M - 107

KING SAUD UNIVERSITY **FULL MARKS: 50**

DEPARTMENT OF MATHEMATICS **TIME: 90min**

(SEMESTER I, 1434-1435) FIRST MID-TERM

Question: 1.(a) By applying Kirchhoff's law to circuit we obtain the following system of linear equations

$$\begin{aligned} i_1 + 2i_2 - 7i_3 &= -4 \\ 2i_1 + i_2 + i_3 &= 13 \\ 3i_1 + 9i_2 - 36i_3 &= -33 \end{aligned}$$

[15]

use Gauss - Jorden method to find i_1, i_2 *and* i_3 .

Question: 2. (a) Find condition on a, b, and c for which the following system is consistent,

[12]

$$\begin{aligned} x - 2y + 3z &= a \\ 3x - y + 5z &= b \\ 2x + 6y - 2z &= c \end{aligned}$$

(b) For what values of λ**, A is not invertible**

$$A = \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda - 1 & 0 \\ -1 & 0 & \lambda - 1 \end{bmatrix}$$

[8]

Question: 3. Solve the system of linear equations

$$\begin{aligned} x + 2y - z &= 2 \\ 2x + 3y + 2z &= 5 \\ 3x + 2y + 3z &= 10 \end{aligned}$$

a) **Write the system in the form AX = B**
b) **Find Adj (A),**
c) **Find A^{-1} by using Adj (A),**
d) **Use A^{-1} to find the solution of the given system** **[15]**

SOLUTION OF FIRST MID-TERM

(SEMESTER I, 1434-1435) FIRST MID-TERM

FULL MARKS: 50

Question: 1.(a) By applying Kirchhoff's law to circuit we obtain the following system of linear equations

$$\begin{aligned} i_1 + 2i_2 - 7i_3 &= -4 \\ 2i_1 + i_2 + i_3 &= 13 \\ 3i_1 + 9i_2 - 36i_3 &= -33 \end{aligned}$$

[15]

use Gauss - Jorden method to find i_1, i_2 *and* i_3 .

Solution : Augmented Matrix is

$$[A \mid b] = \left[\begin{array}{ccc|c} 1 & 2 & -7 & -4 \\ 2 & 1 & 1 & 13 \\ 3 & 9 & -36 & -33 \end{array}\right]$$

$$\equiv \left[\begin{array}{ccc|c} 1 & 2 & -7 & -4 \\ 0 & -3 & 15 & 21 \\ 0 & 3 & -15 & -21 \end{array}\right] \begin{array}{l} \\ -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\equiv \left[\begin{array}{ccc|c} 1 & 2 & -7 & -4 \\ 0 & 1 & -5 & -7 \\ 0 & 3 & -15 & -21 \end{array}\right] \begin{array}{l} \\ R_2/{-3} \\ \\ \end{array}$$

$$\equiv \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} -2R_2 + R_1 \\ \\ -3R_2 + R_3 \end{array}$$

$i_1 + 3i_3 = 10$

$i_2 - 5i_3 = -7$

Let $i_3 = t$ then $i_1 = 10 - 3t_3 = 10 - 3t$

$i_2 = -7 + 5t_3 = -7 + 5t$

There are infinite number of solutions, depending on value of i_3

$i_1 = 10 - 3t$

$i_2 = -7 + 5t$

$i_3 = t$, $t \in \mathbb{R}$

Question: 2. (a) Find condition on a, b, and c for which the following system is consistent, **[12]**

$$x-2y+3z=a$$
$$3x-y+5z=b$$
$$2x+6y-2z=c$$

Solution. Augmented matrix Form.

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & a \\ 3 & -1 & 5 & b \\ 2 & 6 & -2 & c \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & -2 & 3 & a \\ 0 & 5 & -4 & b-3a \\ 0 & 10 & -8 & c-2a \end{array}\right] \begin{array}{l} \\ -3R_1+R_2 \\ -2R_1+R_3 \end{array}$$

$$\equiv \left[\begin{array}{ccc|c} 1 & -2 & 3 & a \\ 0 & 5 & -4 & b-3a \\ 0 & 0 & 0 & c-2b+4a \end{array}\right] \begin{array}{l} \\ \\ -2R_2+R_3 \end{array}$$

The system will be consistent, iff $c-2b+4a=0$
or
$c=2b-4a$.

(b) For what values of λ, A is not invertible

$$A=\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda-1 & 0 \\ -1 & 0 & \lambda-1 \end{bmatrix}$$ **[8]**

Solution A is not invertible iff $\det A = 0$

$$\det A = \lambda \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} + 1 \begin{vmatrix} 0 & 0 \\ -1 & \lambda-1 \end{vmatrix} + 0$$

$$= \lambda(\lambda-1)(\lambda-1)$$

$\det A = 0 \Rightarrow \lambda(\lambda-1)(\lambda-1)=0 \Rightarrow \lambda=0, \lambda=1$

Question: 3. Solve the system of linear equations

$$x + 2y - z = 2$$
$$2x + 3y + 2z = 5$$
$$3x + 2y + 3z = 10$$

a) Write the system in the form AX = B
b) Find Adj (A),
c) Find A^{-1} by using Adj (A),
d) Use A^{-1} to find the solution of the given system [15]

Solution:

(a) $$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 2 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 10 \end{bmatrix}$$

(b) Cofactors

$c_{11} = 5$, $c_{12} = 0$, $c_{13} = -5$

$c_{21} = -8$, $c_{22} = 6$, $c_{23} = 4$

$c_{31} = 7$, $c_{32} = -4$, $c_{33} = -1$

Matrix of cofactors $$C = \begin{bmatrix} 5 & 0 & -5 \\ -8 & 6 & 4 \\ 7 & -4 & -1 \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} 5 & -8 & 7 \\ 0 & 6 & -4 \\ -5 & 4 & -1 \end{bmatrix}$$

(c) $\det A = a_{11} c_{11} + a_{12} c_{12} + a_{13} c_{13} = 5 + 0 + 5 = 10$

$$A^{-1} = \frac{1}{\det A} \text{Adj } A = \frac{1}{10} \begin{bmatrix} 5 & -8 & 7 \\ 0 & 6 & -4 \\ -5 & 4 & -1 \end{bmatrix}$$

(d) $$X = \frac{1}{10} \begin{bmatrix} 5 & -8 & 7 \\ 0 & 6 & -4 \\ -5 & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 10 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 40 \\ -10 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$$

Solution is $x = 4$, $y = -1$, $z = 0$