King Saud University
College of Sciences
Department of Mathematics
Second Semester (1434/1435)
M-106
Second Midterm-Exam

Programmable Calculators are Not Authorized

The Exam paper contains 5 pages
(**5 Multiple choice questions and 5 Full questions**)

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 25 Time: 90 minutes

Marks:

Multiple Choice (1-5)	
Question # 6	
Question # 7	
Question # 8	
Question # 9	
Question # 10	
Total	

Multiple Choice

Q.No:	1	2	3	4	5
$\{a, b, c, d\}$	a	a	a	d	b

Q. No: 1 The integral of $\int_{-1}^{0} \frac{1}{x^4} dx$, is equal to:

$(a)\quad +\infty \quad (b) -\infty \qquad (c)\quad 0 \quad (d)\quad 1$

Q. No: 2 If we used the substitution $u = \tan(\frac{x}{2})$, then the integral $\int \frac{dx}{\sin(x)}$ is equal:

$(a)\quad \ln|u| + c \quad (b)\quad 2\tanh^{-1} u + c \quad (c)\quad \tanh^{-1} u + c \quad (d)\quad \ln|u+1| + c$

Q. No: 3 The Integral $\int \ln(x^\alpha)dx$, where α is a real number and x is positive, is equal to:

$(a)\ x\alpha\,(\ln(x) - 1) + c \quad (b)\ \alpha x \ln(x) - x + c \quad (c)\ \alpha x \ln(x) + \alpha x + c \quad (d)\ \alpha(x\ln(x) - 1) + c$

Q. No: 4 To evaluate the integral $\int \sin^2(x)\cos^3(x)dx$, the best substitution that can be used is:

$(a)\ u = \sec(x) \quad (b)\ u = \tan(x) \quad (c)\ u = \cos(x) \quad (d)\ u = \sin(x)$

Q. No: 5 $\lim_{x \longrightarrow \infty} \frac{\ln x}{\sqrt[3]{x}}$ is equal to:

$(a)\ 1 \qquad (b)\ 0 \qquad (c)\ 3 \quad (d)\ \infty$

Full Questions

Question No: 6. Evaluate $\int x^2 \sinh(x)\, dx$

Let

$$u = x^2 \text{ then } u' = 2x,\ v' = \sinh(x) \text{ then } v = \cosh(x)\ [1]$$

Then

$$\int x^2 \sinh(x)\, dx = x^2 \cosh(x) - 2\int x \cosh(x)\, dx\ [0.5]$$

again let

$$u = x \text{ then } u' = 1,\ v' = \cosh(x) \text{ then } v = \sinh(x)\ [1]$$

we will have

$$\begin{aligned} \int x^2 \sinh(x)\, dx &= x^2 \cosh(x) - 2(x \sinh(x) - \int \sinh(x) dx)[1] \\ &= x^2 \cosh(x) - 2x \sinh(x) + 2\cosh(x) + C\ [0.5] \end{aligned}$$

Question No: 7 Evaluate $\int \frac{\sqrt{x^2-9}}{x} dx$

Let

$$x = 3\sec(\theta) \text{ then } dx = 3\sec(\theta)\tan(\theta)d\theta, [0.5]$$

Consequently

$$\sqrt{x^2-9} = \sqrt{9\sec^2(\theta)-9} = 3\sqrt{\sec^2(\theta)-1} = 3\sqrt{\tan^2(\theta)} = 3\tan(\theta), [0.5]$$

and

$$\begin{aligned} \int \frac{\sqrt{x^2-9}}{x} dx &= \int \frac{3\tan(\theta)}{3\sec(\theta)} 3\sec(\theta)\tan(\theta)d\theta \\ &= 3\int \tan^2(\theta)d\theta\ [0.5] \\ &= 3\int (\sec^2(\theta)-1)d\theta\ [0.5] \\ &= 3\int \sec^2(\theta)d\theta - 3\int d\theta = 3\tan(\theta) - 3\theta + C\ [0.5] \end{aligned}$$

We have

$$\sec(\theta) = \frac{x}{3} \text{ then } \tan(\theta) = \frac{\sqrt{x^2-9}}{3} \text{ and } \theta = \sec^{-1}(\frac{x}{3}) \ [0.5]$$

Then

$$\begin{aligned}\int \frac{\sqrt{x^2-9}}{x}dx &= 3\tan(\theta) - 3\theta + C = 3\frac{\sqrt{x^2-9}}{3} - 3\sec^{-1}(\frac{x}{3}) + C \ [0.5] \\ &= \sqrt{x^2-9} - 3\sec^{-1}(\frac{x}{3}) + C \ [0.5]\end{aligned}$$

Question No: 8. Evaluate the integral $\int \frac{4x^2}{x^4-1}dx$ **[4]**

We have

$$\frac{4x^2}{x^4-1} = \frac{1}{x-1} - \frac{1}{x+1} + \frac{2}{x^2+1} \ [2]$$

Then

$$\begin{aligned}\int \frac{4x^2}{x^4-1}dx &= \int \frac{1}{x-1}dx - \int \frac{1}{x+1}dx + \int \frac{2}{x^2+1}dx \\ &= \ln|x-1| - \ln|x+1| + 2\tan^{-1}(x) + C \ [2]\end{aligned}$$

Question No: 9. Evaluate the integral $\int \frac{1}{\sqrt{3+2x-x^2}}\,dx$ **[4]**

$$\int \frac{1}{\sqrt{3+2x-x^2}}\,dx = \int \frac{1}{\sqrt{4-(x-1)^2}}\,dx \ [1]$$

we have

$$u = x-1 \text{ then } du = dx \ [1]$$

and then

$$\begin{aligned}\int \frac{1}{\sqrt{3+2x-x^2}}\,dx &= \int \frac{1}{\sqrt{4-u^2}}\,du \\ &= \sin^{-1}(\frac{u}{2}) + C \ [1] \\ &= \sin^{-1}(\frac{x-1}{2}) + C \ [1]\end{aligned}$$

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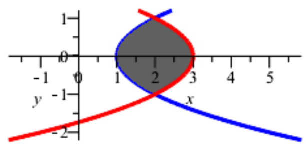

Question No: 10. Sketch the region R bounded by $x = y^2 + 1$, $x = 3 - y^2$. And find the area of R. **[4]**

Graph [1]

The area R bounded by $x = y^2 + 1$, $x = 3 - y^2$ is given by

$$\begin{aligned} R &= \int_{-1}^{1} ((3 - y^2) - (y^2 + 1))dy \ [2] \\ &= \int_{-1}^{1} (2 - 2y^2)dy = \frac{8}{3} \ [1] \end{aligned}$$