Grading scheme

① a) $\ln y = \dfrac{\ln \cos x}{\sin^2 x}$ (1)

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{-\sin x}{2\cos^2 x . \sin x} = -\frac{1}{2} \quad (1,5)$$

so $\lim_{x \to 0} y = e^{-\frac{1}{2}}$ (0.5)

b) $\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx$ (1)

$= x \tan x + \ln|\cos x| + C$ (1)

c) $\displaystyle\int \frac{dx}{x\sqrt{9-x^4}} = \frac{1}{2} \int \frac{2x \, dx}{x^2 \sqrt{9-(x^2)^2}}$ $u = x^2$ (1)

$= -\frac{1}{6} \operatorname{sech}^{-1}\left(\frac{x^2}{3}\right) + C$ (2)

2)

(2) a) $\int \sec^6 x \tan^4 x \, dx = \int (1+\tan^2 x)^2 \tan^4 x \sec^2 x \, dx$

$u = \tan x$ $\qquad = \int (1+u^2)^2 u^4 \, du \quad (1,5)$

$= \int (u^4 + 2u^2 + 1) u^4 \, du$

$(1.5) \qquad = \frac{\tan^9 x}{9} + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$

b) $x = 2\sin\theta \qquad dx = 2\cos\theta \, d\theta \quad (1)$

$\int \frac{x^2}{(4-x^2)^{3/2}} dx = \int \frac{4\sin^2\theta . 2\cos\theta \, d\theta}{8\cos^3\theta}$

$= \int \tan^2\theta \, d\theta = \int (\sec^2\theta - 1) \, d\theta \quad (1)$

$= \tan\theta - \theta + C$

$= \frac{x}{\sqrt{\cancel{x}4 - x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C \quad (1)$

c) $\int \frac{dx}{\sqrt{x^2+4x+3}} = \int \frac{dx}{\sqrt{(x+2)^2 - 1}} \quad (1)$

$= \cosh^{-1}(x+2) + C \quad (1)$

($x+2 = \sec\theta$ is also ok)

③ a)

$$\frac{x+1}{(x-2)(x^2-3x+2)} = \frac{x+1}{(x-1)(x-2)^2}$$

(1.5) $$= \frac{2}{x-1} - \frac{2}{x-2} + \frac{3}{(x-2)^2}$$

$$\int \frac{x+1}{(x-2)(x^2-3x+2)}\,dx = 2\ln|x-1| - 2\ln|x-2| - \frac{3}{x-2} + C$$

(1,5)

b) Set $u = \sqrt{x}$ (1)

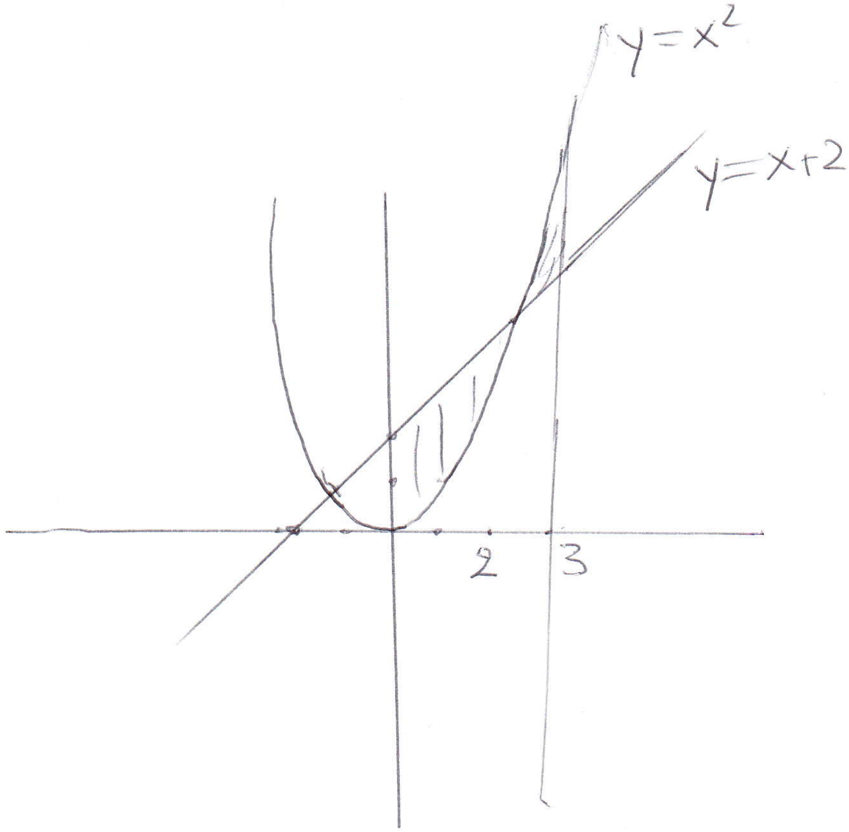
$$\int_0^c e^{-\sqrt{x}}\,dx = \int_0^{\sqrt{c}} 2u e^{-u}\,du$$

$$= 2\left[-ue^{-u} - e^{-u}\right]_0^{\sqrt{c}}$$ (1)

$$= 2\left(-\sqrt{c}\,e^{-\sqrt{c}} - e^{-\sqrt{c}} + 1\right) \xrightarrow[c \to +\infty]{} 2$$

So $\int_0^{+\infty} e^{-\sqrt{x}}\,dx$ converges and $\int_0^{+\infty} e^{-\sqrt{x}}\,dx = 2$ (1)

4)



(1)

intersection points

$$x^2 = x+2 \iff (x-2)(x+1)=0$$

$$x=2 \text{ or } x=-1 \qquad (0.5)$$

$$A = \int_0^2 x+2-x^2\,dx + \int_2^3 x^2-x-2\,dx$$

$$= \left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_0^2 + \left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_2^3$$

$$= \frac{10}{3}+\frac{11}{6} = \frac{31}{6}. \qquad (1.5)$$