2}\,j + \dfrac{1}{t-1}\,k$

$f(t) = \ln(t)$ $\quad D_f = (0, \infty)$

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$g(t) = \sqrt{1-t^2}$ $\quad D_g = -1 \le t \le 1$

$h(t) = \dfrac{1}{t-1}$ $\quad D_h = (-\infty, \infty) - \{1\}$

$D_r = D_f \cap D_g \cap D_h = (0, 1)$

Question: 3. (a) If the acceleration of a moving particle is given by $a(t) = 6ti + (2t+2)j + (\cos t)k$,

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$$r(t) = \left(\tfrac{1}{2}t^2 + 1\right)i + \left(t^2 - 2\right)j + \left(t^3 + 3\right)k\ ,$$

Find the parametric equations of tangent line to curve C at the point t = 1.

Solution (a)

$$v(t) = \int (6ti + (2t+2)j + \cos t\, k)\,dt$$
$$= 3t^2 i + (t^2 + 2t)j + \sin t\, k + c_1 i + c_2 j + c_3 k$$

at $t=0$ $\quad v(0) = c_1 i + c_2 j + c_3 k = i - k \Rightarrow c_1 = 1,\ c_2 = 0,\ c_3 = -1$

$$v(t) = (3t^2 + 1)i + (t^2 + 2t)j + (\sin t - 1)k$$

$$r(t) = \int [(3t^2+1)i + (t^2+2t)j + (\sin t - 1)k]\,dt$$

(6)

$$= (t^3 + t)i + (\tfrac{t^3}{3} + t^2)j + (-\cos t - t)k + c_4 i + c_5 j + c_6 k$$

at $t=0$, $\quad r(0) = -k + c_4 i + c_5 j + c_6 k = 2j + 3k \Rightarrow c_4 = 0,\ c_5 = 2,$

$-1 + c_6 = 3,\ c_6 = 4$

$$r(t) = (t^3 + t)i + (\tfrac{t^3}{3} + t^2 + 2)j + (-\cos t - t + 4)k.$$

(b)

$r(t) = \sin 2t\, i + (\cos 2t + t)j + tk$

$r'(t) = 2\cos 2t\, i + (-2\sin 2t + 1)j + k$

$r''(t) = -4\sin 2t\, i + (-4\cos 2t)j$

$r' \cdot r'' = -4\cos 2t$

$r' \times r'' = \langle 4\cos 2t, -4\sin 2t, 4\sin t\rangle$

(8)

$\|r'(t)\| = \sqrt{6 - 4\sin 2t}$ $\qquad \|r' \times r''\| = 4\sqrt{1 + \sin^2 2t}$

$$a_T = \frac{-4\cos 2t}{\sqrt{6 - 4\sin 2t}}, \quad a_N = \frac{4\sqrt{1+\sin^2 2t}}{\sqrt{6 - 4\sin 2t}}, \quad \kappa = \frac{4\sqrt{1+\sin^2 2t}}{[6 - 4\sin 2t]^{3/2}}$$

(c)

$r' = \langle t, 2t, 3t^2\rangle$

$r'(1) = \langle 1, 2, 3\rangle$, $\qquad$ Point at $t = 1$ $\quad (\frac{3}{2}, -1, 4)$

(4)

Equation of tangent line to curve at $t = 1$ is

$x = \frac{3}{2} + t$

$y = -1 + 2t$

$z = 4 + 3t$, $\quad t \in \mathbb{R}$,