KING SAUD UNIVERSITY **FULL MARKS: 50**

M - 107 **DEPARTMENT OF MATHEMATICS** **TIME: 90min**

(SEMESTER II, 1434-1435) FIRST MID-TERM

Question: 1.(a) Solve the system of linear equations by using Gauss -Jordan method

$$\begin{aligned} x - y + 2z &= \cancel{4}\,5 \\ 3x + 2y + z &= 10 \\ 2x - 3y - 2z &= -10 \end{aligned}$$

[10]

(b) Use row operations to show that the value of det A= 0 if $a+b+c=0$

where $A = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}.$

[5]

Question: 2. (a) Find conditions on a and b, for which the following system ,

[10]

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + a\,z &= b \end{aligned}$$

have (i) no solution, (ii) a unique solution, and (iii) more then one solution?

(b) Let $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$ **and** $p(x) = x^2 - 2$

Find (i) $p(A)$, (ii) A^{-1} and (iii) A^8 **[10]**

Question: 3. Solve the system of linear equations

$$\begin{aligned} 3x - 3y + 4z &= -1 \\ 2x - 3y + 4z &= 4 \\ -y + z &= 3 \end{aligned}$$

a) Write the system in the form of AX = b
b) Find Adj (A),
c) Deduce A^{-1}
d) Use A^{-1} to find the solution of the given system **[15]**

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Question: 1.(a) **Solve the system of linear equations by using Gauss -Jordan method**

$$x - y + 2z = 5$$
$$3x + 2y + z = 10 \qquad [10]$$
$$2x - 3y - 2z = -10$$

Solution

(10)

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 3 & 2 & 1 & 10 \\ 2 & -3 & -2 & -10 \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & 5 & -5 & -5 \\ 0 & -1 & -6 & -20 \end{array}\right]$$

$$\equiv \left[\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & -6 & -20 \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\equiv \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \Longrightarrow \begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}$$

(b)

(5)

$$\det A = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c & a & b \\ b & c & a \end{vmatrix} = 0 \quad \text{because} \quad a+b+c=0$$

Qestion. 2 (a)

(10)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{array}\right] \begin{array}{l} \\ -R_1+R_2 \\ -R_1+R_3 \end{array}$$

$$\equiv \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{array}\right] -R_2+R_3$$

(i) No solution, if $a = 3$

(ii) unique solution $a \neq 3$,

(iii) Many solution $a \neq 3$, $b \neq 10$

(b) Let $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$ and $p(x) = x^2 - 2$

Find (*i*) $p(A)$, (ii) A^{-1} and (iii) A^8 **[10]**

(i) $P(A) = A^2 - 2I$

[4] $A^2 = A \cdot A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$P(A) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

[4] (ii) $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$ $A^{-1} = \dfrac{1}{-1-1}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

[2] (iii) $A^8 = (A^2)^4 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^4 = \begin{bmatrix} 2^4 & 0 \\ 0 & 2^4 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

<u>Question</u> 3

[2] (a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$ $AX = b$ [15]

(b) $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$

[8] $\text{Adj } A = C^T = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

(c) $\det A = 3 + 6 - 8 = 1 \neq 0$

[3] $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

(d) $X = A^{-1} b = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 5 \end{bmatrix}$

[2]

$x = -5, \quad y = 2, \quad z = 5$