M - 107 SOLUTION *(SEMESTER I, 1432-1433) FIRST MID-TERM*

Question: 1 (a) Let

$$A=\begin{bmatrix} x+3y+z & 2x+3y+4z \\ 4x+3y+5z & 8 \end{bmatrix} \text{ and } B=\begin{bmatrix} -y-z & 8-x-y-2z \\ -x-y-z & 3x+4y+6z \end{bmatrix}$$

Use Gauss Jordan method to find x, y *and* z **such that A and B are equal.** **[8]**

[8]

Solution: **As A = B ,**

we can write the system of equations by using equality of matrices

$x+3y+z=-y-z$

$2x+3y+4z=8-x-y-2z$

$4x+3y+5z=-x-y-z$

$8=3x+4y+6z$

rearranging both the sides

$x+4y+2z=0$

$3x+4y+6z=8$

$5x+4y+6z=0$

$3x+4y+6z=8$

Equation 2 and equation 4 are same so the sytem is reduced to three equations

$x+4y+2z=0$

$3x+4y+6z=8$

$5x+4y+6z=0$

Solving system by Gauss Jorden method, rewritting the system in Augmented form and reducing the augmented matrix to reduced row echlon form

$$\begin{bmatrix} 1 & 4 & 2 & 0 \\ 3 & 4 & 6 & 8 \\ 5 & 4 & 6 & 0 \end{bmatrix} \xrightarrow{\text{performing row operations}} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Solution of the system is $x=-4, y=-1$ and $z=4$.

1 (b) Find the values of λ for which the system of linear equations

$$2x+y=5$$
$$x-3y=-1 \qquad \text{will have solutions and find the solutions.} \qquad [6]$$
$$3x+4y=\lambda$$

Solution:

$$2x+y=5$$
$$x-3y=-1$$
$$3x+4y=\lambda$$

Augmented form of the system is

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & -3 & -1 \\ 3 & 4 & \lambda \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & -1 \\ 2 & 1 & 5 \\ 3 & 4 & \lambda \end{bmatrix} \xrightarrow{R_2-2R_1, R_3-3R_1} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 7 & 7 \\ 0 & 13 & \lambda+3 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{7}R_2, R_3-13R_2, R_1+3R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & \lambda-10 \end{bmatrix}$$

The system will have solution if and only if $\lambda - 10 = 0$ or $\lambda = 10$

Sytem is reduced to the form

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \text{ hence the solutionis } x=2 \textit{ and } y=1$$

Question: 2 (a)

If $A=\begin{bmatrix} 1 & 1 & 2 \\ 4 & 2 & 0 \\ 11 & 3 & 2 \end{bmatrix}$**, find the inverse of A by using Elementary matrix method. [8]**

Solution:

$$[A|I] \approx$$

$$\left[\begin{array}{cccccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 4 & 2 & 0 & 0 & 1 & 0 \\ 11 & 3 & 2 & 0 & 0 & 1 \end{array}\right] \approx \ldots\ldots \approx \left[\begin{array}{cccccc} 1 & 0 & 0 & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{6} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{5}{12} & -\frac{1}{3} & \frac{1}{12} \end{array}\right] \approx [I : A^{-1}]$$

$$A^{-1} = \frac{1}{12}\begin{bmatrix} -2 & -2 & 2 \\ 4 & 10 & -4 \\ 5 & -4 & 1 \end{bmatrix}$$

2(b) Let $A=\begin{bmatrix} a^2 & 1 \\ 4a & 16 \end{bmatrix}$ **, find all values of a for which A is not invertible.**

[4]

Solution:

If matrix A is not invertible then detA=0

$\det A = 16a^2 - 4a = 0$

$4a(4a-1) = 0 \Rightarrow \quad a = 0 \;\; or \;\; a = \dfrac{1}{4}$

Question: 3 (a) Evaluate the determinant by using elementary row operations

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \end{vmatrix}$$

[6]

Solution:

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \end{vmatrix}$$

using row opertions,it is reduced to upper triagular form

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{vmatrix} = 5$$

$\lambda 7$

3(b) **Solve the system of linear equations by Cramer's Rule**

$$\begin{aligned} 4x+5y \quad &= 2 \\ 11x+y+2z &= 3 \\ x+5y+2z &= 1 \end{aligned}$$

[8]

Solution:

$$\begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow AX = B$$

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, \det(A) = -132,$$

$$A_1 = \begin{bmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, \det(A_1) = -36$$

$$A_2 = \begin{bmatrix} 4 & 2 & 0 \\ 11 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}, \det(A_2) = -24,$$

$$A_3 = \begin{bmatrix} 4 & 5 & 2 \\ 11 & 1 & 3 \\ 1 & 5 & 1 \end{bmatrix}, \det(A_3) = 12$$

By using Crammer's Rule

$$x = \frac{\det(A_1)}{\det(A)} = \frac{3}{11}, \quad y = \frac{\det(A_2)}{\det(A)} = \frac{2}{11}, \quad z = \frac{\det(A_3)}{\det(A)} = -\frac{1}{11}$$