

**Final Exam, M-106, TM I, (2015-16), Department of Mathematics,  
King Saud University**

**1. [2+2 Marks]**

1.1) Find  $\frac{dy(x)}{dx}$  where  $y(x) = (5^x + 5^{-x})^5$ .

1.2) Evaluate the indefinite integral  $\int x^2 \sinh(x^3) dx$ .

**2. [3+3 Marks]**

2.1) Determine whether the improper integral  $\int_1^{\infty} (1-x)e^{-x} dx$  converges or diverges, and evaluate it if it converges.

2.2) Evaluate the indefinite integral  $\int \cos^{11}(x) \sin^3(x) dx$ .

**3. [3+3+3 Marks]**

3.1) Evaluate the indefinite integral  $\int \frac{5x^2+20x+6}{x^3+2x^2+x} dx$ .

3.2) Sketch the region bounded by the graphs of the equations  $x = 3 - y^2$ ,  $x = y + 1$ , and find its area.

3.3) Evaluate the indefinite integral  $\int \frac{x^2}{(1+x^2)^2} dx$ .

**4. [3+3 Marks]**

4.1) Evaluate the indefinite integral  $\int \frac{x}{\sqrt{6x+1}} dx$ .

4.2) Evaluate the indefinite integral  $\int \frac{1}{\sqrt{8+2x-x^2}} dx$ .

**5. [3+3 Marks]**

5.1) Sketch the region  $R$  bounded by the graphs of the equations

$y = \sqrt{x}$ ,  $y = x^2$ , and find the volume of the solid generated by revolving  $R$  about the  $x$ -axis.

5.2) Find the arc length of the graph of the equation  $y(x) = \ln(\cos(x))$

from  $x = 0$  to  $x = \frac{\pi}{4}$ .

**6. [5+4 Marks]**

6.1) Sketch the region which is outside the graph of the equation  $r = 3$  and inside the graph of the equation  $r = 2 + 2 \sin(\theta)$ , and find its area.

6.2) Sketch the graph of the equation  $r = 2 \cos(\theta)$ , and find the area of the surface generated by revolving it about the line  $\phi = \frac{\pi}{2}$ .