

103 PHYS Formula Sheet

Motion in One Dimension

$$\text{average speed} \equiv \frac{\text{total distance}}{\text{total time}}$$

$$\Delta x \equiv x_f - x_i$$

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t}$$

$$v_x \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t}$$

$$a_x \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$v_{xf} = v_{xi} + a_x t$$

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$

$$x_f - x_i = \bar{v}_x t = \frac{1}{2} (v_{xi} + v_{xf}) t$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i)$$

Vectors

$$A_x = A \cos q$$

$$A_y = A \sin q$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$q = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

Motion in Two Dimensions

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\Delta \vec{r} \equiv \vec{r}_f - \vec{r}_i$$

$$\vec{v}_{avg} \equiv \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j}$$

$$\bar{a}_{avg} \equiv \frac{\Delta \vec{v}}{\Delta t}$$

$$\bar{a} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a}t^2$$

$$y = (\tan q_i) x - \left(\frac{g}{2v_i^2 \cos^2 q_i} \right) x^2$$

$$h = \frac{v_i^2 \sin^2 q_i}{2g} \quad R = \frac{v_i^2 \sin 2q_i}{g}$$

$$a_r = \frac{v^2}{r}, \quad a_t = \frac{dv}{dt}$$

$$\vec{a} = \frac{d|\vec{v}|}{dt} \hat{q} - \frac{v^2}{r} \hat{r}$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

$$\vec{a}' = \vec{a}$$

The Laws of Motion

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_g = m\vec{g}$$

$$f_k = \mu_k n$$

$$f_s \leq \mu_s n$$

$$F_{spring} = -kx$$

Circular Motion

$$\sum F_r = ma_r = \frac{mv^2}{r}$$

Work and Kinetic Energy

$$W \equiv Fd \cos q$$

$$\vec{A} \cdot \vec{B} \equiv AB \cos q = A_x B_x + A_y B_y + A_z B_z$$

$$W = \int_{x_i}^{x_f} F_x dx$$

$$K \equiv \frac{1}{2} mv^2$$

$$\sum W = \Delta K$$

$$K_i + \sum W_{other} - f_k d = K_f$$

$$\bar{P} \equiv \frac{W}{\Delta t}$$

$$P \equiv \lim_{\Delta t \rightarrow 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Potential Energy and Energy Conservation

$$\Delta U = U_f - U_i \equiv - \int_{x_i}^{x_f} F_x dx$$

$$U_g = mgy$$

$$U_s = \frac{1}{2} kx^2$$

$$E \equiv K + U$$

$$K_i + \sum U_i = K_f + \sum U_f$$

$$W_{app} = \Delta K + \Delta U$$

$$F_x = - \frac{dU}{dx}$$

Linear Momentum and Collisions

$$\vec{p} \equiv m\vec{v}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$\vec{I} \equiv \int_{t_i}^{t_f} \vec{F} dt = \Delta \vec{p}$$

$$\vec{F}_{avg} = \frac{\vec{I}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}$$

$$\vec{r}_{CM} \equiv \frac{\sum m_i \vec{r}_i}{M}$$

$$\vec{v}_{CM} \equiv \frac{d\vec{r}_{CM}}{dt} = \frac{\sum m_i \vec{v}_i}{M}$$

$$\vec{a}_{CM} \equiv \frac{d\vec{v}_{CM}}{dt} = \frac{\sum m_i \vec{a}_i}{M}$$

$$\sum \vec{F}_{ext} = M\vec{a}_{CM} = \frac{d\vec{p}_{tot}}{dt}$$

Rotation About a Fixed Axis

$$s = r\theta$$

$$\vec{\omega} \equiv \frac{\Delta \vec{q}}{\Delta t}$$

$$\vec{\omega} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{q}}{\Delta t} = \frac{d\vec{q}}{dt}$$

$$v = \omega r$$

$$\bar{a} \equiv \frac{\Delta \omega}{\Delta t}$$

$$\mathbf{a} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

$$a_t = \frac{dv}{dt} = r\alpha$$

$$a_r = \frac{v^2}{r} = \omega^2 r$$

$$\mathbf{w}_f = \mathbf{w}_i + \mathbf{a}t$$

$$\mathbf{q}_f = \mathbf{q}_i + \mathbf{w}_i t + \frac{1}{2} \mathbf{a} t^2$$

$$\mathbf{q}_f = \mathbf{q}_i + \frac{1}{2} (\mathbf{w}_i + \mathbf{w}_f) t$$

$$\mathbf{w}_f^2 = \mathbf{w}_i^2 + 2\mathbf{a}(\mathbf{q}_f - \mathbf{q}_i)$$

$$I = \sum m_i r_i^2$$

$$I = I_{CM} + MD^2$$

$$K_R = \frac{1}{2} I \omega^2$$

$$\mathbf{t} \equiv rF \sin \theta = Fd$$

$$\sum \mathbf{t}_{ext} = I\alpha$$

$$\mathbf{P} = \mathbf{t}\omega$$

$$\sum W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

Rolling Motion and Angular Momentum

$$K = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} M v_{CM}^2$$

$$v_{CM} = R\omega$$

$$\bar{a} \equiv \frac{d\bar{\omega}}{dt}$$

$$\bar{\boldsymbol{\tau}} \equiv \bar{\mathbf{r}} \times \bar{\mathbf{F}}$$

$$\bar{\mathbf{L}} \equiv \bar{\mathbf{r}} \times \bar{\mathbf{p}}$$

$$|\bar{\mathbf{A}} \times \bar{\mathbf{B}}| = AB \sin \theta$$

$$L_z = I\omega$$

$$\sum \bar{\boldsymbol{\tau}}_{ext} = \frac{d\bar{\mathbf{L}}}{dt}$$

$$\sum \bar{\boldsymbol{\tau}}_{ext} = 0 \Rightarrow \bar{\mathbf{L}} = \text{constant}$$

Static Equilibrium

$$\sum \bar{\mathbf{F}} = 0$$

$$\sum \bar{\boldsymbol{\tau}} = 0$$

The Law of Gravity

$$F_g = \frac{Gm_1 m_2}{r^2}$$

$$g' = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E + h)^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\bar{g} \equiv \frac{\bar{\mathbf{F}}_g}{m}$$

$$U = -\frac{Gm_1 m_2}{r}$$

$$E = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r}$$

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

Simple Harmonic Motion

$$x = A \cos(\omega t + \phi)$$

$$v = \pm \omega \sqrt{A^2 - x^2}$$

$$f = \frac{\omega}{2\pi}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{k}{I}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

$$E = \frac{1}{2} k A^2$$

Wave Motion

$$y = f(x \pm vt)$$

$$v = \sqrt{T/\mu}$$

$$y = A \sin(kx - \omega t)$$

$$v = \lambda f$$

$$k \equiv 2\pi/\lambda \quad \omega \equiv 2\pi f$$

$$P = \frac{1}{2} \mu \omega^2 A^2 v$$

Sound Waves

$$I = \frac{P_{av}}{A} = \frac{P_{av}}{4\pi r^2}$$

$$b = 10 \log(I/I_0)$$

$$f' = \left(\frac{v \pm v_o}{v \mp v_s} \right) f \quad \left(\begin{array}{l} \text{upper : approach} \\ \text{lower : recession} \end{array} \right)$$

$$\sin \theta = \frac{v}{v_s}$$

Superposition and Standing Waves

$$\Delta r = \frac{f}{2\pi} \lambda$$

$$f_n = n \frac{v}{2L} \quad n = 1, 2, 3, \dots$$

$$f_n = n \frac{v}{4L} \quad n = 1, 3, 5, \dots$$

$$f_b = |f_1 - f_2|$$

Physical Constants

$$g = 9.80 \text{ m/s}^2$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$

$$M_{Earth} = 5.98 \times 10^{24} \text{ kg}$$

$$R_{Earth} = 6.37 \times 10^6 \text{ m}$$

$$N_A = 6.02 \times 10^{23}$$

$$I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$$