First Mid Term Exam., Summer, 1435
M 107 Full Marks: 25 Time: **90 Min.**

Question 1. [Mark: 5]
Solve the following system of linear equations by Gauss-Jordan elimination:

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 &= -1 \\ 2x_1 + x_2 - 2x_3 - 2x_4 &= -2 \\ -x_1 + 2x_2 - 4x_3 + x_4 &= 1 \\ 3x_1 \quad - 3x_4 &= -3. \end{cases}$$

Question 2. [Marks: 2+4+2=8]
(a) Consider the following system of linear equations:

$$\begin{cases} x_1 + 2x_2 + 3x_3 &= 3 \\ 2x_1 + 5x_2 + 3x_3 &= 5 \\ x_1 + \quad +8x_3 &= 12. \end{cases}$$

(1) Write the system in the form of $AX = B$, and then find A^{-1} by elementary row operation.
(2) Solve the system by using A^{-1}.
(b) If $f(x) = 2x^2 - 3x + 5$ and

$$A = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$$

then find $f(A)$.
Question 3. [Mark: 4]
Find the determinant of the following matrix by reducing the matrix to row-echelon form:

$$A = \begin{pmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{pmatrix}$$

Question 4. [Mark: 4]
For which value(s) of β the following matrix fails to be invertible?

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 5 & 1 & 6 \\ \beta & 5 & 2 \end{pmatrix}.$$

Question 5. [Mark: 4]
Solve the system of linear equations by using Cramer's Rule:

$$\begin{cases} x + \ +2z & = & 6 \\ -3x + 4y + 6z & = & 30 \\ -x - 2y + 3z & = & 8 \end{cases}$$