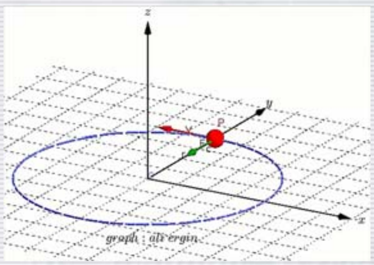
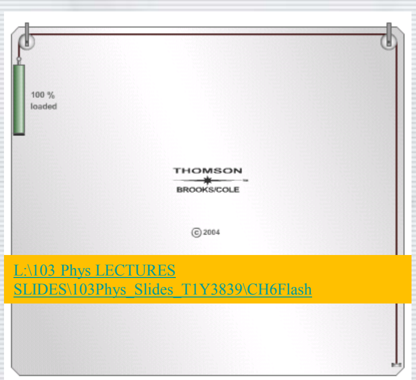
Chapter 6

NEWTON'S 2nd LAW

AND

UNIFORM CIRCULAR MOTION





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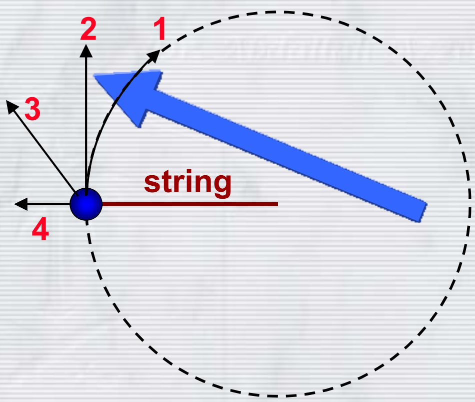
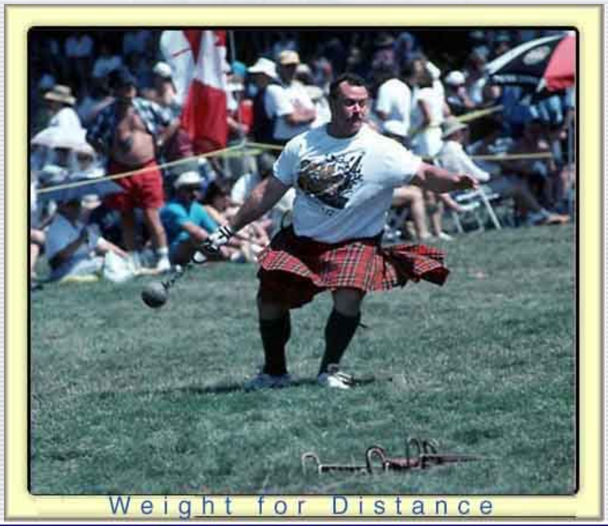


Question: A ball attached to the end of a string is whirled in a horizontal plane. At the point indicated, the string breaks. Looking down on the ball from above, which path does it take?

Thus far we have applied Newton's law, $F = ma$ to linear motion.

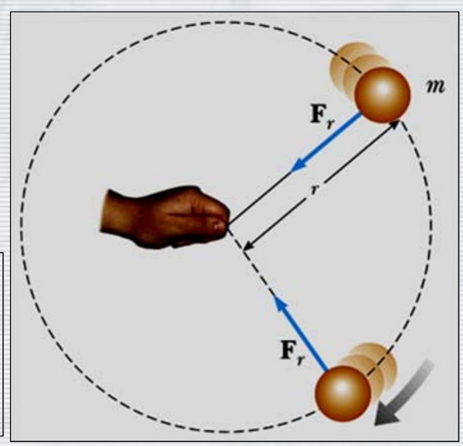
Now we'll apply it to rotational motion



Particle moving with uniform speed v in a circular path with radius r has an acceleration a_c:

$$a_c = \frac{v^2}{r}$$

(Derivation: see Chapter 4.4)



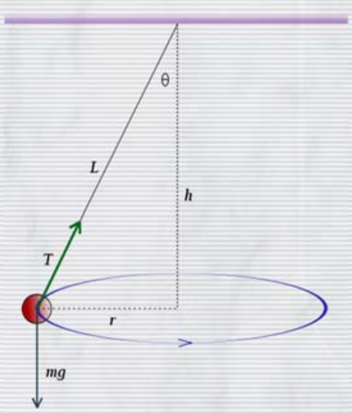
- The acceleration points towards the center of the circle!
- Centripetal acceleration

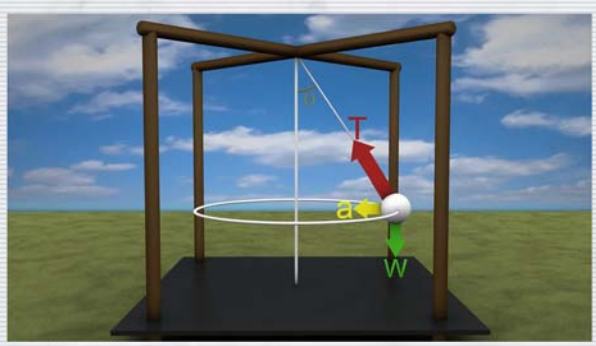
Newton's law along the radial direction (along r):

$$\sum F = m a_c = m \frac{v^2}{r}$$

The direction of the centripetal force points toward the center of the circle.

Conical Pendulum

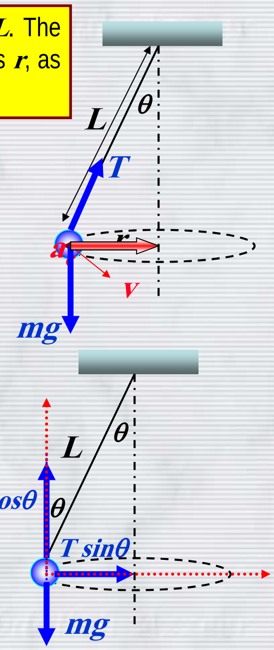




Example 6.2 Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolve with constant speed v in a horizontal circle of radius r, as shown in the Figure. Find an expression for v

Where is the acceleration vector?



FREE BODY DIAGRAM

$$\sum F_x = m a_x = m \frac{v^2}{r}$$

$$T \sin\theta = \frac{m v^2}{r} \qquad (1)$$

$$\sum F_y = m a_y = 0$$

$$T \cos\theta - m g = 0$$

$$T \cos\theta = m g \qquad (2)$$

$$\tan\theta = \frac{v^2}{g r}$$

$$v = \sqrt{r g \tan\theta}$$

$\because\ r = L \sin\theta$

$\therefore\ v = \sqrt{L g \sin\theta \cos\theta}$

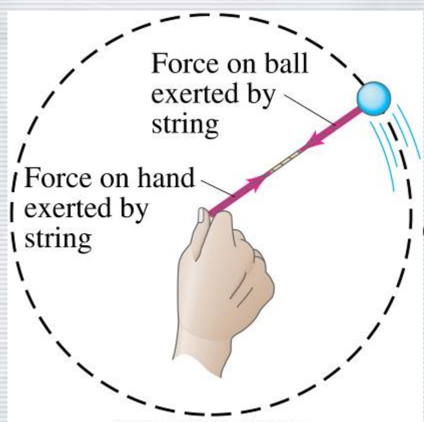
EXAMPLE 6.3

A ball of mass 0.5 kg is attached to the end of a string 1.5 m long and whirled in a horizontal plane at constant speed. If the cord can withstand a maximum tension of 50 N, what is the maximum speed of the ball before the string breaks?

$$\sum F_{hor} = m a_c = \frac{m v^2}{r}$$

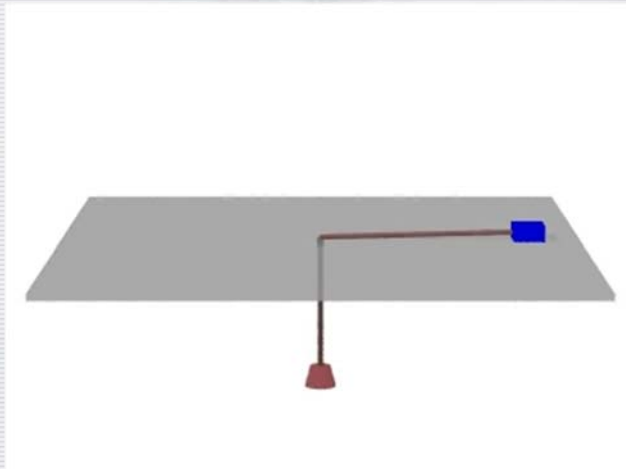
$$T = \frac{m v^2}{r}$$

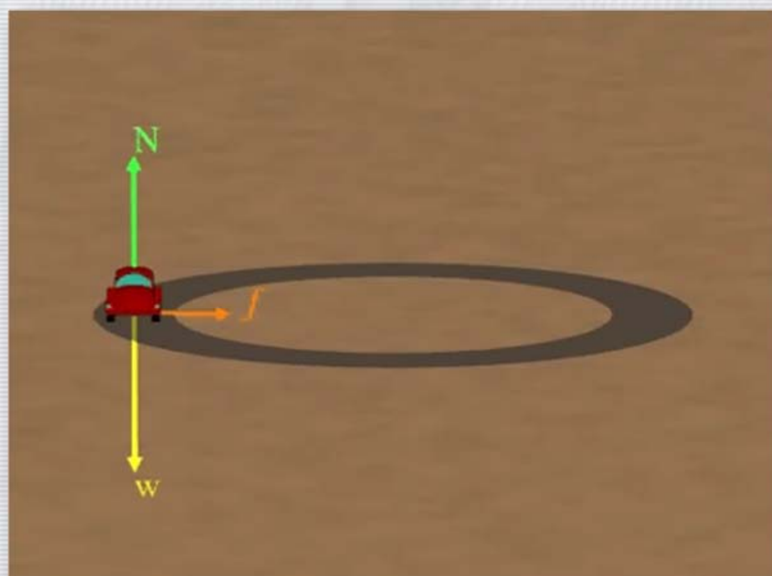
$$v_{max} = \sqrt{\frac{T_{max} r}{m}} = 12.2\ m/s$$



What happen when increase the length of the cord
Read the rest of the example

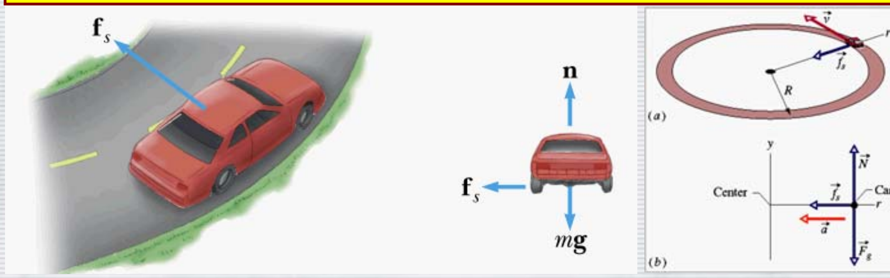
TRY to solve





EXAMPLE 6.4 what is the max speed

A car takes a bend on a flat, horizontal road. If the radius of the bend is 35 m and the coefficient of static friction between the tires and dry pavement is 0.5, what maximum speed can the car safely have?



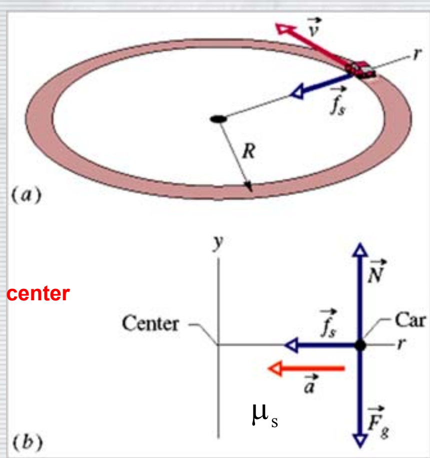
$$\sum F = f_s = m a_c = m\frac{v^2}{r} \quad \Rightarrow f_{s,\max} = m\frac{v_{\max}^2}{r}$$

$$f_{s,\max} = \mu_s n = \mu_s mg$$

$$v_{\max} = \sqrt{\frac{f_{s,\max} r}{m}} = \sqrt{\mu_s g r} = \sqrt{150\, m^2/s^2} = 13.1\ m/s \cong 47 \text{ km/hr}$$

Sample Problem

A car of mass m = 1600 kg traveling at a constant speed v = 20 m/s around a flat, circular track of radius R = 190 m. For what value of μ_s between the track and the tires of the car will the car be on the verge of sliding off the track?



$$f_s = m\frac{v^2}{R}$$

Positive direction towards the center

$$= \mu_s mg$$

$$\mu_s = \frac{mv^2}{mgR} = \frac{v^2}{gR}$$

$$= \frac{(20\, m/s)^2}{(9.8\, m/s^2)(190\, m)} = 0.21$$

In real situations, why is a heavier car less slippery ?

Circular Motion Animation

"Why do I travel to the right inside of a car when the car turns left?"

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by Tony Wayne

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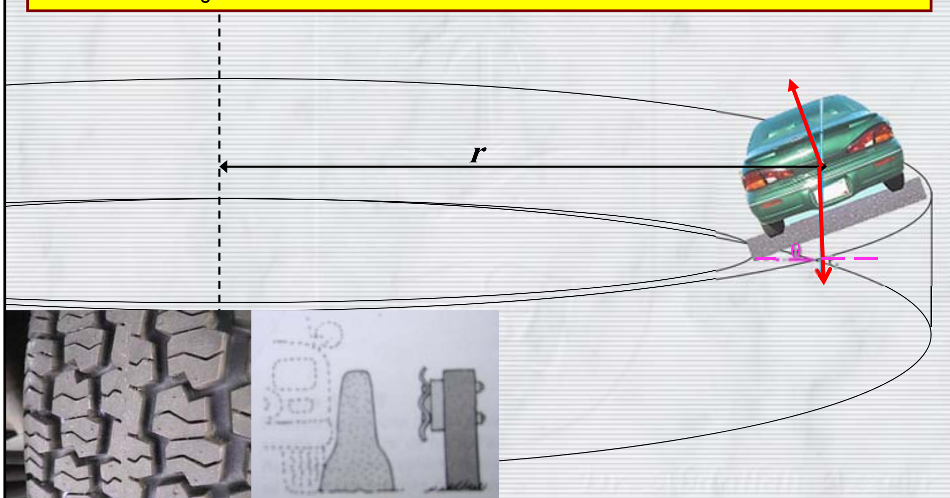
EXAMPLE 6.5 The banked exit ramp

A civil engineer wishes to design a curved exit ramp for a highway in such a way that the car will not have to rely on friction to round the curve without skidding. Suppose the designated speed for the ramp is to be 48 km/hr (13.4 m/s) and the radius of the curve is 50 m. At what angle should the curve be banked?



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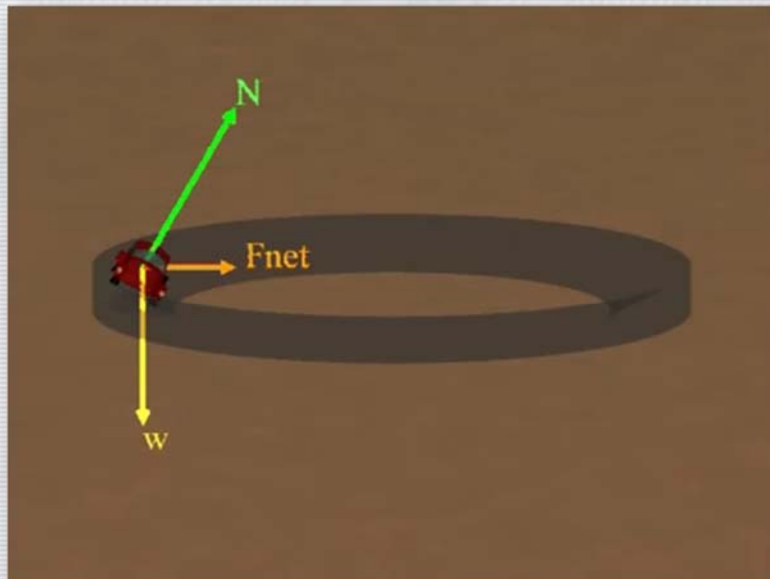


$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$n\ \sin\theta = \frac{mv^2}{r} \qquad (1)$$

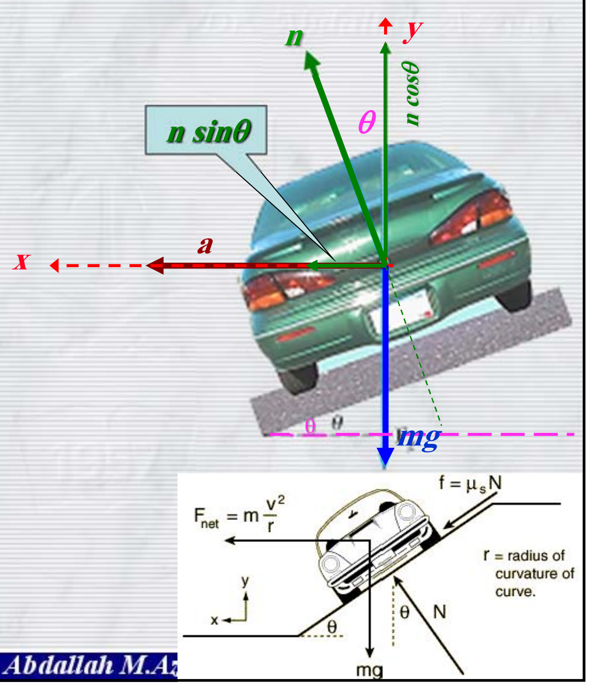
$$\sum F_y = ma_y = 0$$

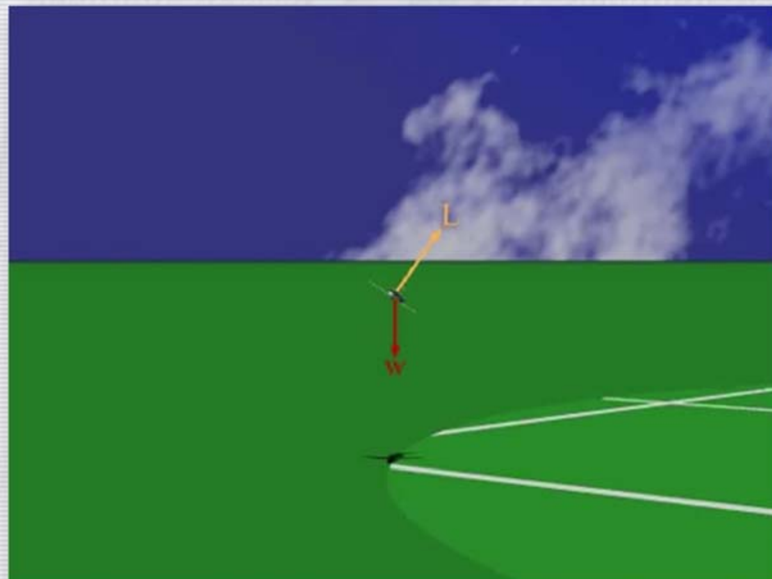
$$n\ \cos\theta - mg = 0$$

$$n\ \cos\theta = mg \qquad (2)$$

$$(1)/(2) \Rightarrow \quad \tan\theta = \frac{v^2}{rg}$$

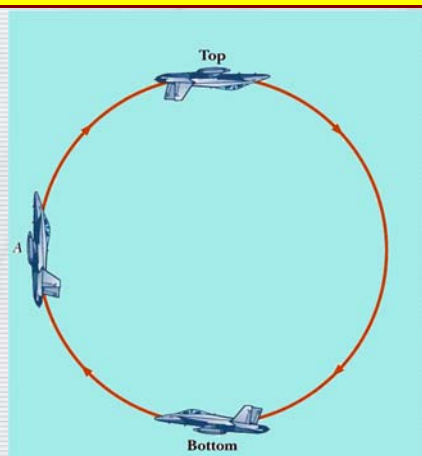
$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = 20.1^{\circ}$$





EXAMPLE 6.6 Let's Go Loop-the-Loop

A pilot of mass m in a jet aircraft executes a loop-the-loop, as shown in the Fig. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot (A) at the bottom of the loop and (B) at the top of the loop. Express your answer in terms of the weight of the pilot mg.



(a)

At Bottom:

$$\sum F = ma_c = m\frac{v^2}{r}$$

$$n_B - mg = \frac{mv^2}{r}$$

$$n_B = \frac{mv^2}{r} + mg = mg\left(1 + \frac{v^2}{rg}\right)$$

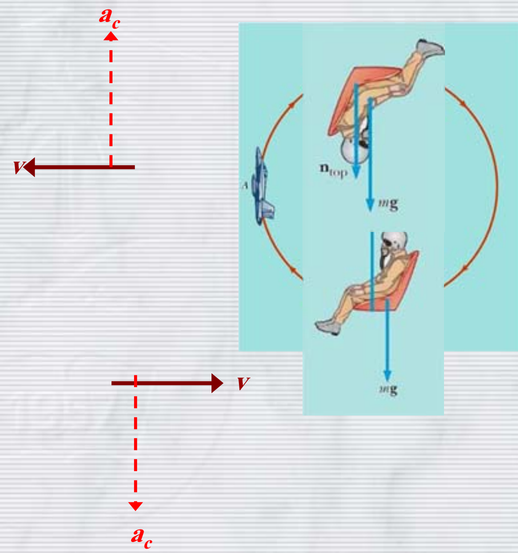
$$= 2.91\ mg$$

At Top:

$$n_T + mg = \frac{mv^2}{r}$$

$$n_T = \frac{mv^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$

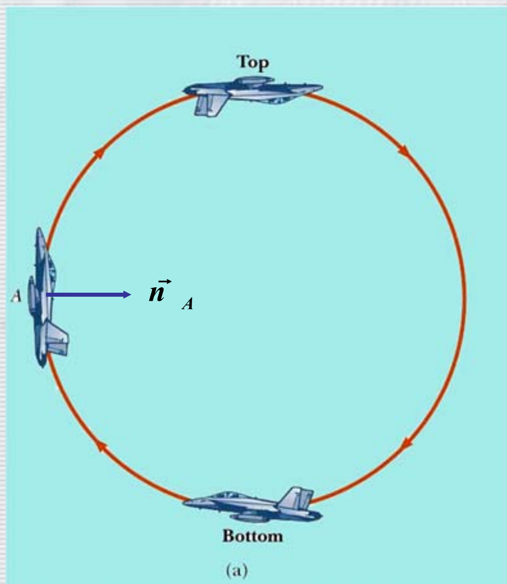
$$= 0.913\ mg$$



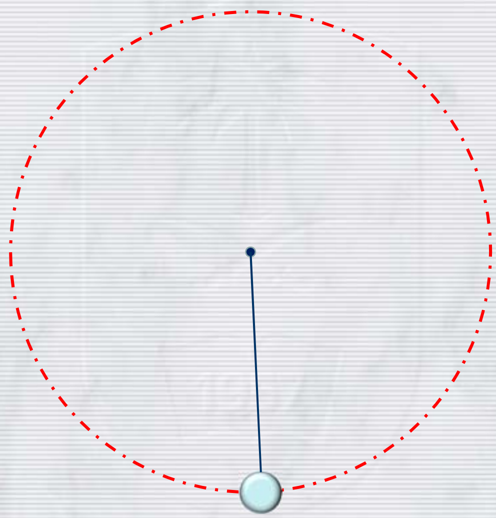
Q: what is n at point A

At A-point:

$$n_A = \frac{mv^2}{r}$$



A Mass moving in a vertical circle



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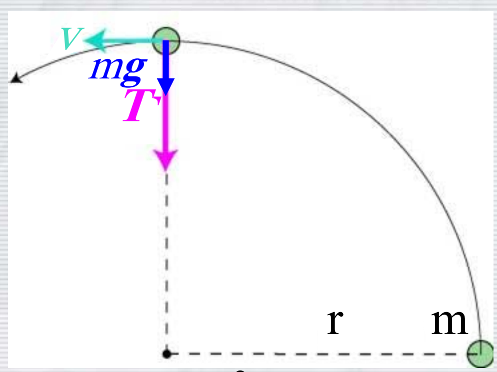
Problem: A ball (m = 0.5 kg) on the end of a r =1 m long string swings in a vertical plane. At the ***top*** of its swing its speed is v = 4.5 m/s, and the tension T in the string is closest to…

1) Zero
2) 5 N
3) 10 N
4) 15 N
5) 20 N

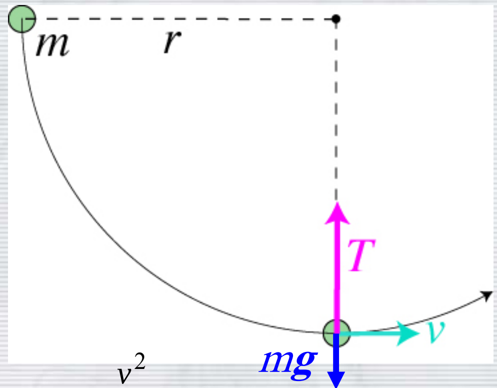


$$\sum F_y = T + mg = ma_r = m\frac{v^2}{r}$$

$$\Rightarrow T = m\frac{v^2}{r} - mg \quad \Rightarrow Could\ be\ zero!$$

Problem: A ball (m = 0.5 kg) on the end of a r =1 m long string swings in a vertical plane. At the ***bottom*** of its swing its speed is v = 4.5 m/s, and the tension T in the string is closest to…

1) Zero
2) 5 N
3) 10 N
4) 15 N
5) 20 N



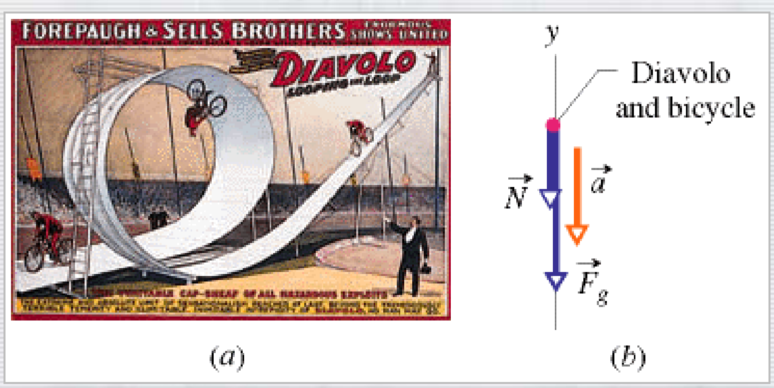
$$\Sigma F_y = T - mg = ma_r = m\frac{v^2}{r}$$

$$\Rightarrow T = m\frac{v^2}{r} + mg \quad \Rightarrow T \geq mg$$

$$= 4.9\,\text{N} + 10.1\,\text{N} = 15.0\,\text{N}$$

Sample Problem

In a 1901 circus performance, Allo "Dare Devil" Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. 6-10a). Assuming that the loop is a circle with radius R = 2.7 m, what is the least speed v Diavolo could have at the top of the loop to remain in contact with it there?



(a) (b)

SOLUTION:

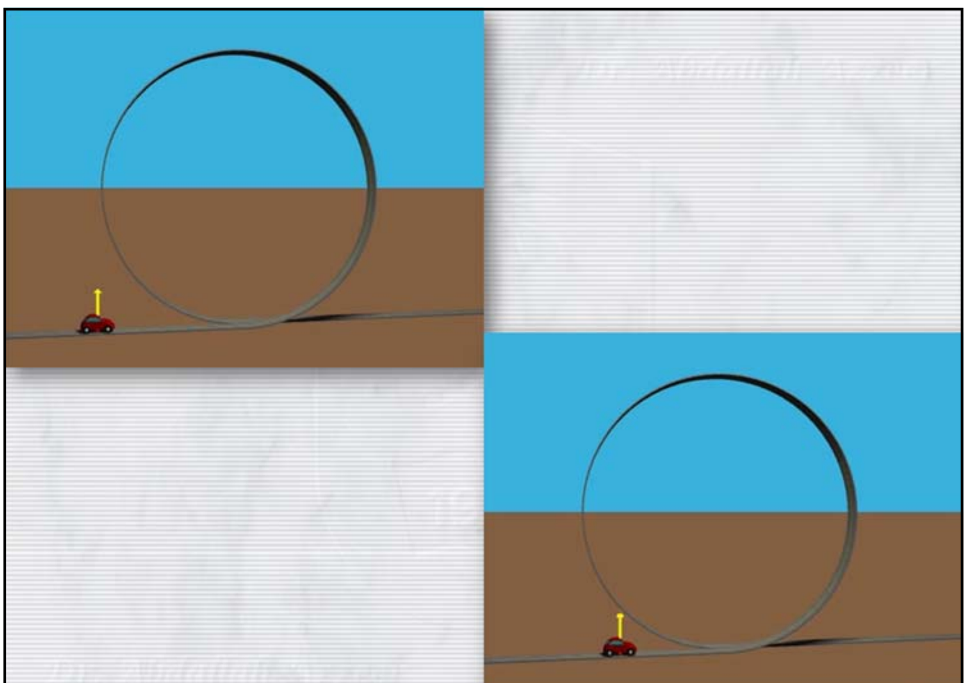
$$N + mg = m\left(\frac{v^2}{R}\right)$$

Positive direction towards center

If N=0, then

$$v = \sqrt{gR} = \sqrt{(9.8\,m/s^2)(2.7\,m)}$$
$$= 5.1\,m/s$$

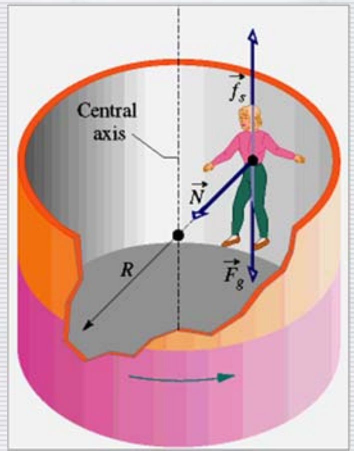
Therefore, he must maintain at least 5.1 m/s at the top of the loop. Otherwise, he'll fall off the track.



Sample Problem

Even some seasoned roller-coaster riders blanch at the thought of riding the Rotor, which is essentially a large, hollow cylinder that is rotated rapidly around its central axis.

Before the ride begins, a rider enters the cylinder through a door on the side and stands on a floor, up against a canvas-covered wall. The door is closed, and as the cylinder begins to turn, the rider, wall, and floor move in unison. When the rider's speed reaches some predetermined value, the floor abruptly and alarmingly falls away.



The rider does not fall with it but instead is pinned to the wall while the cylinder rotates, as if an unseen (and somewhat unfriendly) agent is pressing the body to the wall. Later, the floor is eased back to the rider's feet, the cylinder slows, and the rider sinks a few centimeters to regain footing on the floor. (Some riders consider all this to be fun.)

Suppose that the coefficient of static friction μ_s between the rider's clothing and the canvas is 0.40 and that the cylinder's radius R is 2.1 m.

(a) What minimum speed v must the cylinder and rider have if the rider is not to fall when the floor drops?

SOLUTION:

$$f_s - mg = 0$$

$$\mu_s N - mg = 0 \qquad N = \frac{mg}{\mu_s}$$

$$N = m\frac{v^2}{R}$$

Positive direction towards the center

$$v = \sqrt{\frac{g\,R}{\mu_s}} = \sqrt{\frac{(9.8m/s^2)(2.1\,m)}{0.40}} = 7.17\,m/s \approx 7.2\,m/s$$

(b) If the rider's mass is 49 kg, what is the magnitude of the centripetal force on her?

$$N = m\frac{v^2}{R} = (49\,kg)\frac{(7.17\,m/s)^2}{2.1\,m}$$

$$\approx 1200\,N$$

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