FINAL EXAMINATION, SUMMER SEM. 1434-1435

Department of Mathematics

King Saud University

MATH: 107 Time: 3 Hours Full Marks: 40

Question # 1. Marks: 4+4=8

(a) Solve the following system of linear equations by Gauss-Jordan elimination:

$$\begin{cases} -x_1 - x_2 + 2x_3 - 3x_4 + x_5 & = 0 \\ 2x_1 + 2x_2 - x_3 \quad + x_5 & = 0 \\ x_1 + x_2 - 2x_3 \quad - x_5 & = 0 \\ \qquad x_3 + x_4 + x_5 & = 0. \end{cases}$$

(b) Given the matrices

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix},$$

$B = (1\ -2\quad 3)$ and $X = (x\quad y\quad z)$, solve the equation: $AX^T = 3X^T + 2B^T$ for x, y, z.

Question # 2. Marks: 4+2+3=9

(a) Consider the following system of linear equations:

$$\begin{cases} 3x + 2y - z & = & 64 \\ x + 6y + 3z & = & 128 \\ 2x - 4y + 0z & = & 192. \end{cases}$$

(i) Write the system in the form of $AX = B$, and then find A^{-1} by **adjoint** formula.

(ii) Solve the system by using A^{-1} that you found by **adjoint** in (i).

(b) Find the parametric equations of the line of intersection of the planes: $x - 2y + 4z = 2$ and $x + y - 2z = 5$.

(c) Find the distance from the point $A(3, 1, -1)$ to the line: $x = 1 + 4t$, $y = 3 - t$, $z = 4t$.

Or, (d) Find an equation of the plane that contains the point $P(5, 0, 2)$ and the line: $x = 3t + 1$, $y = -2t + 4$, $z = t - 3$.

Question # 3. Marks: 4+2+3=9

(a) A particle moves along the curve $r(t) = (t^3 + 1)i + 2tj + t^2k$, where t is time. Find velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ at $t = 1$. Also, find the component

of velocity $Com_{\mathbf{b}}\mathbf{v}$, and component of acceleration $Com_{\mathbf{b}}\mathbf{a}$ in the direction of the vector $b = i + j + 2k$.
(b) The position vector of a moving particle at time t is given by $\mathbf{r}(t) = 4\cos t\mathbf{i} + 9\sin t\mathbf{j} + 2t\mathbf{k}$. Find the tangential component of acceleration.

(c) Show that the function $z(x,t) = (\sin n\pi x)(\cos n\pi at)$ satisfies the wave equation $\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

Or, (d) Find $\frac{dw}{dt}$, if $w = x^2y^3z^4$, $x = 2t + 1$, $y = 3t - 2$ $z = 5t + 4$.

Question # 4. Marks: 3+4+4+3=14

(a) Suppose that the height of a right circular cone decreases from 11 to 10.98 inches, while the radius increases from 6 to 6.02 inches. use differentials to approximate the change in volume of the cone.

(b) The electric potential V at (x, y, z) is given by $V(x, y, z) = x^4yz - xy^3 + z$. Find the rate of change of V at $P(1, 1, -3)$ in the direction from P to origin. Also, find in what direction does V increase most rapidly at P.

(c) Identify the surface, and find the equations of the tangent plane, and normal line at the point $P(-2, 1, -3)$ to the surface $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.

(d) Find the local extrema and saddle points, if any, of the function $f(x, y) = 2x^2 + 2xy + y^2 + 2x - 5$.

Or, (e) Use Lagrange multipliers to find the extrema of $f(x, y, z) = x^2 + y^2 + z^2$ subject to constraint $x + 3y - 2z = 7$.