TIME: 3 Hours **DEPARTMENT OF MATHEMATICS** **FULL MARKS: 80**

M - 107 **Final Examination (First Semester 1436-1437)**

Question:1: (a) Find all solutions of the homogeneous system of equations by using the Gauss- Jordan method.

[6+6+6]

$$\begin{aligned} w \quad - 3y - z &= 0 \\ x + 2y + z &= 0 \\ 2w + 3x + y + z &= 0 \\ -2w + x - 2y + 3z &= 0 \end{aligned}$$

(b) Show that the matrix A is invertible:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(c) Solve the system of equations:

$$\begin{aligned} x + y \quad &= 1 \\ 2x - y + z &= 2 \\ -x + y - z &= -1 \end{aligned}$$

finding A^{-1} by method of cofactors, where A is the coefficient matrix of the system.

Question:2(a) Given three points $P(2,1,3)$, $R(3,2,1)$ and $S(1,2,3)$, find the following

[6+6+6]

(i) The distance from P to the line through R and S.

(ii) An equation of the plane through points P, R and S.

(b) Find the domain of the vector valued function

$$r(t) = \sqrt{t}\, i + \frac{1}{t^2 - 1} j + \ln(t) k.$$

(c) The position vector of a moving particle at time t is given by

$r(t) = 2e^{t} i + 3e^{-t} j + 2\sqrt{3}\, t\, k$. **Find at point (1, 1, 0) the tangential and the normal components of acceleration and the curvature.**

Question:3. (a) Show that $f(x,y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & \text{if } (x,y) \neq (0,0); \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

[6+6]

is not continuous at (0,0)

(b) If $z = x - \ln(xy)$, use differentials to approximate Δz when (x,y) moves from (3.00, 0.33) to (2.97, 0.32).

Question:4. (a) Show that $f(x,y) = \sin(2\pi x)\cos(2\pi y)$ satisfies the equation $\dfrac{\partial^2 f}{\partial y^2} = \dfrac{\partial^2 f}{\partial x^2}$.

[6+6+8]

(b) Find equations of the tangent plane and the normal line to the surface $xyz = 12$ at the point $(2,-2,-3)$.

(c) Find the directional derivative of $f(x,y,z) = x^2yz + 4xz^2$ at the point $P(1,-2,-1)$ in the direction of the vector $< 2,-1,-2 >$.

In which direction at P the function f increases most rapidly ?

Question:5. (a) Find local extrema and saddle points of $f(x,y) = x^2 + y^3 - 4xy + 4y$.

[6+6]

(b) If $f(x,y) = 4x + 4y$,use Lagrange multipliers to find maximum and minimum values of $f(x,y)$ subject to condition $x^2 + 4y^2 = 5$.