INTEGRAL CALCULUS (MATH 106)

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Antiderivative and indefinite integral

Definition 2.1

A function G is called an antiderivative of the function f on the interval I if $G'(x) = f(x)$ for all $x \in I$.

Example 2.1

What is the antiderivative of the function $f(x) = 2x$?
- The antiderivative is $G(x) = x^2 + c$, where c is a constant.

Definition 2.2

If $G(x)$ is the antiderivative of $f(x)$ then, $\int f(x)dx = G(x) + C$, $\int f(x)dx$ is called the indefinite integral of function $f(x)$.

Basic Rules of integration

1. $\int 1 dx = x + C$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1, n \in \mathbb{Q}$
3. $\int \cos x \ dx = \sin x + C$
4. $\int \sin x \ dx = -\cos x + C$
5. $\int \sec^2 x \ dx = \tan x + C$
6. $\int \csc^2 x \ dx = -\cot x + C$
7. $\int \sec x \tan x \ dx = \sec x + C$
8. $\int \csc x \cot x \ dx = -\csc x + C$

Basic Rules of integration

1. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C, (n \in \mathbb{Q}, n \neq -1)$
2. $\int \sin(f(x))f'(x)dx = -\cos(f(x)) + C$
3. $\int \cos(f(x))f'(x)dx = \sin(f(x)) + C$
4. $\int \sec^2(f(x))f'(x)dx = \tan(f(x)) + C$
5. $\int \csc^2(f(x))f'(x)dx = -\cot(f(x)) + C$
6. $\int \sec(f(x))\tan(f(x))f'(x)dx = \sec(f(x)) + C$
7. $\int csc(f(x))\cot(f(x))f'(x)dx = -\csc(f(x)) + C$

Example 2.2

$$\int cos(3x+4)dx = \frac{1}{3}\int cos(3x+4)\ 3dx = \frac{1}{3}sin(3x+4) + C$$

$$\int \tan^2 x \sec^2 x\ dx = \int (\tan x)^2 sec^2 x\ dx = \frac{(\tan x)^3}{3} + C$$

CHANGE OF VARIABLE

Example 2.3

Solve $\int (4x+1)^2 \ dx$

Put $u = 4x + 1$ *then* $du = 4dx$ *hence* $\frac{1}{4}du = dx$

$$\int (4x+1)^2 \ dx = \int u^2 \frac{1}{4}du = \frac{1}{4}\int u^2 du = \frac{1}{4}\frac{u^3}{3} + C = \frac{1}{4}\frac{(4x+1)^3}{3} + C$$

Properties of indefinite integral

1. $\int af(x)dx = a\int f(x)dx$ where $a \in \mathbb{R}$
2. $\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$

Remark 2.1

1. $\int f(x)dx = G(x) + C$, $\int \frac{d}{dx} G(x)dx = G(x) + C$
2. $\frac{d}{dx} \int f(x)dx = f(x)$

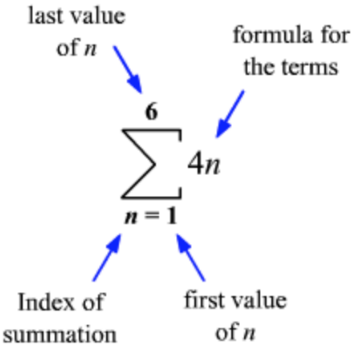
Sums and sigma notation

- A series can be represented in a compact form, called summation or sigma notation.
- The Greek capital letter, $\sum$, is used to represent the sum.
- The series $4 + 8 + 12 + 16 + 20 + 24$ can be expressed as

$$\sum_{n=1}^{6} 4n$$

- The expression is read as the sum of 4n as n goes from 1 to 6 .
- The variable n is called the index of summation.

Sums and sigma notation



So if $a_1, a_2, \dots, a_n \in \mathbb{R}$, then $\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$

Sums and sigma notation

Theorem 3.1

If $c, a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n \in \mathbb{R}$, *Then,*

1. $\sum_{i=1}^{n} C = Cn$
2. $\sum_{i=1}^{n} Ca_i = C \sum_{i=1}^{n} a_i$
3. $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$
4. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
5. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
6. $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Sums and sigma notation (Examples)

Example 3.1

Use the properties in the theorem 3.1 to find the value of:

$$\sum_{i=1}^{4}(k^3 - k + 2)$$

Solution 1

$$\sum_{i=1}^{4}(k^3 - k + 2) = \sum_{i=1}^{4} k^3 - \sum_{i=1}^{4} k + \sum_{i=1}^{4} 2 = \sum_{i=1}^{4} k^3 - \sum_{i=1}^{4} k + 2\sum_{i=1}^{4}$$
$$= \left(\frac{4(4+1)}{2}\right)^2 - \frac{4(4+1)}{2} + (2 \times 4) = 98$$

Sums and sigma notation (Exercises)

Exercise 1

Using the formulas and properties from above determine the value of the following summations.

1. $\sum_{i=1}^{100} (3-2i)^2$ 1293700
2. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2$ $\frac{1}{3}$
3. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{5k}{n^2}$ $\frac{5}{2}$

Riemann Sum

In this section we assume that the function $f(x) \geq 0$ on the interval $[a, b]$.

Definition 3.1

The set $\{a = x_0, x_1, \ldots, x_n = b\}$ *is called a* **regular partition** *of the interval* $[a, b]$ *if* $x_i = x_0 + i\triangle x$ *for every* $i = 1, 2, \ldots, n$, *and* $\triangle x = \frac{b-a}{n}$.
This regular partition divides the interval $[a, b]$ *into* n *subintervals of the form* $[x_{i-1}, x_i]$ *where* $i = 1, 2, \ldots, n$.

- **Area under the graph of a function :**

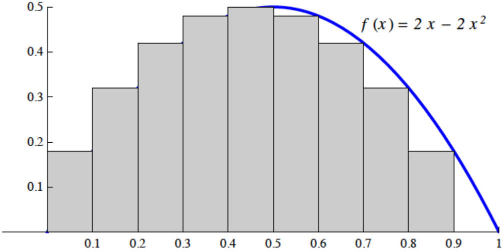
If $f(x) \geq 0$ on the interval $[a, b]$ and $\{x_0 = a, x_1, \ldots, x_n = b\}$ is a regular partition of $[a, b]$, then the area under the graph of $f(x)$ can be approximated by n rectangles using the formula:

$$A_n = \sum^{n} f(x_i)\triangle x$$

Riemann Sum

Example 3.2

Approximate the area under the graph of $f(x) = 2x - 2x^2$ *on the interval* $[0, 1]$ *using 10 rectangles .*



Riemann Sum

solution

1. $\triangle x = \frac{1-0}{10} = 0.1$
2. $x_0 = 0, x_1 = 0.1, x_2 = 0.2, \ldots, x_9 = 0.9, x_{10} = 1$
3. $A_{10} = \sum_{i=1}^{10} f(x_i)\triangle x = \sum_{i=1}^{10}(2x_i - 2x_i^2)0.1$
4. $A_{10} = 0.1[0.18+0.32+0.42+0.48+0.5+0.48+0.42+0.32+0.18+0]$
5. $A_{10} = 0.1(3.3) = 0.33$

Riemann Sum

Let $\{x_0 = a, x_1, \ldots, x_n = b\}$ be a **regular partition** of the interval $[a, b]$ with $\triangle x = \frac{b-a}{n}$. Pick points $c_1, c_2, \ldots, c_n$ where c_i is any point in the subintrval $[x - i - 1, x_i], i = 1, 2, \ldots, n$.

The Riemann sum is:

$$R_n = \sum_{i=1}^{n} f(c_i)\triangle x$$

The area under the curve of $f(x)$ is the limit of the Riemann sum.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\triangle x$$

Example 3.3

Find the area under the curve of the function $f(x) = 3x + 1$ on the interval $[1, 3]$ using Riemann sum and c_i is the middle *point of the subinterval.*

Solution

1. $\triangle x = \frac{b-a}{n} = \frac{2}{n}$
2. $x_0 = 1, x_i = x_0 + i\triangle x = 1 + \frac{2i}{n}$ for every $i = 1, 2, \ldots, n$
3. For every $i = 1, 2, \ldots, n, c_i \in [x_{i-1}, x_i], c_i = \frac{x_i + x_{i-1}}{2} = 1 + \frac{2i-1}{n}$.
4. $R_n = \sum_{i=1}^{n} f(c_i)\triangle x = \sum_{i=1}^{n} [3(1 + \frac{2i-1}{n}) + 1]\frac{2}{n} = 8 + 6\frac{n(n+1)}{n^2} - \frac{6}{n}$.
5. The desired area A is: $A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} 8 + 6\frac{n(n+1)}{n^2} - \frac{6}{n} = 14$

Riemann Sum

Exercise 2

Do the last example where ci is the end point of the subinterval.

The definite Integral

Definition 3.2

For any continuous function f defined on the interval $[a, b]$ the definite integral of f from a to b is:

$$\int_a^b f(x)dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\triangle x, \text{ whenever the limit exists.}$$

(where c_i is any point in the subintrval $[x_{i-1}, x_i], i = 1, 2, \ldots, n$).

Remark 3.1

1. *Rieman Sum is the same for any choice of the points* $c_1, c_2, \ldots, c_n$
2. *When the limit exists we say that the function f is integrable.*

The definite Integral

Remark 4.1

If the function f is continuous on $[a, b]$ and $f(x) \geq 0$ for every $x \in [a, b]$, then

1. $\int_a^b f(x)dx \geq 0$
2. $\int_a^b f(x)dx =$ *The area under the curve of f*

Example 4.1

$$\int_1^3 (3x+1)dx = \text{Area under the curve of } f = \lim_{x\to\infty} R_n = 14$$

The definite Integral

Theorem 4.1

If the function f is continuous on the interval $[a, b]$ then f is integrable on $[a, b]$.

Properties of the definite integral:

1. $\int_b^a kf(x)dx = k\int_b^a f(x)dx$ for every $k \in \mathbb{R}$.
2. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
3. For every $c \in [a, b]$, $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
4. If $f(x) \leq g(x)$ for every $x \in [a, b]$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$
5. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

The definite Integral (Examples)

1. $\int_7^2 3(x^2-3)dx = 3\int_7^2 (x^2-3)dx = -3\int_2^7 (x^2-3)dx = -290$
2. $\int_7^2 3(x^2-3)dx = -3\int_2^7 (x^2-3)dx =$
   $-3\int_2^5 (x^2-3)dx - 3\int_5^7 (x^2-3)dx = -290$
3. $x^2 \geq \frac{x^2}{x^2+4}$ then, $\int_{-1}^1 x^2dx \geq \int_{-1}^1 \frac{x^2}{x^2+4}dx$

Fundamental Theorem of Calculus (Part I)

If f is a continuous function on the interval $[a, b]$, and $G(x)$ is the antiderivative of $f(x)$ on $[a, b]$ then:

$$\int_a^b f(x)dx = [G(x)]_a^b = G(b) - G(a)$$

Remark

$$\int_a^b \frac{d}{dx} G(x)dx = G(b) - G(a)$$

Example 4.2

1. $\int_0^2 (x^2 - 2x)dx = [\frac{x^3}{3} - x^2]_0^2 = (\frac{8}{3} - 4) - (\frac{0}{3} - 0) = -\frac{4}{3}$
2. *Find the area under the graph of* $f(x) = \sin x$ *on* $[0, \pi]$
   The area
   $= \int_0^\pi \sin x \ dx = [-\cos x]_0^\pi = (-\cos \pi) - (-\cos 0) = 2$

Fundamental Theorem of Calculus (Part II)

If f is a continuous function on the interval $[a, b]$ and $G(x) = \int_a^x f(t)dt$ for every $x \in [a, b]$ then $G'(x) = f(x)$ for every $x \in [a, b]$.

Example 4.3

1. $\frac{d}{dx} \int_0^x \sqrt{t^2+1}\, dt = \sqrt{x^2+1}$
2. $\frac{d}{dx} \int_1^x \frac{1}{t^2+1}\, dt = \frac{1}{x^2+1}$

Fundamental Theorem of Calculus

Theorem 4.2

If f is a continuous function , g and h are deifferentiable functions then

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\, dt = f(h(x))h'(x) - f(g(x))g'(x)$$

Remark 4.2

1. *If* $g(x) = a$ *and* $h(x) = b$ *then*
   $\frac{d}{dx}\int_{a}^{b} f(t)\, dt = f(b)(0) - f(a)(0) = 0$
2. *If* $g(x) = a$ *and* $h(x) = x$ *then*
   $\frac{d}{dx}\int_{a}^{x} f(t)\, dt = f(x)(1) - f(a)(0) = f(x)$

Fundamental Theorem of Calculus (Examples)

1. Find $G'(x)$, if $G(x) = \int_{1-x}^{x^2} \frac{1}{4+3t^2} dt$

$$G'(x) = \frac{d}{dx} \int_{1-x}^{x^2} \frac{1}{4+3t^2}\, dt = \frac{1}{4+3(x^2)^2}(2x) - \frac{1}{4+3(1-x)^2}(-1)$$

$$G'(x) = \frac{2x}{4+3(x^2)^2} + \frac{1}{4+3(1-x)^2}$$

2. Find $F'(2)$, if $F(x) = \int_{1}^{x^2} \frac{1}{t} dt = \frac{1}{x^2}(2x) - 0 = \frac{2x}{x^2} = \frac{2}{x}$ Hence

$$F'(2) = \frac{2}{2} = 1$$

Average value of a function

Definition 4.1

Let f be a continuous function on $[a, b]$ then the average value of f on $[a, b]$ is $f_{av} = \frac{\int_a^b f(x)dx}{b-a}$

Example 4.4

Find f_{av} of the following function: $f(x) = x^2 - 2x$ *on the interval* $[1, 4]$

$\int_1^4 (x^2 - 2x)dx = [\frac{x^3}{3} - x^2]_1^4 = 6$

Hence $f_{av} = \frac{\int_1^4 (x^2-2x)dx}{4-1} = \frac{6}{3} = 2$

Average value of a function (Exercises)

Exercise 3

1. *Find* f_{av} *of the function* $f(x) = (2x+1)^2$ *on the interval* $[0, 1]$
2. *Find* f_{av} *of the function* $f(x) = \sin^2 x \cos x$ *on the interval* $[0, \frac{\pi}{2}]$

Integral Mean Value Theorem

Theorem 4.3

If f is a continuous function on the interval $[a, b]$ then there exists a number $c \in (a, b)$ for which $f(c) = \dfrac{\int_a^b f(x)dx}{b-a}$

Example 4.5

Find the value that satisfies the integral Mean value theorem for the function $f(x) = 4x^3 - 1$ *on the interval* $[1, 2]$

$$f(c) = \frac{\int_1^2 (4x^3-1)}{2-1} \Rightarrow 4c^3-1 = [x^4-x]_1^2 \Rightarrow 4c^3-1 = 14 \Rightarrow c = \sqrt[3]{\frac{15}{4}}$$

Note that $\sqrt[3]{\frac{15}{4}} \in [1, 2]$

Numerical Integration

1. The Trapezoidal Rule: It is used to approximate $\int_a^b f(x)dx$ with a regular partition of the interval $[a, b]$, where $\triangle x = \frac{b-a}{n}$, by using the formula

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

2. Simpson's Rule: It is used to approximate $\int_a^b f(x)dx$ with a regular partition of the interval $[a, b]$, where $\triangle x = \frac{b-a}{n}$, and n even, by using the formula

$$\int_a^b f(x)dx \approx \frac{b-a}{3n}[f(x_0)+4f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+4f(x_{n-1})+$$

Numerical Integration(Examples)

Example 4.6

Approximate the integral $\int_0^1 \sqrt{x+x^2}\, dx$ *using Trapezoidal rule with* $n = 4$.

Answer:

$[a, b] = [0, 1], f(x) = \sqrt{x+x^2},\ \triangle x = \frac{1-0}{4} = 0.25$

n	x_n	$f(x_n)$	m	$mf(x_n)$
0	0	0	1	0
1	0.25	0.559017	2	1.11803
2	0.5	0.86625	2	1.73205
3	0.75	1.14564	2	2.29129
4	1	1.41421	1	1.41421
				6.55559

$$\int_0^1 \sqrt{x+x^2}\, dx \approx \frac{0-1}{2(4)}[f(0) + 2f(0.25) + 2f(0.5) + 2f(0.75) + f(1)]$$

$$\int_0^1 \sqrt{x+x^2}\, dx \approx 0.819448$$

Numerical Integration(Examples)

Example 4.7

Approximate the integral $\int_0^{10} \sqrt{10x - x^2}\, dx$ *using Simpson's rule with* $n = 4$.

Answer:

$[a, b] = [0, 10], f(x) = \sqrt{10x - x^2}, \ \triangle x = \frac{10-0}{4} = 2.5$

n	x_n	$f(x_n)$	m	$mf(x_n)$
0	0	0	1	0
1	2.5	4.33013	4	17.3204
2	5	5	2	10
3	7.5	4.33013	4	17.3204
4	10	0	1	0
				44.6408

$$\int_0^{10} \sqrt{10x - x^2}\, dx \approx \frac{10-0}{3(4)}[f(0) + 4f(2.5) + 2f(5) + 4f(7.5) + f(10)]$$

$$\int_0^{10} \sqrt{10x - x^2}\, dx \approx 37.2007$$

Numerical Integration(Exercises)

Exercise 4

1. *Approximate the integral* $\int_2^4 \frac{1}{x-1}dx$ *using Trapezoidal rule with* $n = 4$.
2. *Approximate the integral* $\int_0^2 \frac{x}{x+1}dx$ *using Simpson's rule with* $n = 4$.

The natural logarithmic function

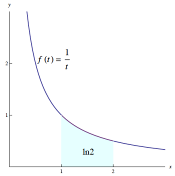
Definition 4.2

For $x > 0$, the natural logarithmic function is defined by

$$\ln x = \int_1^x \frac{1}{t}\,dt$$

Remark

The domain of the function $\ln x$ is the open interval $(0, \infty)$



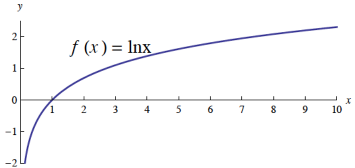
properties of natural logarithmic function

1. If $x > 1$ then $\ln x > 0$
2. $\ln 1 = 0$
3. If $0 < x < 1$ then $\ln x < 0$
4. The range of the function $\ln x$ is $\mathbb{R}$
5. $\lim_{x \to \infty} \ln x = \infty$ and $\lim_{x \to 0+} \ln x = -\infty$
6. $\frac{d}{dx} \ln |x| = \frac{1}{x}$ and $\frac{d}{dx} \ln |f(x)| = \frac{f'(x)}{f(x)}$
7. $\ln |x|$ is the antiderivative of $\frac{1}{x}$
8. $\int \frac{1}{x} dx = \ln |x| + c$ and $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$
9. $\ln(xy) = \ln x + \ln y$, $\ln(\frac{x}{y}) = \ln x - \ln y$ and $\ln x^r = r \ln x$

The natural logarithmic function

The graph of $\ln x$**:**

1. First derivative test :
   $\frac{d}{dx}\ln x = \frac{d}{dx}\int_1^x \frac{1}{t}dt = \frac{1}{x} > 0$ for every $x \in (0, \infty)$
   Hence $\ln x$ is an increasing function on $(0, \infty)$
2. Second derivative test :
   $\frac{d^2}{dx^2}\ln x = \frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2} < 0$ for every $x \in (0, \infty)$
   Hence $\ln x$ is a convex function on $(0, \infty)$



The natural logarithmic function

Basic Rules of Integration :

1. $\int \tan x \; dx = \ln|\sec x| + c$
2. $\int \cot x \; dx = \ln|\sin x| + c$
3. $\int \sec x \; dx = \ln|\sec x + \tan x| + c$
4. $\int \csc x \; dx = \ln|\csc x - \cot x| + c$

The natural logarithmic function (Examples)

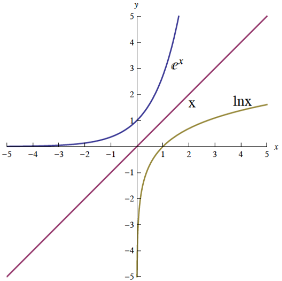
Example 4.8

1. $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\ dx = \frac{1}{3}\int \frac{3x^2+6x+9}{x^3+3x^2+9x}\ dx = \frac{1}{3}\ln|x^3+3x^2+9x| + c$
2. $\int \frac{1}{x\sqrt{\ln x}}\ dx = \int (\ln x)^{-\frac{1}{2}}\frac{1}{x}\ dx = \frac{(\ln x)^{\frac{1}{2}}}{\frac{1}{2}} + c$

The natural exponential function

Definition 4.3

The natural exponential function is the inverse of the natural logarithmic function, and it is denoted by e^x



Properties of the natural exponential function

1. The domain of the function e^x is $\mathbb{R}$
2. The range of the function e^x is the open interval $(0, \infty)$
3. $e^x > 0$ for every $x \in \mathbb{R}$, $e^0 = 1$, and $e \approx 2.71828$ and $\ln e = 1$
4. $\lim_{x\to\infty} e^x = \infty$, $\lim_{x\to-\infty} e^x = 0$
5. $\ln(e^x) = x$ and $e^{\ln x} = x$
6. If $x, y \in \mathbb{R}$, then $e^x e^y = e^{x+y}$, $\frac{e^x}{e^y} = e^{x-y}$, and $(e^x)^y = e^{xy}$
7. $\frac{d}{dx}e^x = e^x$, and $\frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x)$
8. $\int e^x dx = e^x + c$ and $\int e^{f(x)}f'(x)dx = e^{f(x)} + c$

The natural exponential function (Examples)

Example 4.9

1. *Find the value of x that satisfies the equation* $\ln\frac{1}{x} = 2$*?*

$$x = e^{-2} = \frac{1}{e^2}$$

2. *Find* $f'(x)$ *If* $f(x) = e^{5x} + \frac{1}{e^x}$

$f'(x) = 5e^{5x} - e^{-x}$

3. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} = 2\int e^{\sqrt{x}}\frac{1}{2\sqrt{x}} = 2e^{\sqrt{x}} + c$

4. $\int_1^e \frac{\sqrt[3]{\ln x}}{x}dx = \int_1^e (\ln x)^{\frac{1}{3}}\frac{1}{x}dx =$

$[\frac{(\ln x)^{\frac{4}{3}}}{\frac{4}{3}}]_1^e = \frac{3}{4}(\ln e)^{\frac{3}{4}} - \frac{3}{4}(\ln 1)^{\frac{4}{3}} = \frac{3}{4}$

The natural exponential function (Exercises)

Exercise 5

1. *Find the value of x that satisfies the equation* $e^{5x+3} = 4$?
2. $\int \frac{e^{\sin x}}{\sec x} dx =$
3. *Find* $g(x)$ *if* $\int e^{3x^2} g(x) dx = -e^{3x^2} + c$

The general exponential function and logarithmic function

Definition 4.4

It has the form a^x *where* $a > 0$ *and* $a \neq 1$.
Note: $a^x = e^{x \ln a}$

Derivative and Integration of the general exponential function :

1. $\frac{d}{dx} a^x = a^x \ln a$, and $\int a^x dx = \frac{a^x}{\ln a} + c$
2. $\frac{d}{dx} a^{f(x)} = a^{f(x)} f'(x) \ln a$, and $\int a^{f(x)} f'(x) dx = \frac{a^{f(x)}}{\ln a} + c$

The general exponential function and logarithmic function

Definition 4.5

The general logarithmic function of base a where $a > 0$ and $a \neq 1$ is denoted by $\log_a x$ and it is the inverse function of the general exponential function a^x

Note: $\log_a x = y \Leftrightarrow a^y = x$ and $\log_a x = \frac{\ln x}{\ln a}$
Notations: $\log x = \log_{10} x$ and $\ln x = \log_e x$
Derivative of the general logarithmic function :
$\frac{d}{dx} \log_a |x| = \frac{1}{x} \frac{1}{\ln a}$ and $\frac{d}{dx} \log_a |f(x)| = \frac{f'(x)}{f(x)} \frac{1}{\ln a}$

The general exponential function and logarithmic function (Examples)

1. Find the value of x if $\log_2 x = 3$?
   $\log_2 x = 3 \Leftrightarrow x = 2^3 = 8.$
2. Find y' if $y = (\sin x)^x$
   $y = (\sin x)^x \Rightarrow \ln y = \ln(\sin x)^x = x \ln|\sin x|$
   Differentiate both sides :
   $\frac{y'}{y} = \ln|\sin x| + x\frac{\cos x}{\sin x} = \ln|\sin x| + x \cot x$
3. $\int x^2 6^{x^3} dx = \frac{1}{3}\int 6^{x^3}(3x^2)\ dx = \frac{6^{x^3}}{3\ln 6} + c$

The general exponential function and logarithmic function (Exercises)

1. Find $f'(x)$ if $f(x) = (x^2 + 1)^x$
2. Evaluate $\int \frac{3^{\sqrt{x}}}{\sqrt{x}}$?