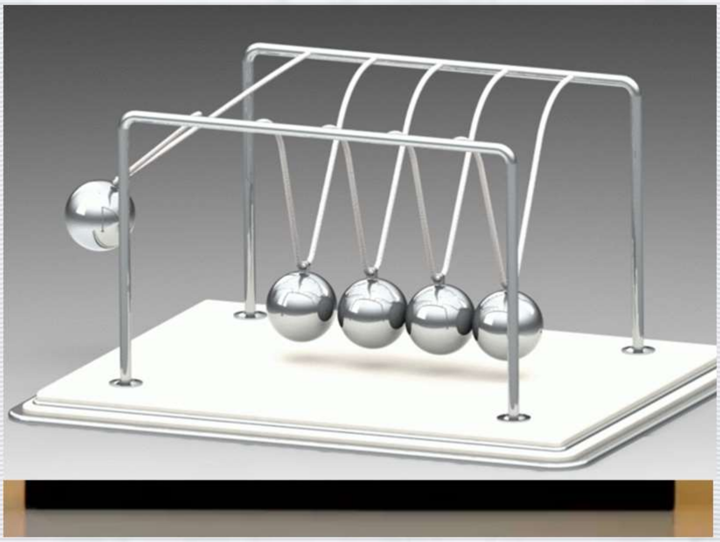
103 Phys 1



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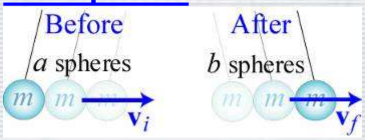
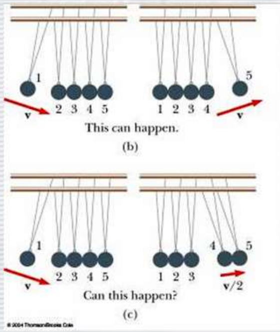
Example 9.5





Momentum: $amv_i = bmv_f$

$av_i = bv_f \quad (1)$

KE: $\frac{1}{2}amv_i^2 = \frac{1}{2}bmv_f^2$

$av_i^2 = bv_f^2 \quad (2)$

Must conserve both momentum and KE to understand results

$$\frac{a^2v_i^2}{av_i^2} = \frac{b^2v_f^2}{bv_f^2} \qquad \frac{(1)^2}{(2)}$$

$$a = b$$

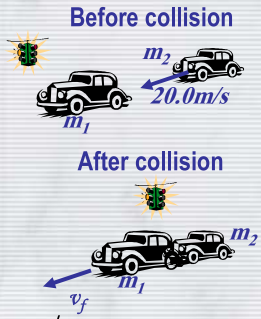
Example 9.6

A car of mass 1800 kg stopped at a traffic light is rear-ended by a 900 kg car, and the two become **entangled**. If the lighter car was moving at 20.0 m/s before the collision what is the velocity of the entangled cars after the collision?

The momenta before and after the collision are

$$\vec{p}_i = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = 0 + m_2\vec{v}_{2i}$$

$$\vec{p}_f = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = (m_1 + m_2)\vec{v}_f$$



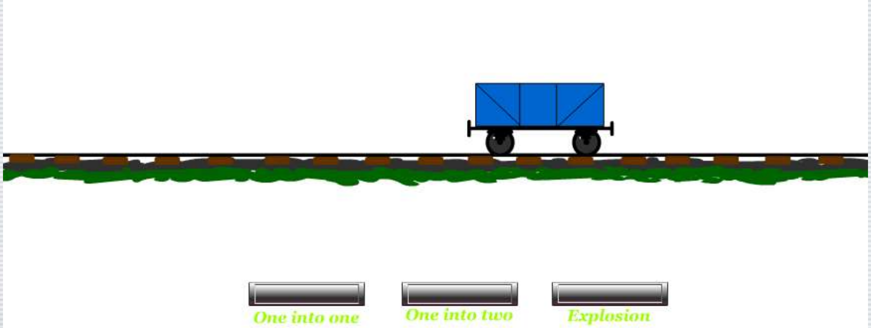
Since momentum of the system must be conserved

$$\vec{p}_i = \vec{p}_f \Rightarrow (m_1 + m_2)\vec{v}_f = m_2\vec{v}_{2i}$$

$$\vec{v}_f = \frac{m_2\vec{v}_{2i}}{(m_1 + m_2)} = \frac{900 \times 20.0\vec{i}}{900 + 1800} = 6.67\vec{i} \ \ m/s$$

READ THE SECOND PART

MOMENTUM CONSERVATION in a COLLISION



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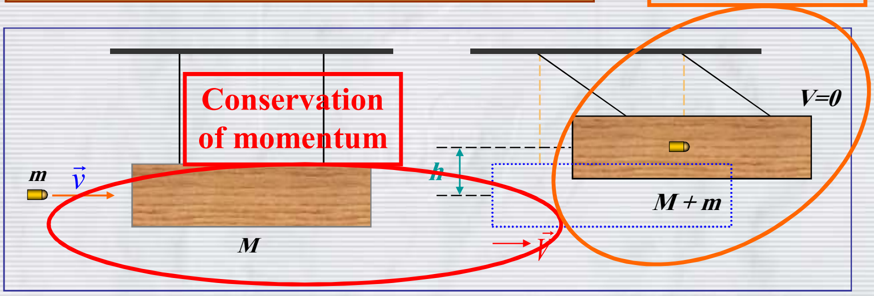
Examples 9.7

The ballistic pendulum uses conservation laws to find the velocity of a bullet

A gun is fired horizontally into a wooden block suspended by strings (see Fig.). The bullet stops in the block, which rises 0.2 m. The mass of the bullet is 0.03 kg, and the mass of the block is 2 kg. (a) What was the velocity of the block just after the bullet stopped in it? (b) What was the velocity of the bullet before it struck the block?



Two stage process:

The first is the rapid "collision" of the bullet and the block.

The second is the subsequent rise of the block plus the embedded bullet.

Stage 1: Momentum is conserved

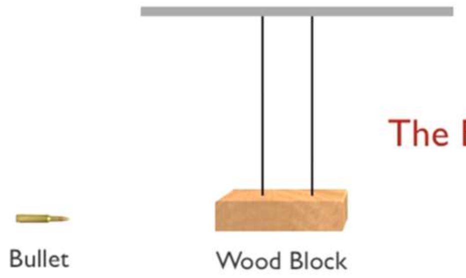
$$mv = (m + M)V \qquad V = \left(\frac{m}{m+M}\right)v$$

Stage 2: K+U Energy is conserved

$$(E_i = E_f)$$

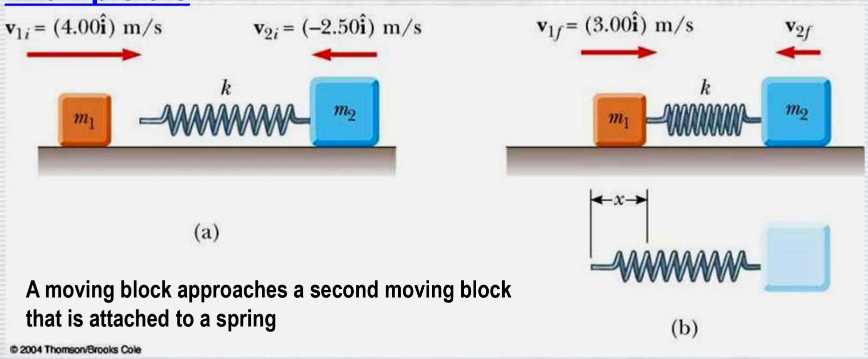
$$\tfrac{1}{2}(m+M)V^2 = (m+M)gh \Rightarrow V^2 = 2gh \Rightarrow V = \sqrt{2gh} = 1.98 \text{ m/s}$$

Eliminating **V** gives: $v = \left(1 + \frac{M}{m}\right)\sqrt{2gh} = 134 \text{ m/s} = 482 \text{ km/hr}$



The Ballistic Pendulum

Example 9.8



A moving block approaches a second moving block that is attached to a spring

READ & try to solve carefully

Example 9.9

READ & try to solve carefully

Sample Problem

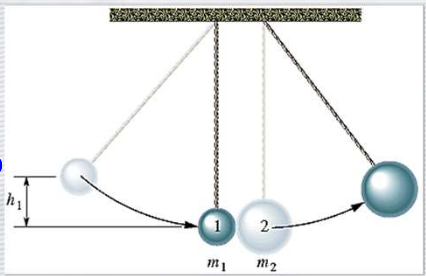
Two metal spheres, suspended by vertical cords, initially just touch, as shown in Fig. 10-15. Sphere 1, with mass m_1 = 30 g, is pulled to the left to height h_1 = 8.0 cm, and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass m_2 = 75 g. What is the velocity v_{1f} of sphere 1 just after the collision?

$$\frac{1}{2} m_1 v_{1i}^2 = m_1 g h_1$$

$$v_{1i} = \sqrt{2gh_1} = \sqrt{(2)(9.8m/s2)(0.080m)} = 1.252\ m/s$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.030\ kg - 0.075\ kg}{0.030\ kg + 0.075\ kg}(1.252\ m/s)$$

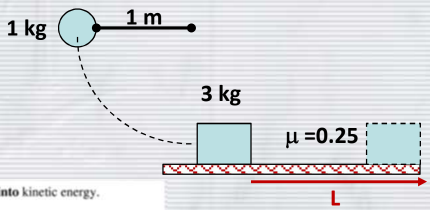
$$= -0.537\ m/s \approx -0.54\ m/s$$



The minus sign means that sphere 1 moves to the left just after the collision

A 1 kg steel ball is attached to a light weight 1 m long rod ball pivoted at the other end. The ball is released at horizontal and the strikes a 3 kg steel block elastically resting on a surface with coefficient of friction 0.25.

How far does the block travel?



First calculate the velocity of the ball as it hits the block. Potential energy **Goesinto** kinetic energy.

$mgh = (1/2)mv^2$ or $v = \sqrt{2gh} = \sqrt{2(9.8 m/s^2)1.0 m} = 4.4 m/s$

The collision is elastic and $v_{B1} = 0$, so

$$v_{B2} = \frac{2m_A}{m_A + m_B} v_{A1} = \frac{2m_A}{m_A + m_B}\sqrt{2gh} = \frac{1}{2}\sqrt{2gh} = 2.2 m/s$$

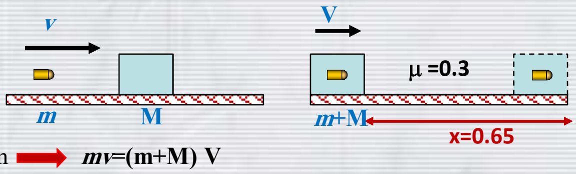
The gives the initial velocity of the struck block. The kinetic energy **Goesinto** work against friction.

$\frac{1}{2} m_B v_{B2}^2 = \mu m_B g L$ or $L = \frac{v_{B2}^2}{2\mu g} = \frac{1}{2\mu g}\frac{2gh}{4} = \frac{h}{4\mu} = \frac{1.0 m}{4 \cdot 0.25} = 1.0 m$

Ans: L = 1 m

A 6 g bullet is fired horizontally into a 2.8 kg block resting on a horizontal surface with coefficient of friction 0.3. The bullet comes to rest in the block and the block slides 0.65 m before coming to stop.

what is the velocity of the bullet?



Conservation of momentum ➡ $mv=(m+M)\,V$

Once the bullet-block combination is moving at V, the kinetic energy, $(1/2)(m+M)V^2$, **Goesinto** work to overcome friction $\mu(m+M)gx$

$$(1/2)(m+M)V^2 = \mu(m+M)gx \quad \text{or} \quad V^2/2 = \mu gx$$

Substituting

$$\frac{1}{2}\left(\frac{mv}{m+M}\right)^2 = \mu gx$$

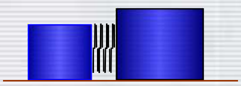
$$v = \frac{m+M}{m}\sqrt{2\mu gx} = \frac{2.806}{0.006}\sqrt{2(0.30)(9.8\,m/s^2)0.65\,m} = 914\,m/s$$

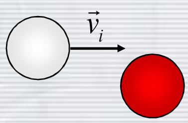
Elastic vs. Inelastic Collisions

A collision is said to be ***elastic*** when kinetic energy as well as momentum is conserved before and after the collision.

$K_{before} = K_{after}$

- Carts colliding with a spring in between, billiard balls, etc.

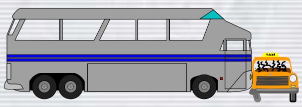




A collision is said to be ***inelastic*** when kinetic energy is not conserved before and after the collision, but momentum is conserved.

$K_{before} \neq K_{after}$

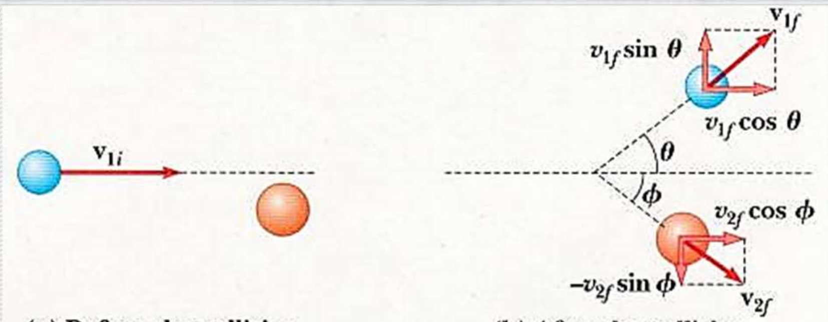
- Car crashes, collisions where objects stick together, etc.



Comment on Energy Conservation

- We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
  - Energy is lost:
    - Heat (bomb)
    - Bending of metal (crashing cars)
- Kinetic energy is not conserved since work is done during the collision!
- Momentum along a certain direction is conserved when there are no external forces acting in this direction.
  - In general, momentum conservation is easier to satisfy than energy conservation.

9.4 Two-Dimensional Collisions



(a) Before the collision (b) After the collision

$$\vec{p}_{tot} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\hat{x}: m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$

$$m_1v_{1i} + 0 = m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi$$

$$\hat{y}: m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

$$0 + 0 = m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi$$

For elastic

$$K_{tot} = \tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_1 v_{2i}^2 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_1 v_{2f}^2$$

$$\tfrac{1}{2}m_1 v_{1i}^2 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2$$

If $m_1 = m_2 = m$

$$m\vec{v}_{1i} = m\vec{v}_{1f} + m\vec{v}_{2f} \qquad \Rightarrow \vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$$

$$\tfrac{1}{2}mv_{1i}^2 = \tfrac{1}{2}m_1\left(\vec{v}_{1f} + \vec{v}_{2f}\right)^2 = \tfrac{1}{2}m\left(v_{1f}^2 + v_{2f}^2\right)$$

$$\vec{v}_{1f} \cdot \vec{v}_{2f} = 0 \qquad \Rightarrow \vec{v}_{1f} \perp \vec{v}_{2f}$$

READ PROBLEM SOLVING HINTS on page 268

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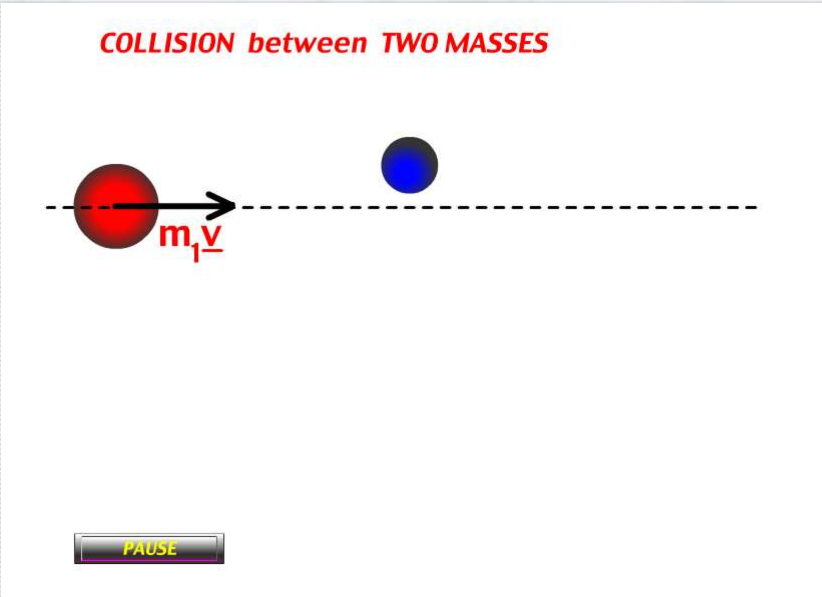

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Example 9.8

A m_c=1500 kg car traveling east at v_c= 25 m/s collides with a m_v= 2500 kg van moving north at v_v= 20 m/s. If the car and van stick together following the collision, what is their velocity (magnitude and direction) immediately after the impact?

$$\Sigma p_{yi} = \Sigma p_{yf}$$

$$(1) \quad m_v v_v = (m_c + m_v) v_f \sin\theta$$

$$\Sigma p_{xi} = \Sigma p_{xf}$$

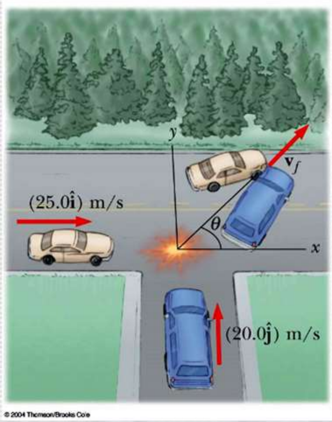
$$(2) \quad m_c v_c = (m_c + m_v) v_f \cos\theta$$

Divide Eq. (1) by Eq. (2)

$$\frac{(1)}{(2)} \quad \frac{m_v v_v}{m_c v_c} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{4}{3} \Rightarrow \theta = 53°$$

Substitute in Eq. (1):

$$v_f = \frac{m_v v_v}{(m_c + m_v)\sin\theta} = 15.6 \text{ m/s}$$



How much did the total KE change when the m_c = 1500 kg car moving at v_c = 25 m/s collided with the m_v = 2500 kg pick-up traveling at v_v 20 m/s?

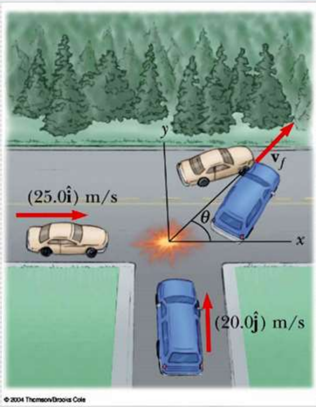
Recall that afterwards the two vehicles moved together at v_f= 15.6 m/s.

$$\Delta K = K_f - K_i$$

$$= \tfrac{1}{2}(m_c + m_v)v_f^2 - \left(\tfrac{1}{2} m_c v_c^2 + \tfrac{1}{2} m_v v_v^2\right)$$

$$= (487 - 969) \text{ kJ} = -482 \text{ kJ}$$

KE is not conserved. Almost half of it was lost!



EXAMPLE 8.1 Preliminary analysis of a collision

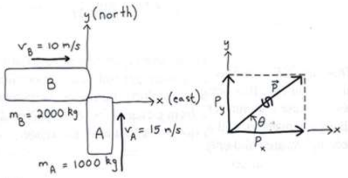
Begin 5

A small compact car with a mass of 1000 kg is traveling north on Morewood Avenue with a speed of 15 m/s. At the intersection of Morewood and Fifth Avenues, it collides with a truck with a mass of 2000 kg that is traveling east on Fifth Avenue at 10 m/s. Treating each vehicle as a particle, find the total momentum (magnitude and direction) just before the collision.

SOLUTION

SET UP Because momentum is a vector quantity, we need coordinate axes. We draw a sketch (Figure 8.2a), labeling the car A and the truck B.

$$p_{A,x} = m_A v_{A,x} = (1000\text{ kg})(0) = 0,$$
$$p_{A,y} = m_A v_{A,y} = (1000\text{ kg})(15\text{ m/s}) = 1.5 \times 10^4\text{ kg}\cdot\text{m/s},$$
$$p_{B,x} = m_B v_{B,x} = (2000\text{ kg})(10\text{ m/s}) = 2.0 \times 10^4\text{ kg}\cdot\text{m/s},$$
$$p_{B,y} = m_B v_{B,y} = (1000\text{ kg})(0) = 0$$



(a)

There are no z components of velocity or momentum.

From Equations 8.5, the components of the total momentum $\vec{P}$ are

$$P_x = m_A v_{A,x} + m_B v_{B,x} = 0 + 2.0 \times 10^4\text{ kg}\cdot\text{m/s} = 2.0 \times 10^4\text{ kg}\cdot\text{m/s},$$
$$P_y = m_A v_{A,y} + m_B v_{B,y} = 1.5 \times 10^4\text{ kg}\cdot\text{m/s} + 0 = 1.5 \times 10^4\text{ kg}\cdot\text{m/s}.$$

The total momentum $\vec{P}$ is a vector quantity with these components. Its magnitude is

$$P = \sqrt{(2.0 \times 10^4\text{ kg}\cdot\text{m/s})^2 + (1.5 \times 10^4\text{ kg}\cdot\text{m/s})^2} = 2.5 \times 10^4\text{ kg}\cdot\text{m/s}.$$

Its direction is given by the angle θ in Figure 8.2b, where

$$\tan\theta = \frac{1.5 \times 10^4\text{ kg}\cdot\text{m/s}}{2.0 \times 10^4\text{ kg}\cdot\text{m/s}} = \frac{3}{4}, \qquad \theta = 36.9^\circ.$$

REFLECT It is essential to treat momentum as a vector. When we add the momenta of the two objects we must do vector addition, using the method of components. Each vehicle has momentum in only one coordinate direction so has only one nonzero component of momentum, but the total momentum of the system has both x and y components.

Example 9.11

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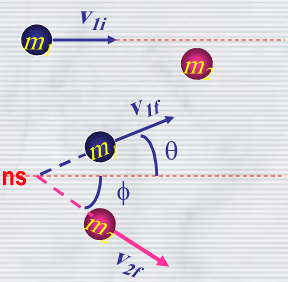
Proton #1 with a speed 3.50x10^5 m/s collides elastically with proton #2 initially at rest. After the collision, proton #1 moves at an angle of 37^o to the horizontal axis and proton #2 deflects at an angle ϕ to the same axis. Find the final speeds of the two protons and the scattering angle of proton #2, ϕ.

Since both the particles are protons $m_1=m_2=m_p$.

Using momentum conservation, one obtains

x-comp. $m_p v_{1i} = m_p v_{1f}\cos\theta + m_p v_{2f}\cos\phi$

y-comp. $m_p v_{1f}\sin\theta - m_p v_{2f}\sin\phi$



Canceling m_p and put in all known quantities, one obtains

$$v_{1f}\cos 37^\circ + v_{2f}\cos\phi = 3.50\times 10^5 \quad (1)$$

$$v_{1f}\sin 37^\circ = v_{2f}\sin\phi \quad (2)$$

From kinetic energy conservation:

$$\left(3.50\times10^{5}\right)^{2}=v_{1f}^{2}+v_{2f}^{2} \quad (3)$$

Solving Eqs. 1-3 equations, one gets

Do this at home☺

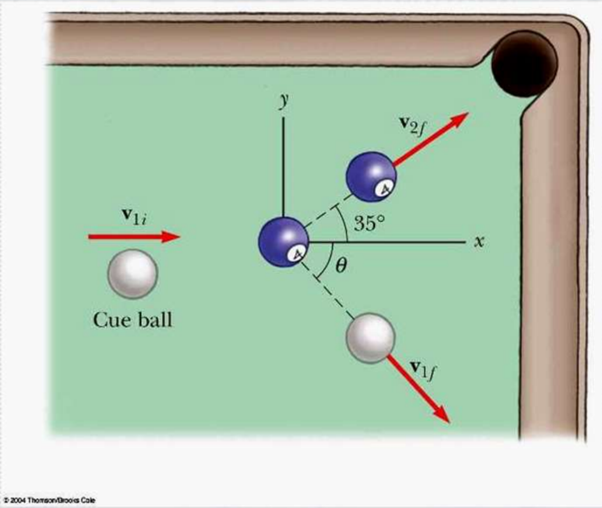
$$v_{1f}=2.80\times10^{5}\,m/s$$

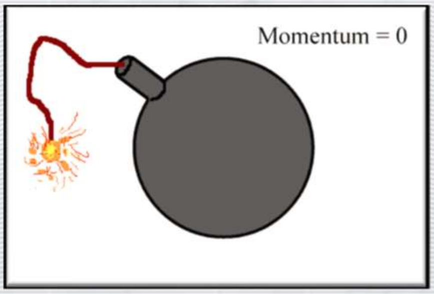
$$v_{2f}=2.11\times10^{5}\,m/s$$

$$\varphi=53.0^{\circ}$$

Example 9.12

Do this at home☺




Momentum = 0