TIME: 3 Hours | DEPARTMENT OF MATHEMATICS | FULL MARKS: 80

M - 107 | Final Examination (Second Semester 1435-1436)

Question: 1: (a) For what values of a the following system of equations

[6+4+6]

$$x + z = 4$$
$$2x + y + 3z = 5$$
$$-3x - 3y + (a^2 - 5a)z = a - 8$$

has (a) unique solution, (b) No solution.

(b) Use properties of determinant to find the det (A+ B) if A and B are matrices satisfying A(A +B) = B and det (A) =2 , det (B) = 6

(c) Use Elementary matrix method to find all values of b for which A^{-1} exists, and find A^{-1}

where $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & b \end{bmatrix}$.

Question:2 (a) Given three points $P(2,-1,1)$, $Q(-3,2,0)$ and $R(4,-5,3)$, find the following

[6+8]

i. a unit normal vector to the plane determined by P, Q and R.

ii. the distance of R to the line through P and Q.

iii. an equation of the plane through points P,Q and R.

(b) The position vector of a moving particle at time t is given by $r(t) = (t^3 + 2)i + (3t^2 + 1)j + (6t - 1)k$. Find the tangential and the normal components of acceleration and the curvature at any time t.

Question: 3. (a) For the surface $y^2 + 4z^2 = x$

[6+6+6]

(i) Write the name of the surface,

(ii) Write the names and the equations of the traces of the surface on the co-ordinate planes, and Sketch the surface.

(b) Show that $\lim_{(x,y)\to(0,0)} \dfrac{4x^3 y}{2x^4 + 3y^4}$ does not exist.

(c) The dimensions of a closed rectangular box are measured as 4ft, 5ft and 6ft with possible error of $\pm\dfrac{1}{48}$ ft. Use differential to approximate the maximum error in calculated value of the volume of the box.

Question: 4. (a) Show that $w = (x - at)^4 + \cos(x + at)$ satisfies the wave equation $\dfrac{\partial^2 w}{\partial t^2} = a^2 \dfrac{\partial^2 w}{\partial x^2}$.

[6+6+8]

(b) Find equations of the tangent plane and the normal line to the hyperboloid $16x^2 - 9y^2 + 36z^2 = 144$ at the point $(3,-4,2)$.

(c) Find the directional derivative of $f(x,y,z) = xy\sin z$ at the point $P(4,9,\frac{\pi}{4})$ in the direction of the line $x = -3 + 2t,\ y = 1 + 3t,\ z = 5 - 2t$,

In which direction at P the function f increases most rapidly ?

Question: 5. (a) If $f(x,y) = x^3 + y^3 - 3xy + 5$, find local extrema and saddle points of $f(x,y)$.

[6+6]

(b) If $f(x,y) = x^2 - 2y$, use Lagrange multipliers to find maximum and minimum values of $f(x,y)$ subject to condition $x^2 + y^2 = 9$.