M 106 - INTEGRAL CALCULUS

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Solution of the Final Exam

First semester 1433-1434 H

Multiple choice questions (15 marks : one mark for each question)

1. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{2k}{n^2}$ is equal to

   (a) 0 (b) 1 (c) 2 (d) ∞

   The answer : $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{2k}{n^2}=\lim\limits_{n\to\infty}\left(\frac{2}{n^2}\sum\limits_{k=1}^{n}k\right)$

   $=\lim\limits_{n\to\infty}\left(\frac{2}{n^2}\;\frac{n(n+1)}{2}\right)=\lim\limits_{n\to\infty}\frac{n^2+n}{n^2}=1$

   The right answer is (b) .

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2. The average value of the function $f(x)=\sin 3x$ on $\left[0,\frac{\pi}{3}\right]$ is equal to

   (a) $-\frac{2}{\pi}$ (b) $\frac{2}{3}$ (c) $\frac{2}{\pi}$ (d) $-\frac{2}{3}$

   The answer : $f_{av}=\dfrac{\int_0^{\frac{\pi}{3}}\sin 3x\;dx}{\frac{\pi}{3}-0}$

   $\int_0^{\frac{\pi}{3}}\sin 3x\;dx=-\frac{1}{3}\int_0^{\frac{\pi}{3}}-(\sin 3x)\;3\;dx=-\frac{1}{3}\left[\cos 3x\right]_0^{\frac{\pi}{3}}$

   $=-\frac{1}{3}\;[\cos\pi-\cos 0]=-\frac{1}{3}\;[-1-1]=\frac{2}{3}$

   $f_{av}=\dfrac{\int_0^{\frac{\pi}{3}}\sin 3x\;dx}{\frac{\pi}{3}-0}=\dfrac{\left(\frac{2}{3}\right)}{\left(\frac{\pi}{3}\right)}=\frac{2}{\pi}$

   The right answer is (c) .

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3. The integral $\int_0^1\left|x-\frac{1}{2}\right|\;dx$ is equal to

   (a) 0 (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

   The answer :

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$$\left|x-\frac{1}{2}\right| = \begin{cases} x-\frac{1}{2} & , \quad x-\frac{1}{2} \geq 0 \\ -\left(x-\frac{1}{2}\right) & , \quad x-\frac{1}{2} < 0 \end{cases}$$

$$\left|x-\frac{1}{2}\right| = \begin{cases} x-\frac{1}{2} & , \quad x \geq \frac{1}{2} \\ \frac{1}{2}-x & , \quad x < \frac{1}{2} \end{cases}$$

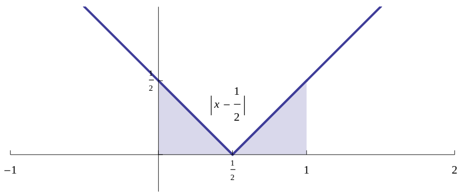
$$\int_0^1 \left|x-\frac{1}{2}\right| \, dx = \int_0^{\frac{1}{2}} \left|x-\frac{1}{2}\right| \, dx + \int_{\frac{1}{2}}^1 \left|x-\frac{1}{2}\right| \, dx$$

$$= \int_0^{\frac{1}{2}} \left(\frac{1}{2}-x\right) \, dx + \int_{\frac{1}{2}}^1 \left(x-\frac{1}{2}\right) \, dx$$

$$= \left[\frac{x}{2}-\frac{x^2}{2}\right]_0^{\frac{1}{2}} + \left[\frac{x^2}{2}-\frac{x}{2}\right]_{\frac{1}{2}}^1 = \left[\left(\frac{1}{4}-\frac{1}{8}\right)-0\right] + \left[0-\left(\frac{1}{8}-\frac{1}{4}\right)\right]$$

$$= \frac{1}{4}-\frac{1}{8}-\frac{1}{8}+\frac{1}{4} = \frac{1}{2}-\frac{1}{4} = \frac{1}{4}$$

Geometric solution :



The definite integral $\int_0^1 \left|x-\frac{1}{2}\right| \, dx$ equals to the area under the graph of the curve $\left|x-\frac{1}{2}\right|$ on the interval $[0,1]$ (the shaded region).

The shaded region consists of two identical triangles each of which has a base equals $\frac{1}{2}$ and height also equals $\frac{1}{2}$.

$$\int_0^1 \left|x-\frac{1}{2}\right| \, dx = \frac{1}{2}\left(\frac{1}{2}\,\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\,\frac{1}{2}\right) = \frac{1}{8}+\frac{1}{8} = \frac{1}{4} \, .$$

The right answer is (c) .

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4. The integral $\int \frac{\sin x}{1+\cos^2 x} \, dx$ is equal to

   (a) $-\tan^{-1}(\sin x)+c$ (b) $-\tan^{-1}(\cos x)+c$
   (c) $\tan^{-1}(\sin x)+c$ (d) $\tan^{-1}(\cos x)+c$

The answer : $\int \frac{\sin x}{1+\cos^2 x} \, dx = -\int \frac{-\sin x}{1+(\cos x)^2} \, dx = -\tan^{-1}(\cos x)+c$

The right answer is (b)

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5. The integral $\int 2^{-x} \tanh(2^{1-x})\ dx$ is equal to

(a) $\frac{1}{-2\ln 2} \ln\cosh(2^{1-x}) + c$ (b) $\frac{1}{-\ln 2} \ln\cosh(2^{1-x}) + c$
(c) $\frac{1}{-2\ln 2} \tanh^2(2^{1-x}) + c$ (d) $\frac{1}{-2\ln 2} \ln\sinh(2^{1-x}) + c$

The answer : $\int 2^{-x} \tanh(2^{1-x})\ dx$

$= -\frac{1}{2}\ \frac{1}{\ln 2} \int \tanh(2^{1-x})\ \left(-2^{-x}\ 2\ \ln 2\right)\ dx$

$= \frac{1}{-2\ln 2} \int \tanh(2^{1-x}) \left(2^{1-x}\ (-1)\ \ln 2\right)\ dx = \frac{1}{-2\ln 2} \ln\cosh(2^{1-x}) + c$

The right answer is (a)

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6. If $F(x) = \pi^x \int_0^{x^2} \tan^{-1} t\ dt$ then $F'(x)$ is equal to

(a) $\pi^x F(x) \ln\pi + 2x\pi^x \tan^{-1} x$ (b) $\pi^x F(x) \ln\pi + 2x\pi^x \tan^{-1} x^2$
(c) $F(x) \ln\pi + 2x\pi^x \tan^{-1} x$ (d) $F(x) \ln\pi + 2x\pi^x \tan^{-1} x^2$

The answer : $F'(x) = \frac{d}{dx}\left(\pi^x \int_0^{x^2} \tan^{-1} t\ dt\right)$

$$F'(x) = \left(\frac{d}{dx}\pi^x\right) \int_0^{x^2} \tan^{-1} t\ dt + \pi^x \left(\frac{d}{dx} \int_0^{x^2} \tan^{-1} t\ dt\right)$$

$$= \pi^x \ln\pi \int_0^{x^2} \tan^{-1} t\ dt + \pi^x \tan^{-1} x^2\ (2x)$$

$$= \ln\pi \left(\pi^x \int_0^{x^2} \tan^{-1} t\ dt\right) + 2x\pi^x \tan^{-1} x^2$$

$$= \ln\pi\ F(x) + 2x\pi^x \tan^{-1} x^2$$

The right answer is (d)

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7. $\lim\limits_{x\to\frac{\pi}{4}} \frac{1-\tan x}{\cos 2x}$ is equal to

(a) 2 (b) -2 (c) -1 (d) 1

The answer : $\lim\limits_{x\to\frac{\pi}{4}} \frac{1-\tan x}{\cos 2x} \quad \left(\frac{0}{0}\right)$

Apply L'Hôpital's rule

$$\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{\cos 2x} = \lim_{x\to\frac{\pi}{4}} \frac{-\sec^2 x}{-2\sin 2x} = \lim_{x\to\frac{\pi}{4}} \frac{1}{2\cos^2 x \ \sin 2x}$$

$$= \frac{1}{2\left(\cos\frac{\pi}{4}\right)^2 \ \sin\frac{\pi}{2}} = \frac{1}{2\left(\frac{1}{\sqrt{2}}\right)^2 \ (1)} = \frac{1}{2\left(\frac{1}{2}\right)} = 1$$

The rightanswer is (d) .

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8. If a point has xy-coordinates $(x, y) = \left(\sqrt{2}, \sqrt{2}\right)$ then one of its polar coordinates (r, θ) is

(a) $\left(1, \frac{\pi}{4}\right)$ (b) $\left(2, \frac{\pi}{4}\right)$ (c) $\left(\sqrt{2}, \frac{\pi}{4}\right)$ (d) $\left(\sqrt{2}, \frac{3\pi}{4}\right)$

The answer : $r = \sqrt{x^2+y^2} = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{2+2} = \sqrt{4} = 2$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{\sqrt{2}} = 1 \quad \Rightarrow \theta = \frac{\pi}{4}$ or $\theta = \frac{5\pi}{4}$

$(r, \theta) = \left(2, \frac{\pi}{4}\right)$

The right answer is (b) .

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9. The integral $\int_1^2 x \ln x \ dx$ is equal to

(a) $2\ln 2 - \frac{5}{4}$ (b) $2\ln 2 + \frac{5}{4}$ (c) $2\ln 2 - \frac{3}{4}$ (d) $2\ln 2 + \frac{3}{4}$

The answer : Using Integration by parts

$u = \ln x \qquad dv = x \ dx$

$du = \frac{1}{x} \ dx \qquad v = \frac{x^2}{2}$

$$\int_1^2 x \ln x \ dx = \left[\frac{x^2}{2} \ \ln x\right]_1^2 - \int_1^2 \frac{x^2}{2} \ \frac{1}{x} \ dx$$

$$= \left[\frac{x^2}{2} \ \ln x\right]_1^2 - \int_1^2 \frac{x}{2} \ dx = \left[\frac{x^2}{2} \ \ln x\right]_1^2 - \left[\frac{x^2}{4}\right]_1^2$$

$$= \left[2\ln 2 - \frac{1}{2}\ln 1\right] - \left[1 - \frac{1}{4}\right] = [2\ln 2 - 0] - \frac{3}{4} = 2\ln 2 - \frac{3}{4}$$

The right answer is (c)

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10. The slope of the tangent line at the point corresponding to $t = 1$ on the curve given parametrically by the equations $x = t^3 + t$, $y = -3t$ is

(a) $-\frac{1}{2}$ (b) $-\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

The answer : $\dfrac{dy}{dt} = -3$, $\dfrac{dx}{dt} = 3t^2 + 1$

The slope $m = \dfrac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \dfrac{-3}{3t^2+1}$

The slope at $t = 1$ is $m|_{t=1} = \dfrac{-3}{3(1)^2+1} = -\dfrac{3}{4}$

The right answer is (b)

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11. If a graph has a polar equation $r = -4\cos\theta$, then its equation in xy-system is

(a) $x^2 + 4x + y^2 = 0$ (b) $x^2 - 4x + y^2 = 0$
(c) $x^2 - x + y^2 = 0$ (d) $x^2 + y^2 = 0$

The answer : Multiplying both sides of the equation by r

$r = -4\cos\theta \Rightarrow r^2 = -4(r\cos\theta) \Rightarrow x^2 + y^2 = -4x \Rightarrow x^2 + 4x + y^2 = 0$

Note that $r^2 = x^2 + y^2$ and $x = r\cos\theta$

The right answer is (a)

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12. The arc length of the curve C : $x = 4\cos t$, $y = 4\sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$ equals

(a) 2π (b) 8π (c) π (d) 4π

The answer : $\dfrac{dx}{dt} = -4\sin t$, $\dfrac{dy}{dt} = 4\cos t$

$$L = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{(-4\sin t)^2 + (4\cos t)^2}\, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{16\sin^2 t + 16\cos^2 t}\, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{16(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{16}\, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\, dt = 4\,[t]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 4\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = 4\pi$$

The right answer is (d) .

Note : The parametric curve represents the right half of a circle of radius 4 , hence its arc length equals half of the perimeter of the circle of radius 4 which is $\dfrac{1}{2}\, 2\pi\ (4) = 4\pi$.

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13. The The surface area resulting by revolving the graph of the equation $x = y$, $0 \le y \le 1$ around the $y-axis$ is equal to

(a) $8\sqrt{2}\pi$ (b) $\sqrt{2}\pi$ (c) $24\sqrt{2}\pi$ (d) $\frac{9}{2}\sqrt{2}\pi$

The answer : $g(y) = y$, $\dfrac{dx}{dy} = g'(y) = 1$.

$S.A = 2\pi \int_0^1 |y|\sqrt{1+(1)^2}\ dy = 2\sqrt{2}\pi \int_0^1 y\ dy$

$= 2\sqrt{2}\pi \left[\dfrac{y^2}{2}\right]_0^1 = 2\sqrt{2}\pi \left[\dfrac{1}{2} - 0\right] = \sqrt{2}\pi$

Note that $|y| = y$ on the interval $[0,1]$.

The right answer is (b)

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14. The improper integral $\displaystyle\int_0^\infty \frac{1}{x+1}\ dx$

(a) converges 0 (b) diverges (c) converges to 1 (d) converges to -1

The answer : $\displaystyle\int_0^\infty \frac{1}{x+1}\ dx = \lim_{t\to\infty} \int_0^t \frac{1}{x+1}\ dx$

$= \lim_{t\to\infty} [\ln|x+1|]_0^t = \lim_{t\to\infty} [\ln|t+1| - \ln 1]$

$= \lim_{t\to\infty} \ln|t+1| = \infty$

The improper integral $\displaystyle\int_0^\infty \frac{1}{x+1}\ dx$ diverges

The right answer is (b)

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15. The graph of the curve C defined paramerically by the parametric equations $x = t+1$, $y = t^2+1$, $-3 \le t \le 1$ is

(a) a straight line (b) a parabola (c) an ellipse (d) a circle

The answer : $x = t+1 \ \Rightarrow\ t = x-1$

$y = t^2+1 \ \Rightarrow\ y = (x-1)^2+1$

The graph of C represents a parabola

The right answer is (b)

Full Questions (25 marks)

16. Evaluate $\int \frac{1}{x^2(x^2+1)}\,dx$ [4 marks]

The answer : Using trigonometric substitutions

Put $x = \tan\theta \;\Rightarrow\; \theta = \tan^{-1} x$

$dx = \sec^2\theta d\theta$

$$\int \frac{\sec^2\theta}{(\tan\theta)^2\left((\tan\theta)^2+1\right)}\,d\theta = \int \frac{\sec^2}{\tan^2\theta\ \left(\tan^2\theta+1\right)}\,d\theta$$

$$= \int \frac{\sec^2\theta}{\tan^2\theta\ \sec^2\theta}\,d\theta = \int \frac{1}{\tan^2\theta}\,d\theta = \int \cot^2\theta\ d\theta = \int \left(\csc^2\theta - 1\right)\,d\theta$$

$$= -\cot\theta - \theta + c = -\frac{1}{\tan\theta} - \theta + c = -\frac{1}{x} - \tan^{-1} x + c$$

Another solution : $\frac{1}{x^2\ (x^2+1)} = \frac{1}{x^2} - \frac{1}{x^2+1}$

$$\int \frac{1}{x^2(x^2+1)}\,dx = \int \left(\frac{1}{x^2} - \frac{1}{x^2+1}\right)\,dx$$

$$= \int x^{-2}\ dx - \int \frac{1}{x^2+1}\,dx = -x^{-1} - \tan^{-1} x + c = -\frac{1}{x} - \tan^{-1} x + c$$

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17. Evaluate $\int \frac{x+2}{\sqrt{x^2+2x+2}}\,dx$. [4 marks]

The answer : $\int \frac{x+2}{\sqrt{x^2+2x+2}}\,dx = \int \frac{(x+1)+1}{\sqrt{x^2+2x+2}}\,dx$

$$= \int \frac{x+1}{\sqrt{x^2+2x+2}}\,dx + \int \frac{1}{\sqrt{x^2+2x+2}}\,dx$$

$$= \frac{1}{2}\int (x^2+2x+2)^{-\frac{1}{2}}2(x+1)\ dx + \int \frac{1}{\sqrt{(x^2+2x+1)+1}}\,dx$$

$$= \frac{1}{2}\int (x^2+2x+2)^{-\frac{1}{2}}(2x+2)\ dx + \int \frac{1}{\sqrt{(x+1)^2+1}}\,dx$$

$$= \frac{1}{2}\frac{(x^2+2x+2)^{\frac{1}{2}}}{\frac{1}{2}} + \sinh^{-1}(x+1) + c$$

$$= \sqrt{x^2+2x+2} + \sinh^{-1}(x+1) + c$$

Another solution : Using trigonometric substitutions

$$\int \frac{x+2}{\sqrt{x^2+2x+2}}\,dx = \int \frac{(x+1)+1}{\sqrt{(x+1)^2+1}}\,dx$$

Put $x+1=\tan\theta$

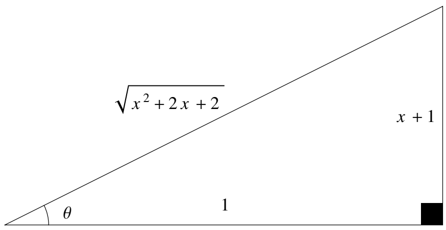
$dx=\sec^2\theta\ d\theta$

$$\int \frac{(x+1)+1}{\sqrt{(x+1)^2+1}}\ dx=\int \frac{(\tan\theta\ +1)\sec^2\theta}{\sqrt{\tan^2\theta+1}}\ d\theta$$

$$=\int \frac{(\tan\theta\ +1)\sec^2\theta}{\sec\theta}\ d\theta=\int \sec\theta\ (\tan\theta\ +1))\ d\theta$$

$$=\int (\sec\theta\ \ \tan\theta+\sec\theta)\ d\theta=\int \sec\theta\tan\theta\ d\theta+\int \sec\theta\ d\theta$$

$$=\sec\theta+\ln|\sec\theta+\tan\theta|+c$$



$$\int \frac{x+2}{\sqrt{x^2+2x+2}}\ dx=\sqrt{x^2+2x+2}+\ln\left|\sqrt{x^2+2x+2}+(x+1)\right|+c$$

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18. (a). Evaluate $F(x)=\dfrac{d}{dx}\left(\sqrt{x}\sinh\sqrt{x}\right)$ [3 marks]

(b). Find $\displaystyle\int \cosh\sqrt{x}\ dx$ by using $F(x)$. [2 marks]

The answer :

(a). $F(x)=\dfrac{d}{dx}\left(\sqrt{x}\sinh\sqrt{x}\right)=\dfrac{1}{2\sqrt{x}}\sinh\sqrt{x}+\sqrt{x}\ \cosh\sqrt{x}\left(\dfrac{1}{2\sqrt{x}}\right)$

$$F(x)=\frac{\sinh\sqrt{x}}{2\sqrt{x}}+\frac{\cosh\sqrt{x}}{2}$$

$$\cosh\sqrt{x}=2F(x)-\frac{\sinh\sqrt{x}}{\sqrt{x}}$$

(b). $\displaystyle\int \cosh\sqrt{x}\ dx=\int\left(2F(x)-\frac{\sinh\sqrt{x}}{\sqrt{x}}\right)\ dx$

$$=2\int F(x)\ dx-\int \frac{\sinh\sqrt{x}}{\sqrt{x}}\ dx$$

$$=2\int \frac{d}{dx}\left(\sqrt{x}\sinh\sqrt{x}\right)\ dx-2\int \sinh\sqrt{x}\ \frac{1}{2\sqrt{x}}\ dx$$

$$=2\sqrt{x}\sinh\sqrt{x}-2\cosh\sqrt{x}+c$$

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19. Evaluate the integral $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$ [4 marks]

The answer : Using Integration by parts

$$u = \sin^{-1} x \qquad dv = \frac{x}{\sqrt{1-x^2}}\, dx$$
$$du = \frac{1}{\sqrt{1-x^2}}\, dx \qquad v = -\sqrt{1-x^2}$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx = -\sqrt{1-x^2}\sin^{-1} x - \int (-\sqrt{1-x^2})\frac{1}{\sqrt{1-x^2}}\, dx$$

$$= -\sqrt{1-x^2}\sin^{-1} x + \int dx = -\sqrt{1-x^2}\sin^{-1} x + x + c$$

Another solution : Using trigonometric substitutions , then integration by parts

Put $x = \sin\theta \quad \Rightarrow \quad dx = \cos\theta\, d\theta$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx = \int \frac{\sin\theta\ \sin^{-1}(\sin\theta)\ \cos\theta}{\sqrt{1-\sin^2\theta}}\, d\theta$$

$$= \int \frac{\theta\ \sin\theta\ \cos\theta}{\cos\theta}\, d\theta = \int \theta\ \sin\theta\ d\theta$$

$$u = \theta \qquad dv = \sin\theta\ d\theta$$
$$du = d\theta \qquad v = -\cos\theta$$

$$\int \theta\ \sin\theta\ d\theta = -\theta\cos\theta - \int -\cos\theta\ d\theta$$

$$= -\theta\ \sqrt{1-\sin^2\theta} + \int \cos\theta\ d\theta$$

$$= -\theta\ \sqrt{1-\sin^2\theta} + \sin\theta + c$$

$$= -\sin^{-1} x\ \sqrt{1-x^2} + x + c$$

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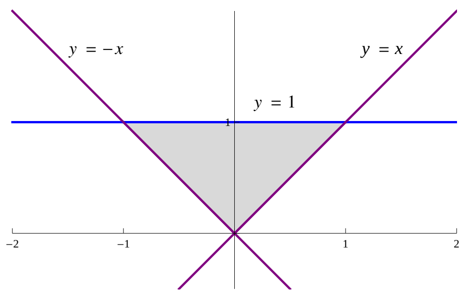
20. Let R be the region bounded by the graphs $y = x$, $y = -x$ and $y = 1$. **Sketch** the region R and Find the **volume** of the solid generated by revolving the region R about the $x-$axis . [4 marks]

The answer :



Note that the line $y = x$ intersects the line $y = 1$ at $(1, 1)$, and the line $y = -x$ intersects the line $y = 1$ at $(-1, 1)$.

(1) Using cylindrical shells :

$$V = 2\pi \int_0^1 y(y - (-y))\ dy = 2\pi \int_0^1 y(2y)\ dy$$

$$V = 4\pi \int_0^1 y^2\ dy = 4\pi \left[\frac{y^3}{3}\right]_0^1 = 4\pi \left[\frac{1}{3} - 0\right] = \frac{4\pi}{3}$$

(2) Using Washer Method :

$$V = \pi \int_{-1}^0 \left[(1)^2 - (-x)^2\right]\ dx + \pi \int_0^1 \left[(1)^2 - (x)^2\right]\ dx$$

$$V = \pi \int_{-1}^0 (1 - x^2)\ dx + \pi \int_0^1 (1 - x^2)\ dx = \pi \int_{-1}^1 (1 - x^2)\ dx$$

$$V = \pi \left[x - \frac{x^3}{3}\right]_{-1}^1 = \pi \left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right]$$

$$V = \pi \left[\frac{2}{3} - \left(-\frac{2}{3}\right)\right] = \pi \left(\frac{2}{3} + \frac{2}{3}\right) = \frac{4\pi}{3}$$

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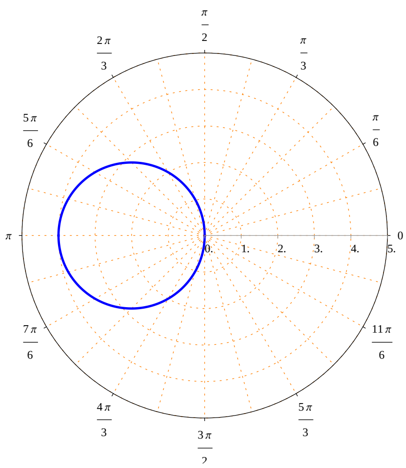
21. **Sketch** and **Find** the surface area generated by revolving the graph of the polar equation $r = -4\cos\theta$ about the vertical line $\theta = \frac{\pi}{2}$. [4 marks]

The answer :



$$S.A = 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |r(\theta)\cos\theta| \sqrt{[r(\theta)]^2 + \left[\frac{dr}{d\theta}\right]^2}\ d\theta$$

$$S.A = 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |-4\cos\theta\ \ \cos\theta|\ \sqrt{[-4\cos\theta]^2 + [4\sin\theta]^2}\ d\theta$$

$$S.A = 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |-4\cos^2\theta|\ \sqrt{16\cos^2\theta + 16\sin^2\theta}\ d\theta$$

$$S.A = 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 4\cos^2\theta\ \sqrt{16(\cos^2\theta + \sin^2\theta)}\ d\theta$$

$$S.A = 2\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 4\cos^2\theta\ \sqrt{16}\ d\theta = 32\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2\theta\ d\theta$$

$$S.A = 32\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}(1+\cos 2\theta)\ d\theta = 16\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1+\cos 2\theta)\ d\theta$$

$$S.A = 16\pi \left[\theta + \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = 16\pi \left[\left(\frac{3\pi}{2} + \frac{\sin 3\pi}{2}\right) - \left(\frac{\pi}{2} + \frac{\sin \pi}{2}\right)\right]$$

$$S.A = 16\pi \left[\left(\frac{3\pi}{2} - 0\right) - \left(\frac{\pi}{2} - 0\right)\right] = 16\pi \left(\frac{3\pi}{2} - \frac{\pi}{2}\right) = 16\pi(\pi) = 16\pi^2$$