

Question 1(2+4)

a) If $F(x) = \int_{4x^2}^{x^4} \frac{dt}{\sqrt{t+9}}$ find $F'(2)$.

b) Use Simpson's rule, with $n = 4$, to approximate $\int_1^2 \frac{1}{x} dx$ and estimate the error

Question 2(2+3+3)

a) Evaluate the integral $\int (x + 1)3^{-x^2-2x} dx$

b) Compute the integral $\int \frac{x^2 dx}{\cosh^2 x^3}$

c) Find the indefinite integral $\int \frac{dx}{x\sqrt{x-1}}$, $x > 1$

Question 3(3+3+2)

a) Use the substitution $u = \sqrt{x}$ to compute the integral $\int e^{\sqrt{x}} dx$

b) Evaluate the integral $\int (\sin x)^{1/2} \cos^5 x dx$

c) Evaluate the definite integral $\int_0^{\pi/2} \sin(3x)\cos(2x) dx$

Question 4(3+4+2)

a) Sketch the region bounded by $y = 2x^2 + 1$, $y = 4x + 1$
 $x = 0$, $x = 3$, and find its area.

b) i) Sketch the region R bounded by $y = \sqrt{x-1}$, $y = x-1$, and find the volume of the solid obtained by revolving R about the x-axis.

ii) Set up an integral for the volume of the solid obtained by revolving R about the line $x = 3$

c) Evaluate the integral $\int \frac{2x+1}{x(x+1)^2} dx$

Question5(3+3+3)

a) Find the arc length of the parametric curve given by: $x = \frac{t^2}{2} + 1$

$$y = \frac{t^3}{3} - 1, \quad 0 \leq t \leq 1$$

b) Find the area of the surface obtained by revolving the curve $y = 2x^{1/2}$, $x \in [1,2]$, about the x-axis.

c) Sketch the region R that lies in the first quadrant inside $r = 2\cos\theta$ and outside $r = 2\sin\theta$, and find its area.