Example;

An object is fired from the ground at 100 m/s at an angle of 30° with the horizontal

A. Calculate the horizontal and vertical components of the initial velocity

B. After 2.0 seconds, how far has the object traveled in the horizontal direction?

C. How high is the object at this point?

A.

$$v_{ix} = v_i \cos\theta = (100\ m/s)(\cos 30^0) = 87\ m/s$$

$$v_{iy} = v_i \sin\theta = (100\ m/s)(\sin 30^0) = 50\ m/s$$

B.

$$v_{ix} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x t = (87\ m/s)(2.0\ s) = 174\ m$$

C.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2} g\,(\Delta t^2) = (50\ m/s)(2.0\ s) + \frac{1}{2}(9.8\ m/s^2)(2.0\ s)^2$$

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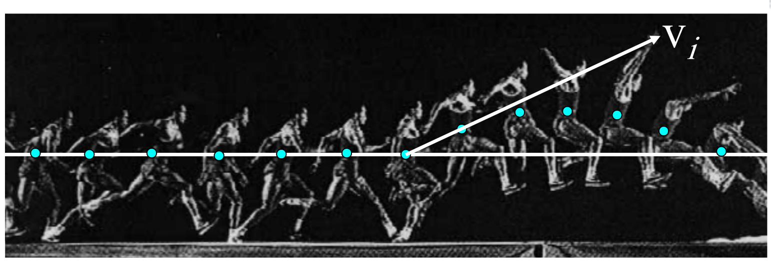
Example;

Strobe photo of Mike Powell, world record holder for running long jump.



Suppose he runs at the speed of a good sprinter, **v = 10 m/s**.

Also suppose he takes off at an angle of 25° .

What is the horizontal distance of the jump?

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horizontal distance:

$$\Delta x = \mathrm{v}_i \cos\theta \cdot t$$

get the time from …

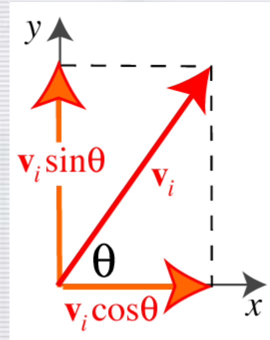
$$\Delta y = \mathrm{v}_i \sin\theta \cdot t - \frac{1}{2} g t^2$$

so for $\Delta y = 0$, $\quad t = \dfrac{2\mathrm{v}_i \sin\theta}{g}$

plug back in…

$$\Delta x = \mathrm{v}_i \cos\theta \cdot t$$

$$= \mathrm{v}_i \cos\theta \cdot \frac{2\mathrm{v}_i \sin\theta}{g} = \frac{2\mathrm{v}_i^2}{g} \cos\theta \sin\theta$$



This is only true if $\Delta y = 0$, i.e. $y_f = y_i$.

$$\Delta x = \frac{2\mathrm{v}_i^2}{g} \cos\theta \cdot \sin\theta$$

$$= \frac{2\ (10\ \mathrm{m/s})^2}{9.8\ \mathrm{m/s}^2} 0.91 \cdot 0.42 = 7.8\ \mathrm{m}$$

Not his best jump…
World record 8.95 m !

How to set the world record long jump

At what angle should Powell have jumped?

$$\Delta x = \frac{2v_i^2}{g}\cos\theta\cdot\sin\theta$$

A little calculus tells us that
cosθ·sinθ has its maximal value of 0.5 when θ = 45°.

A jump at the same speed but at 45° would give:

$$\Delta x = \frac{2v_i^2}{g}\cos\theta\cdot\sin\theta = \frac{2\ (10\ \text{m/s})^2}{9.8\ \text{m/s}^2}0.5 = 10.2\ \text{m}$$

More than a meter **over** the world record!!!

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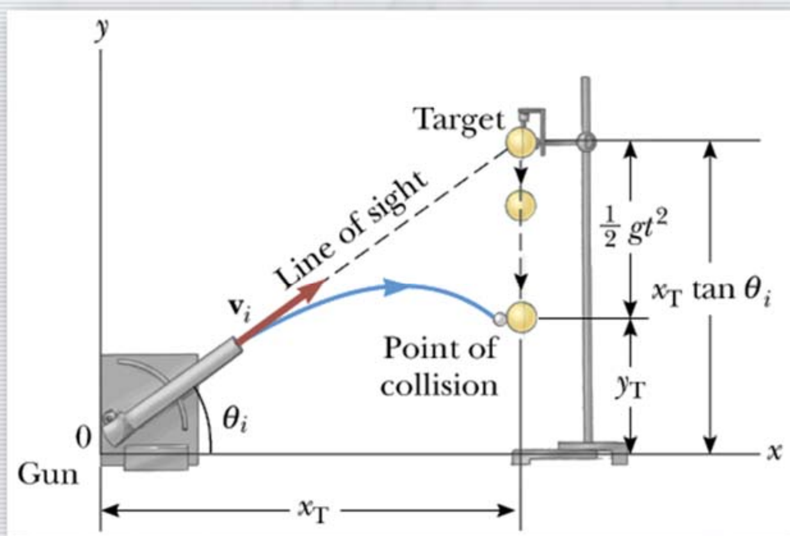
Example :Hitting the bull's eye. How's that?



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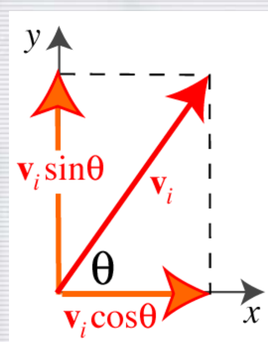
PROJECTILE MOTION

Neglecting air resistance…

Projectiles follow ***parabolic*** trajectories:



Parabolic proof

$$v_x = v_i \cos\theta \text{ remains constant so}$$

$$\Delta x = x = v_i \cos\theta \cdot t \Rightarrow t = \frac{x}{v_i \cos\theta}$$

$$\left.\begin{aligned} v_{yi} &= v_i \sin\theta \\ a_y &= -g \\ \Delta t &= t \\ \Delta y &= y = ? \end{aligned}\right\} \quad y = v_i \sin\theta \cdot t - \frac{1}{2} g \cdot t^2$$

$$\Rightarrow y = (\tan\theta)x - \left(\frac{g}{2v_i^2 \cos^2\theta}\right)x^2$$

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

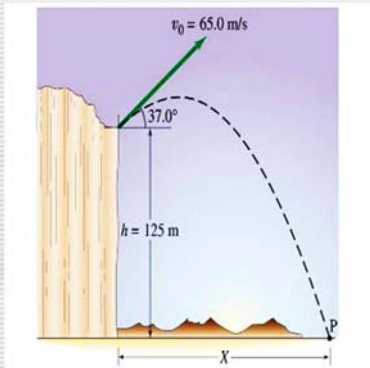
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43\,s \quad \text{or} \quad t = 10.4\,s$$

$$t = 10.4\,s$$

Since negative time does not exist.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^{\,2} + v_{yf}^{\,2}} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

Do these yourselves at home for fun!!!

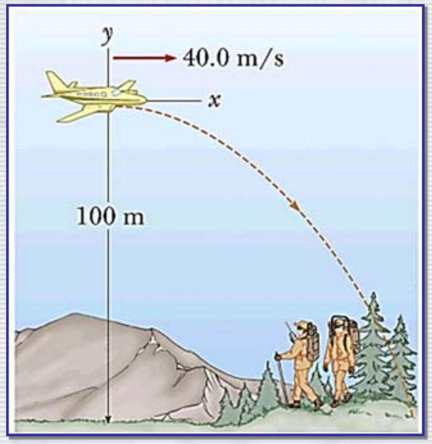
Example:

A rescue plane drops a package to stranded explorers. The plane is traveling horizontally at 40.0 m/s and is 100 m above the ground.

A. Where does the package strike the ground relative to the point at which it was released.

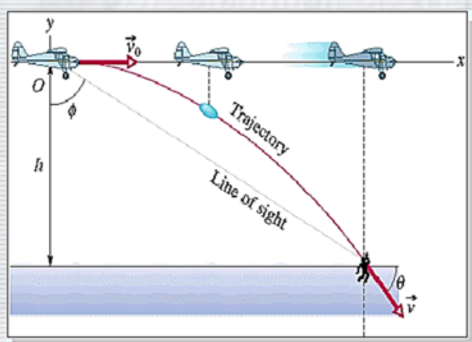
B. What are the horizontal and vertical components of the velocity of the package just before it hits the ground? What is the speed of the package as it hits the ground?

C. Where is the plane when the package hits the ground? (Assume that the plane does not change its speed or course.)



Sample Problem 4-6

In the Fig., a rescue plane flies at 198 km/h (= 55.0 m/s) and a constant elevation of 500 m toward a point directly over a boating accident victim struggling in the water. The pilot wants to release a rescue capsule so that it hits the water very close to the victim.



(a) What should be the angle φ of the pilot's line of sight to the victim when the release is made?

Solution

$$\varphi = \tan^{-1}\frac{x}{h}$$

$$x - x_0 = (v_0 \cos\theta_0)t$$

$$y - y_0 = (v_0 \sin\theta_0)t - \frac{1}{2}g t^2$$

$$-500\,m = (55.0\,m/s)(\sin 0^\circ)t - \frac{1}{2}(9.8\,m/s^2)t^2$$

Solving for t, we find $t = \pm\ 10.1$ s (take the positive root).

$$x - 0 = (55.0\,m/s)(\cos 0^\circ)(10.1\,s)$$

$$x = 555.5\,m$$

$$\varphi = \tan^{-1}\frac{555.5\,m}{500\,m} = 48^\circ$$

(b) As the capsule reaches the water, what is its velocity $\vec{V}$ in unit-vector notation and as a magnitude and an angle?

When the capsule reaches the water,

$$v_x = v_0 \cos\theta_0 = (55.0\,m/s)(\cos 0^\circ) = 55.0\,m/s$$

$$\begin{aligned} v_y &= v_0 \sin\theta_0 - g\,t \\ &= (55.0\,m/s)(\sin 0^\circ) - (9.8\,m/s^2)(10.1\,s) \\ &= -99.0\,m/s \end{aligned}$$

$$\vec{v} = (55.0\,m/s)\hat{i} - (99.0\,m/s)\hat{j}$$

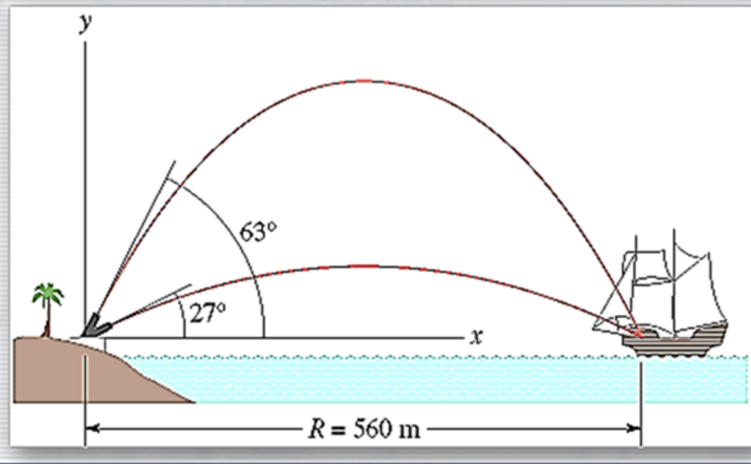
$$v = 113\,m/s \quad and \quad \theta = -61^\circ$$

Sample Problem

Figure shows a pirate ship 560 m from a fort defending the harbor entrance of an island. A defense cannon, located at sea level, fires balls at initial speed v_0 = 82 m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

(b) How far should the pirate ship be from the cannon if it is to be beyond the maximum range of the cannonballs?



SOLUTION:

$(a)\ R = \frac{v_0^2}{g} \sin 2\theta_0$ Which gives

$$2\theta_0 = \sin^{-1} \frac{g\,R}{v_0^2} = \sin^{-1} \frac{(9.8\,m/s^2)(560\,m)}{(82\,m/s)^2} = \sin^{-1} 0.816$$

There are two solutions

$$\theta_0 = \frac{1}{2}(54.7^\circ) \approx 27^\circ$$

$$\theta_0 = \frac{1}{2}(125.3^\circ) \approx 63^\circ$$

(b) Maximum range is :-

$$R = \frac{v_0^2}{g} \sin 2\theta_0 = \frac{(82\,m/s)^2}{9.8\,m/s} \sin(2\times 45^\circ)$$

$$= 686\,m \approx 690m.$$

The maximum range is 690m. Beyond that distance, the ship is safe from the cannon.

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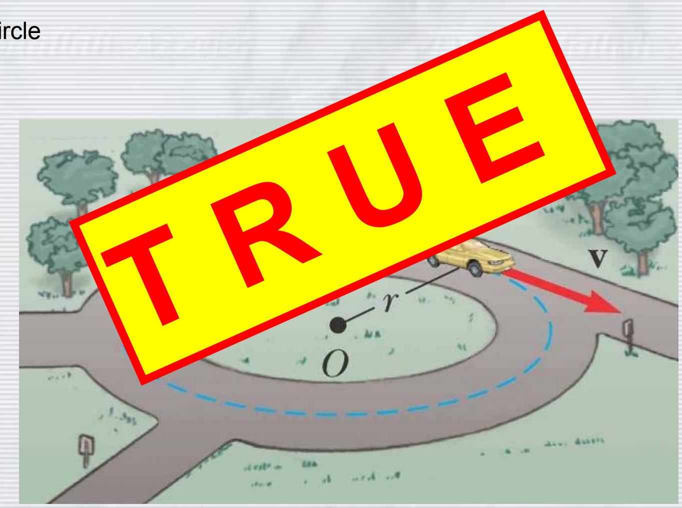
Car in a circle

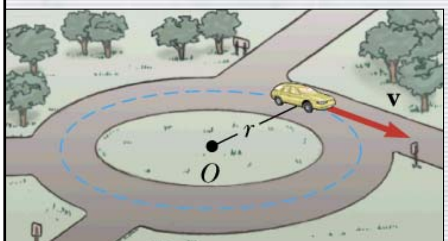


TRUE OR FALSE? **– A car moving in a circle at constant speed is accelerated.**

UNIFORM CIRCULAR MOTION

→ Motion in a circular path at constant speed.

- Velocity is changing, thus there is an acceleration!!
- Acceleration is perpendicular to velocity
- **Centripetal** acceleration is towards the center of the circle
- Magnitude of acceleration is $\boxed{a_r = \frac{v^2}{r}}$
- ***r*** is radius of circle



UNIFORM CIRCULAR MOTION

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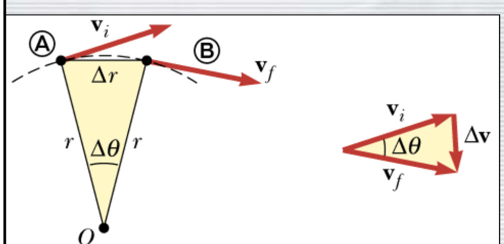
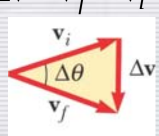


Acceleration is in the same direction as the change in velocity.

$$\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$$



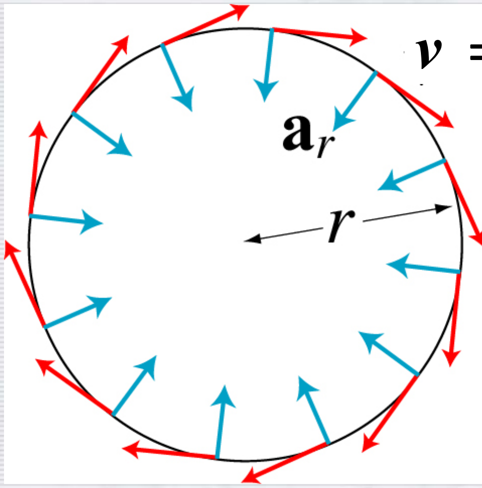
$$a = \frac{\Delta \mathbf{v}}{\Delta t}$$

An object moving in a circle at constant speed has an instantaneous acceleration, which …

… is called the ***radial*** or ***centripetal*** acceleration, …

… and which points towards the ***center*** of the circle.

UNIFORM CIRCULAR MOTION

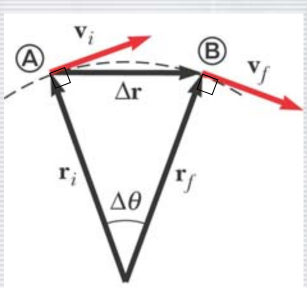


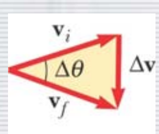
$$v = \frac{\Delta r}{\Delta t}$$

Velocity is always tangent to the circle, hence perpendicular to the radius.

Acceleration derivation

$$\textit{Similar triangles: } \frac{\Delta v}{v} = \frac{\Delta r}{r} \Rightarrow \Delta v = \frac{v\,\Delta r}{r}$$





$$a_r = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{r} \cdot \frac{\Delta r}{\Delta t} = \frac{v^2}{r}$$

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EXAMPLE;
Approximately what is the centripetal acceleration of someone standing at the Earth's equator?
A. 7×10^{-5} m/s^2
B. 0.03 m/s^2
C. 9.8 m/s^2
D. 3×10^{8} m/s^2
E. No idea

$$v = \frac{2\pi r}{T} = \frac{2\pi(6.4\times10^6\ m)}{24\times60\times60\ s} = 465\ m/s$$

$$a = \frac{v^2}{r} = \frac{(465\ m/s)^2}{6.4\times10^6\ m} = 0.034\ m/s^2$$

$$r_{earth} = 6.4\times10^6\ m$$

EXAMPLE;
If a particular San Francisco hill has a radius of curvature of 20 m, what speed does the driver have to reach at the top of the hill in order to become airborne?

$$\frac{v^2}{r} = g \Rightarrow$$

$$v = \sqrt{r \cdot g} = \sqrt{20\ m \cdot 9.8\ m/s^2} = 14\ m/s$$

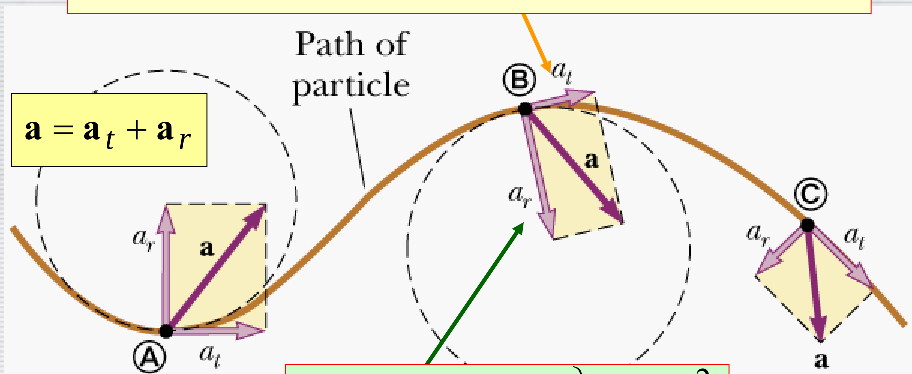
$$= 14\ m/s\ (3600\ s/hr)(1\ km/1000\ m)$$

$$= 50\ km/hr$$

MOTION ALONG CURVED PATH WITH CHANGING SPEED

Magnitude of tangential acceleration $\Big\}\ a_t = \lim_{\Delta t \to 0} \frac{\Delta Speed}{\Delta t} = \frac{d|\mathbf{v}|}{dt}$

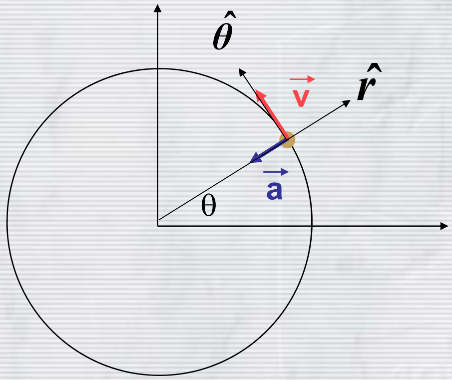
$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r$



Magnitude of radial acceleration $\Big\}\ a_r = \frac{v^2}{r}$

Radial and Tangential Quantities

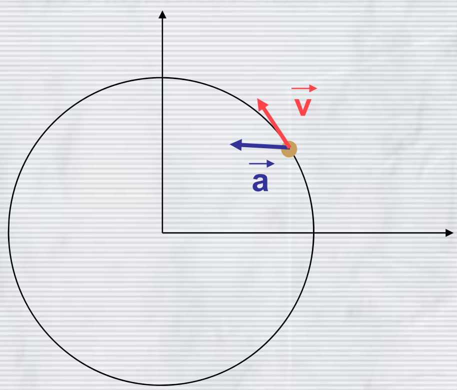
For **uniform** circular motion



$$\vec{v} = 0\,\hat{r} + v\,\hat{\theta}$$

$$\vec{a} = a\,\hat{r} + 0\,\hat{\theta}$$

What about **non-uniform** circular motion ?

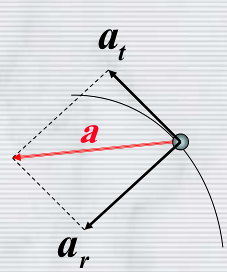


$$\vec{v} = 0\,\hat{r} + v\,\hat{\theta}$$

$$\vec{a} = a_r\,\hat{r} + a_t\,\hat{\theta}$$

a_t is along the direction of motion

$$\boxed{a_t = \frac{d\,|\vec{v}|}{dt}}$$



a_r is perpendicular to the direction of motion

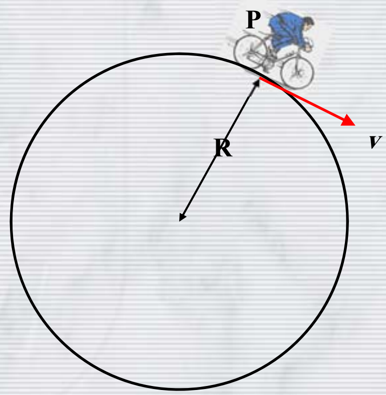
$$\boxed{a_r = \frac{v^2}{r}}$$

READ EXAMPLE 4.9

More Example:

(*a*) When at point P on a circular track of radius 50 m, a bicyclist has a speed of 5 m/s. In units of m/s^2, the magnitude of his centripetal acceleration is closest to...

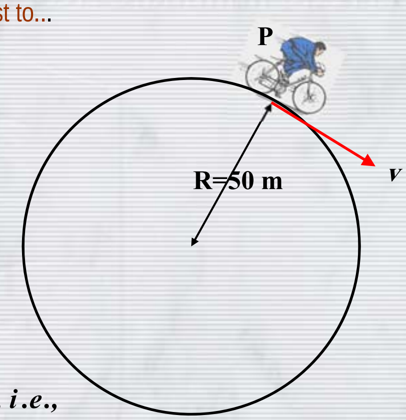
A. 0.1
B. 0.5
C. 10
D. 55
E. 250



$$a_r = \frac{v^2}{r} = \frac{(5\ m/s)^2}{50\ m} = 0.50\ \frac{m}{s^2}$$

(*b*) He increases his speed at a constant rate during the next lap, and 44.9 s later is back at P. His average speed for the lap was closest to...

A. 6 m/s
B. 7 m/s
C. 9 m/s
D. 12 m/s
E. 15 m/s



$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{2\pi R}{\Delta t} = \frac{2\pi \times 50\ m}{44.9\ s} = 7.0\ \frac{m}{s},$$

where Δs is the distance around one lap, i.e.,

the circumference of the circular track.

(c) His *new* instantaneous speed at P is closest to...

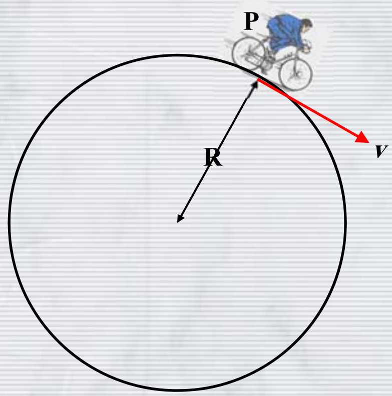
A. 6 m/s

B. 7 m/s

C. 9 m/s

D. 12 m/s

E. 15 m/s



Because he increases his speed at a constant rate:

$$\bar{v} = \frac{v_i + v_f}{2} \Rightarrow v_f = 2\bar{v} - v_i = (2 \times 7.0 - 5.0)\frac{m}{s} = 9.0\frac{m}{s}$$

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(d) While he is speeding up, his tangential acceleration in m/s^2 is closest to...

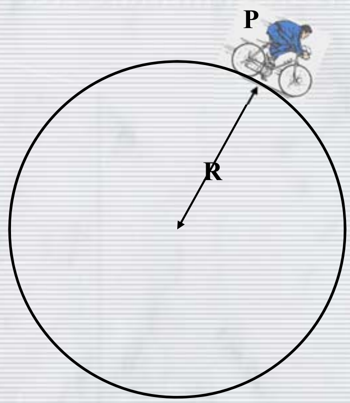
A. 0.09

B. 0.21

C. 0.34

D. 0.38

E. 0.55



$$a_t = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t} = \frac{(9.0 - 5.0)\ m/s}{44.9\ s} = 0.089\frac{m}{s^2}$$

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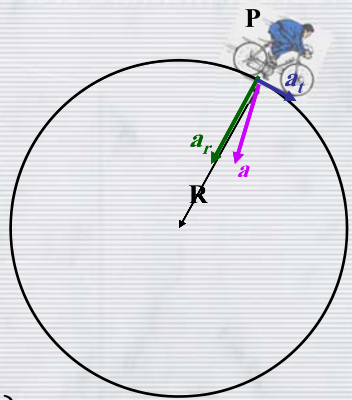
(*e*) When her velocity is 6.0 m/s the magnitude of her total acceleration in m/s² is closest to...

A. 0.1

B. 0.2

C. 0.3

D. 0.7



$$a_r = \frac{v^2}{r} = \frac{(6.0\ m/s)^2}{50\ m} = 0.720\ \frac{m}{s^2}$$

$$a_t = 0.089\ \frac{m}{s^2}$$

$$\left.\begin{array}{l} a_r = 0.720\ \frac{m}{s^2} \\ a_t = 0.089\ \frac{m}{s^2} \end{array}\right\} a = \sqrt{a_r^2 + a_t^2} = 0.725\ \frac{m}{s^2}$$