جامعة الملك سعود
King Saud University

TIME: 3 Hours | **DEPARTMENT OF MATHEMATICS** | **FULL MARKS: 80**

M - 107 | **Final Examination (Second Semester 1436-1437)**

Question 1: (a) What conditions must a, b and c satisfy in order for the system

[6+6+6]

$$\begin{aligned} x + z &= a \\ y - z &= b \\ 2x + y + z &= c \end{aligned}$$

to be consistent.

(b) Suppose the points P(3, 0), Q(1, 2) and R(5, 2) lie on the circle

$$x^2 + y^2 + ax + by + c = 0$$

i. **Write the system of linear equations in a, b and c.**
ii. **Solve the system by Gauss - Jordon method.**
iii. **Write the equation of circle.**

(c) Solve the system of equations

$$\begin{aligned} 2x - 2y + 3z &= 3 \\ x - y + z &= 1 \\ x + y - z &= 1 \end{aligned}$$

by finding A^{-1}, using method of cofactors, where A is the coefficient matrix of the system.

Question 2: (a) Let A(1, 0, 2), B(1, 2, 2), C(1, 1, -1) and D(x, 3, -2) be points in space.

[6+6+6] **Find x, if the volume of the parallelepiped having adjacent sides AB, AC, and AD is 5 unit^3.**

(b) Show that the line L: $x = 2-3t$, $y = 1+t$, **and** $z = 1-t$, $t \in R$ **and the plane P:** $x + 2y - z + 1 = 0$ **are parallel. Hence find the distance between the line and the plane.**

(c) Let the curve C is given by $y = 4 - x^2$ **with -2< x < 2. Sketch the curve C and find radius of curvature and center of curvature at the point P (0, 4)**

Question 3: (a) Show that $\lim_{(x,y)\to(0,0)} \left(\frac{x^2 y}{x^4 + y^2} \right)$ **does not exist.**

[6+6] **(b) The temperature distribution T at any point P(x, y, z) in xyz coordinate system is given by** $T(x,y,z) = 8(2x^2 + 4y^2 + 9z^2)^{\frac{1}{2}}$, **use differentials to approximate the temperature difference between the points (6, 3, 2) to (6.1, 3.3, 1.98).**

Question 4: (a) Show that $v(x,t) = (x - at)^4 + \cos(x + at)$ **satisfies the wave equation**

[6+6+8]

$$\frac{\partial^2 v}{\partial t^2} = a^2 \frac{\partial^2 v}{\partial x^2}, \quad a \in R .$$

(b) Find equations of the tangent plane and the normal line to the surface $xy^2 + 3x - z^2 = 4$ **at the point** $(2,1,-2)$.

(c) Let $f(x,y) = x^2 - 4xy$.

(i) Find the gradient of $f(x,y)$ **at the point** $P(1,2)$.

(ii) Use gradient to find the directional derivative of $f(x,y)$ **at** $P(1,2)$ **in the direction from** $P(1,2)$ **to** $Q(2,5)$.

Question 5: (a) Find local extrema and saddle points of $f(x,y) = 2x^3 + 6xy^2 - 3y^3 - 150x$

[6+6] **(b) If** $f(x,y,z) = 2x^2 + y^2 + 3z^2$, **use Lagrange multipliers to find maximum and minimum values of** $f(x,y,z)$ **subject to condition** $2x - 3y - 4z = 49$.