## Example;

A 5 kg block is placed on a frictionless inclined plane of angle $30^{\circ}$ and pushed up the plane with a horizontal force of magnitude 30 N .


What are the direction and magnitude of block's acceleration?
x соmp. $\quad \sum \boldsymbol{F}_{\boldsymbol{x}}=\boldsymbol{m a}$

$$
\begin{aligned}
& F \cos \theta-m g \sin \theta=m a \\
& \therefore \quad a=\frac{F \cos \theta-m g \sin \theta}{m}=0.3 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

## Example: Accelerometer

A weight of mass $m$ is hung from the ceiling of a car with a massless string. The car travels on a horizontal road, and has an acceleration $a$ in the $x$ direction. The string makes an angle $\theta$ with respect to the vertical $(y)$ axis. Solve for $\theta$ in terms of $a$ and $g$.


## 103 PHYS -

Draw a free body diagram for the mass:
What are all of the forces acting?


- Using components
$x: F_{X}=T_{X}=T \sin \theta=m a$
$y: F_{Y}=T_{Y}-m g$

$$
=T \cos \theta-m g=0
$$

- Eliminate $T$ :



## Accelerometer

$\tan \theta=\frac{a}{g}$

- Let's put in some numbers:
- Say the car goes from 0 to 60 mph in 10 seconds:
$-60 \mathrm{mph}=60 \times 0.45 \mathrm{~m} / \mathrm{s}=27 \mathrm{~m} / \mathrm{s}$.
- Acceleration $a=\Delta v / \Delta t=2.7 \mathrm{~m} / \mathrm{s}^{2}$.
- So $\quad a / g=2.7 / 9.8=0.28$.
$-\theta=\tan ^{-1}(a / g)=15.6$ dea


READ EXAMPLES 5.5 to 5.10
103 PHYS -
Dr. Abinllah M.Azzeer

### 5.8 The Frictional Force:

The frictional force is also one component of the force that a surface exerts on an object with which it is in contact, namely, the component PARALLEL to the surface.
$>$ The magnitude of the frictional force is related to the Normal force (how hard two objects push against one another).
$>$ The magnitude of the frictional force also depends on whether or not the object is already in motion. It is more difficult to accelerate a stationary object than it is to accelerate an object already in motion when there is friction.

Magnitude of "static" frictional force $=f_{s}^{\text {MAX }}=\mu_{s} \boldsymbol{F}_{N}$


Magnitude of "kinetic" frictional force $=\boldsymbol{f}_{\boldsymbol{k}}=\mu_{\mathbf{k}} \boldsymbol{F}_{\boldsymbol{N}}$

The following empirical laws hold true about friction:
$\Rightarrow$ Friction force, f , is proportional to normal force, n.

$$
f_{s} \leq \mu_{s} n \quad f_{k}=\mu_{k} n
$$

$\Rightarrow \mu_{\mathrm{s}}$ and $\mu_{\mathrm{k}}$ : coefficients of static and kinetic friction, respectively

$\Rightarrow$ Direction of frictional force is opposite to direction of relative motion
$\Rightarrow$ Values of $\mu_{\mathrm{s}}$ and $\mu_{\mathrm{k}}$ depend on nature of surface.

$\Rightarrow \mu_{\mathrm{s}}$ and $\mu_{\mathrm{k}}$ don't depend on the area of contact
$\Rightarrow \mu_{\mathrm{s}}$ and $\mu_{\mathrm{k}}$ don't depend on speed.

## Motion Along an Inclined Plane

## Example 5.12

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block just starts to move. Show that by measuring this critical angle, $\theta_{c}$, one can determine coefficient of static friction, $\mu_{s}$


## Net force $\quad \sum \overrightarrow{\boldsymbol{F}}=\boldsymbol{m} \boldsymbol{a}$

$x$ component $\quad \sum \boldsymbol{F}_{\mathrm{x}}=\boldsymbol{m g} \sin \theta_{\mathrm{c}}-\mathrm{f}_{\mathrm{s}}=\mathbf{0}$ (1) Just to satar or move with constant velecity
$y$ component. $\quad \sum \boldsymbol{F}_{\boldsymbol{y}}=\mathbf{n} \boldsymbol{m g} \cos \theta_{c}=\mathbf{0} \Rightarrow \boldsymbol{n}=\boldsymbol{m g} \cos \theta_{c}$ (2)

$$
\because f_{s}=\mu_{s} n
$$

Eq. (1) becomes;

$$
m g \sin \theta_{c}-\mu_{s} m g \cos \theta_{c}=0
$$

$$
\mu_{s}=\frac{m g \sin \theta_{c}}{m g \cos \theta_{c}}=\tan \theta_{c}
$$

## READ EXAMPLES 5.13 and 5.14



## More questions

The body that is suspended by a rope in the figure has a weight of 75 N .
Is $T$ equal to, greater than or less than 75 N when the body is moving downward at...
(a) increasing speed

1. less than
2. equal to
3. greater than
(b) decreasing speed
4. less than
5. greater than
6. equal to



The figure shows a train of four block being pulled across a frictionless floor by force $\boldsymbol{F}$. What total mass is accelerated to the right by...
(a) Force $F$

1. 20
2.2
3.7
2. 10
(b) cord 3
1.8
3. 20
4. 3
5. 18

cord 1
6. 20
7. 13
8. 18
9. 10

(d) Rank the blocks according to their accelerations, greatest first.
10. all tie

11. $1,2,3,4$
12. $1,3,2,4$
13. 4,3,2,1
(e) Rank the cords according to their tensions, greatest first.
14. all tie
15. $3,2,1$
16. $1,3,2$
17. $1,2,3$

The figure shows a group of three blocks being pushed across a frictionless floor by a horizontal force $\boldsymbol{F}$. What total mass is accelerated to the right ...
(a) by force $F$

1. 17

(b) by force $\boldsymbol{F}_{21}$ on block 2 from block 1
2. 5
3. 2
4. 7
5. 12
(c) by force $\boldsymbol{F}_{32}$ on block $\mathbf{3}$ from block 2
6. 17
7. 12

8. 2
9. 10
(d) Rank the blocks according to the magnitude of their accelerations, greatest first.
10. $3,2,1$
11. $1,2,3$
12. all tie
13. $2,1,3$
(e) Rank forces $\boldsymbol{F}, \boldsymbol{F}_{21}$ and $\boldsymbol{F}_{32}$ according to their magnitude, greatest first.
14. all tie
15. $\mathrm{F}_{32}, \mathrm{~F}_{21}, \mathrm{~F}$
16. $\mathrm{F}_{21}, \mathrm{~F}, \mathrm{~F}_{32}$
17. $F, F_{21}, F_{32}$

## 103 PHYS -



$$
\begin{aligned}
& F_{x}=\mu_{k} \times F_{N} \\
& \boldsymbol{F}_{x}=\mu_{k} \times m \times\left(-a_{g r a v}\right) \\
& \boldsymbol{F}_{x}=(0.0500)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(\mathrm{m})
\end{aligned}
$$

Frictional force is in the negative $x$-direction!

$$
\begin{aligned}
& a_{x}=\frac{\boldsymbol{F}_{x}}{\boldsymbol{m}}=\frac{-(0.0500)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(m)}{m} \\
& a_{x}=-0.49 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$$
\begin{aligned}
& v_{x}^{2}=v_{0, x}^{2}+2 a x \\
& x=\frac{v_{x}^{2}-v_{0, x}^{2}}{2 a}=16.3 \mathrm{~m}
\end{aligned}
$$

## Example:

A jet travels with constant speed at an angle of 30 degrees above the horizontal. The plane has a weight whose magnitude is $\mathrm{W}=86,500 \mathrm{~N}$. The engine provides a forward thrust of magnitude $\mathrm{T}=103,000 \mathrm{~N}$. A lift force from the air (like a normal force) acts on the plane in a direction perpendicular to the wings. A drag force (like a kinetic friction) acts on the plane in a direction opposite the motion. Find the magnitude of the lift force and the drag force.

Constant Velocity $\rightarrow$ No Acceleration $\rightarrow$ Net Force is Zero


## Example (continued)

$$
\begin{aligned}
\sum F_{x} & =0=T-R-W \sin 30^{\circ} \\
R & =T-W \sin 30^{\circ} \\
& =59750 \mathrm{~N} \\
\sum F_{y} & =0=L-W \cos 30^{\circ} \\
L & =W \cos 30^{\circ} \\
& =74900 \mathrm{~N}
\end{aligned}
$$



## Previous Exams problems

The horizontal surface on which the block slides is frictionless. If $\mathrm{F}=30 \mathrm{~N}$ and $\mathrm{m}=3 \mathrm{~kg}$, what is the magnitude of the acceleration of the block?
(a) $6.4 \mathrm{~m} / \mathrm{s}^{2}$
(b) $5.7 \mathrm{~m} / \mathrm{s}^{2}$
(c) $6.1 \mathrm{~m} / \mathrm{s}^{2}$
(d) $5.3 \mathrm{~m} / \mathrm{s}^{2}$

(e) $2.8 \mathrm{~m} / \mathrm{s}^{2}$


$$
\begin{aligned}
& \sum F=m a \\
& 2 F \cos \theta-F=m a \\
& \therefore a=\frac{1}{m}(2 F \cos \theta-F)=5.32 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

A 5 kg box is pushed up a $30^{\circ}$ rough incline by a 50 N force (as shown). If the box moves 5 m at a constant speed, calculate

1) Show all the forces acting on the box.
2) Force of gravity.
3) The coefficient of friction between the box and the incline

4) The friction force
(2) $W_{g}=m g \sin \theta=24.5 \mathrm{~N}$
(3) $\sum F_{x}=\boldsymbol{m a}, v=$ constant $\Rightarrow a=0$ $F \cos \theta-m g \sin \theta-f_{k}=0$
$\therefore \quad f_{k}=F \cos \theta-m g \sin \theta=\mu_{k} N$
but $\sum F_{y}=N-F \sin \theta-m g \cos \theta=0$
$\therefore \quad N=F \sin \theta+m g \cos \theta=67.435 N$
$\Rightarrow \quad \mu_{k}=\frac{F \cos \theta-m g \sin \theta}{N}=0.279$
(4) $f_{k}=\mu_{k} N=18.8 \mathrm{~N}$

