Midterm1106 Time: 90mn

Question 1(2+3+3)

a) Let $F(x) = sinhx. \int_0^{2x} (1+t^2)^5\, dt$ .Compute $F'(0)$

b) Evaluate the indefinite integral $\int_0^{\pi/4} \frac{(1+tanx)^6}{cos^2x} dx$

c) Find the values of z that satisfy the integral mean value theorem for the function $f(x) = \sqrt{x+2}$ on [-1, 2].

Question 2(3+2)

a) Approximate the integral $\int_0^1 \sqrt{1+x^2}\, dx$ using the trapezoid rule with n=4

b) If $f(x) = log_{10}(cosh^{-1}(2x))$, find $f'(x)$ for $x > 1/2$.

Question 3(3+3)

a) Use logarithmic differentiation to find y' if $y = \frac{x^x . \sqrt[3]{1+4x}}{sin^{-1}x}$,$0 < x < 1$

b) Evaluate the integral $\int \frac{2xln(1+x^2)dx}{1+x^2}$

Question 4(3+3)

a) Compute the integral $\int \frac{3^x dx}{2+3^{2x}}$

b) Find $\int \frac{dx}{\sqrt{9-e^{6x}}}$, with $x < \frac{ln3}{3}$

106Midterm1 solutions(Sem1-37/38)

Question1 a) $F'(x) = coshx.\int_0^{2x}(1+t^2)^5dt + 2sinhx(1+4x^2)^5$ (1)

$F'(0) = 0$ (1)

b) $\int_0^{\pi/4}\frac{(1+tanx)^6}{cos^2x}dx = \int_0^{\pi/4}(1+tanx)^6sec^2xdx$ (1)

$= \int_1^2 u^6du \quad u = 1+tanx, du = sec^2xdx$ (1)

$= [u^7/7]_1^2 = \frac{1}{7}(2^7-1)$ (1)

c) $\int_{-1}^{2}\sqrt{x+2}\,dx = \frac{2}{3}[(x+2)^{\frac{3}{2}}]_{-1}^{2} = \frac{14}{3}$ (1)

$\frac{14}{9} = \sqrt{c+2}$ (1) so $c = \frac{34}{81}$ (1)

Question2 a) $x_0 = 0, x_1 = \frac{1}{4}, x_2 = \frac{1}{2}, \; x_3 = \frac{3}{4}, x_4 = 1$ (1)

$T_4 = \frac{1}{8}\left(1 + 2\sqrt{\frac{17}{16}} + 2\sqrt{\frac{5}{4}} + 2\frac{5}{4} + \sqrt{2}\right)$ (1)

$T_4 \approx \frac{1}{8}(1 + 2.061553 + 2.236068 + 2.5 + 1.414213) \approx 1.151479$

(1)

b) $f'(x) = \frac{(cosh^{-1}(2x))'}{ln10.cosh^{-1}(2x)} = \frac{2}{ln10.cosh^{-1}(2x).\sqrt{4x^2-1}}$ (1) + (1)

Question3 a) $y = \frac{x^x.\sqrt[3]{1+4x}}{sin^{-1}x}$

$lny = xlnx + \frac{1}{3}ln(1+4x) - ln(sin^{-1}(x))$ (1)

$$\frac{y'}{y} = lnx + 1 + \frac{4}{3(1+4x)} - \frac{1}{sin^{-1}(x).\sqrt{1-x^2}} \quad (1,5)$$

$$y' = \left(lnx + 1 + \frac{4}{3(1+4x)} - \frac{1}{sin^{-1}(x).\sqrt{1-x^2}} \right) y \quad (0,5)$$

b) $\int \frac{2xln(1+x^2)dx}{1+x^2} = \int udu \qquad u = \ln(1+x^2) \quad du = \frac{2xdx}{1+x^2} \quad (2)$

$$= \frac{u^2}{2} + C = \frac{1}{2}(\ln(1+x^2))^2 + C \quad (1)$$

Question3

a) $\int \frac{3^x dx}{2+3^{2x}} = \frac{1}{ln3}\int \frac{du}{2+u^2} \qquad u = 3^x \text{ , } du = ln3.3^x dx \ (2)$

$$= \frac{1}{\sqrt{2}ln3} tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C = \frac{1}{\sqrt{2}ln3} tan^{-1}\left(\frac{3^x}{\sqrt{2}}\right) + C$$

(1)

b) $\int \frac{dx}{\sqrt{9-e^{6x}}} = \frac{1}{3}\int \frac{du}{u\sqrt{3^2-u^2}} \qquad u = e^{3x} \ dx = \frac{du}{3u} \quad (2)$

$$= -\frac{1}{9} sech^{-1}\left(\frac{e^{3x}}{3}\right) + C \quad (1)$$