Final Examination, Summer, 1434

M 107 Full Marks: 40 Time: **3 Hours**

Question 1. [**Mark: 5**]
Let a and b be two real numbers.
(1) Find the determinant of the matrix A given by

$$A = \begin{pmatrix} a & 0 & 3 \\ 0 & 3 & -2 \\ 1 & 0 & b \end{pmatrix}.$$

(2) Find a relation between a and b so that the system of linear equations

$$\begin{cases} ax + 3z = 2 \\ 3y - 2z = 1 \\ x + bz = 2 \end{cases}$$

has a unique solution. Solve this system in this case, using Cramer's rule.

Question 2. [**Mark: 5**]
(1) Find A^{-1} by the method of cofactors, where A is the matrix given by

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{pmatrix}.$$

(2) Solve the system of linear equations

$$\begin{cases} 2x + 3z = 1 \\ 3y + 2z = 0 \\ -2x - 4z = 0 \end{cases}$$

Question 3. [**Mark: 5**]
(1) Find an equation of the line passing through the points $A(-3, 1, -1)$ and $B(7, 11, -8)$.
(2) Find the distance from the point $C(1, -1, 2)$ to the plane with equation $3x - 7y + z - 5 = 0$.

Question 4. [**Mark: 8**]
(1) Let C be the curve with parametric equations:

$$x = t, \; y = t^2, \; z = t^3, \; t \geq 0.$$

Find the parametric equations for the tangent line to C at the point corresponding to $t = 1$.
(2) Find the curvature, radius of curvature and center of curvature of the curve $y = x^4$ at $P(1, 1)$.

Question 5. [**Mark: 8**]
(1) Show that the $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2 + y^2}$ does not exist.
(2) The radius and altitude of a right circular cylinder are measured as 5 inches and 9 inches, respectively, with possible error in measurement of ± 0.03 inches. Use differentials to approximate the maximum error in the calculated volume of the cylinder.

Question 6. [**Mark: 9**]
(1) Find the directional derivative of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $P(1, 0, 2)$ in the direction of the vector $\mathbf{a} = \langle -1, 1, 1 \rangle$.
(2) Identify the surface and find the equations of the tangent plane and normal line at the point $P(-2, 1, -3)$ to the surface $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.
(3) Find the local extrema and saddle points, if any, of the function

$$f(x, y) = 3x^2 - 12xy + 4y^3 - 48$$