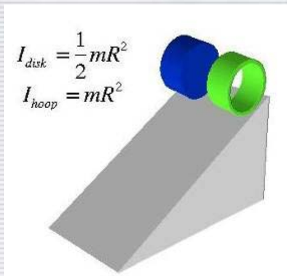
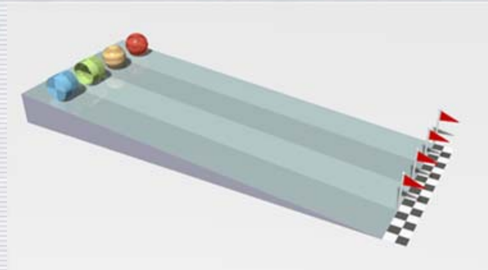
The moment of inertia of an object provides <u>resistance to a rotational force</u> in the same way the mass of an object resists linear acceleration.

The race between the hoop and the disk

which will get to the bottom of the ramp first?





L:\103 Phys LECTURES SLIDES\103Phys_Slides_T1Y3839\CH10Flash

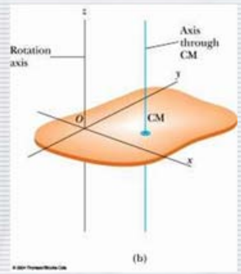
Newton's second law for rotation helps us predict a winner. The moment of inertia of an object provides resistance to a rotational force in the same way the mass of an object resists linear acceleration. If we inspect the moment of inertia for the disk and the hoop in the above image we find that the disk has half the moment of inertia of a hoop of equal mass. Since it has less resistance to rotation, it will speed up quicker and get to the bottom of the ramp first as seen in the

Parallel Axis Theorem

$$I=\int r^2\,dm \quad (rotational\ inertia, continuous\ body)$$

Let D be the perpendicular distance between the axis that we need and the axis through the center of mass (CM) (remember these two axes must be parallel). Then the rotational inertia I about the required axis is

$$\boxed{I=I_{CM}+MD^2}$$



Moment of inertia of any object about any arbitrary axis are the same as the sum of moment of inertia for a rotation about the CM and that of the CM about the rotation axis.

Proof of the Parallel-Axis Theorem

Moment of inertia is defined

$$I=\int r^2dm=\int\sqrt{(x^2+y^2)}\,dm \quad (1)$$

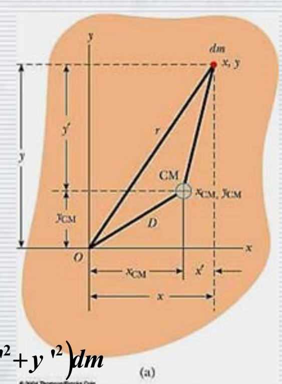
Since x and y are $x=x_{CM}+x'$

$$y=y_{CM}+y'$$

One can substitute x and y in Eq. 1 to obtain

$$I=\int\left[(x_{CM}+x')^2+(y_{CM}+y')^2\right]dm$$

$$=(x_{CM}^2+y_{CM}^2)\int dm+2x_{CM}\int x'dm+2y_{CM}\int y'dm+\int(x'^2+y'^2)dm$$



Since the x' and y' are the distance from CM, by definition $\int x'dm=0$ $\int y'dm=0$

Therefore, the parallel-axis theorem

$$I=(x_{CM}^2+y_{CM}^2)\int dm+\int(x'^2+y'^2)dm=MD^2+I_{CM}$$

Example 10.8

Calculate the moment of inertia of a uniform rigid rod of length L and mass M about an axis that goes through one end of the rod, using parallel-axis theorem.

The moment of inertia about the CM (example 10.6)

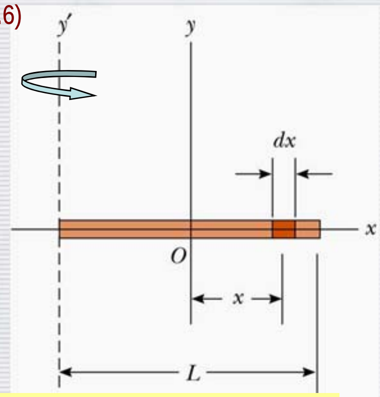
$$I_y = I_{CM} = \frac{ML^2}{12}$$

Using the parallel axis theorem

$$I = I_{CM} + D^2 M = \frac{ML^2}{12} + \left(\frac{L}{2}\right)^2 M$$

$$= \frac{ML^2}{12} + \frac{ML^2}{4}$$

$$= \frac{ML^2}{3}$$

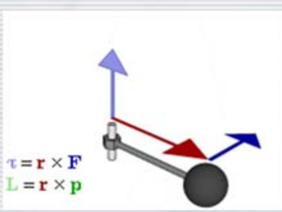


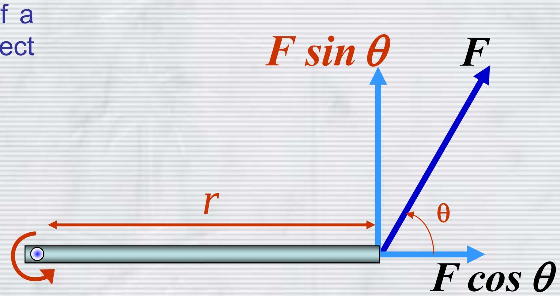
The result is the same as using the definition of moment of inertia.

Parallel-axis theorem is useful to compute moment of inertia of a rotation of a rigid object with complicated shape about an arbitrary axis

What is the Torque

Torque is the tendency of a force to rotate an object about an axis.





A force F is acting at an angle θ on a lever that is rotating around a pivot point. r is the distance between F and the pivot point.

This force-lever pair results in a torque τ on the lever

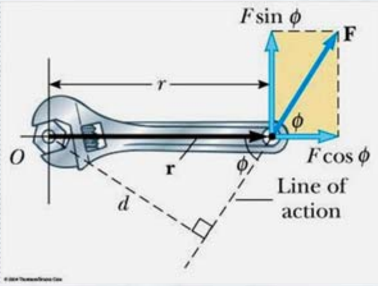
$$\boxed{\tau = rF \sin\theta}$$

10-6. Torque

Torque is the tendency of a force to rotate an object about an axis.

Torque, τ, is a vector quantity.

Magnitude of torque is defined as the product of the force exerted on the object to rotate it and the moment arm.



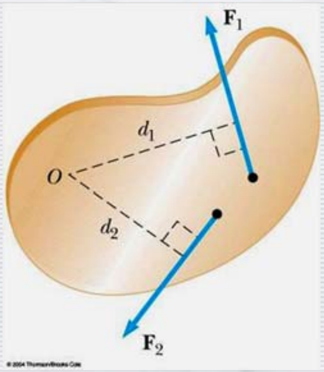
$$\tau \equiv rF\sin\varphi = Fd$$

The perpendicular distance (d) from the pivoting point O to the line of action is called Moment arm.

$$d = r\sin\varphi$$

When there are more than one force being exerted on certain points of the object, one can sum up the torque generated by each force vectorially.
The convention for sign of the torque is positive if rotation is in counter-clockwise and negative if clockwise.

$$\sum\tau = \tau_1 + \tau_2$$
$$= F_1 d_1 - F_2 d_2$$



The SI unit of torque is N.m
(Note that the unit of work **J** is also N.m . However, the name **J** is exclusively reserved for work).

Example 10.9

A one piece cylinder is shaped as in the figure with core section protruding from the larger drum. The cylinder is free to rotate around the central axis shown in the picture. A rope wrapped around the drum whose radius is R_1 exerts force F_1 to the right on the cylinder, and another force exerts F_2 on the core whose radius is R_2 downward on the cylinder.

(A) What is the net torque acting on the cylinder about the rotation axis?

The torque due to F_1

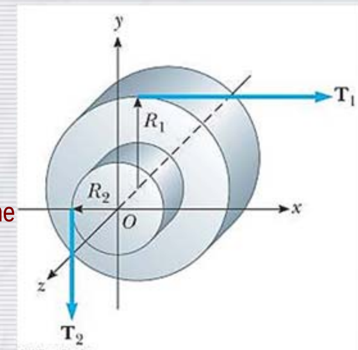
$$\tau_1 = -R_1F_1$$

and due to F_2

$$\tau_2 = R_2F_2$$

So the total torque acting on the system by the forces is

$$\sum \tau = \tau_1 + \tau_2 = -R_1F_1 + R_2F_2$$



(B) Suppose F_1=5.0 N, R_1=1.0 m, F_2= 15.0 N, and R_2=0.50 m. What is the net torque about the rotation axis and which way does the cylinder rotate from the rest?

$$\sum \tau = -R_1F_1 + R_2F_2 = -5.0 \times 1.0 + 15.0 \times 0.50 = 2.5\ Nm$$

The cylinder rotates in counter-clockwise.

10-7. Relationship between Torque & Angular Acceleration

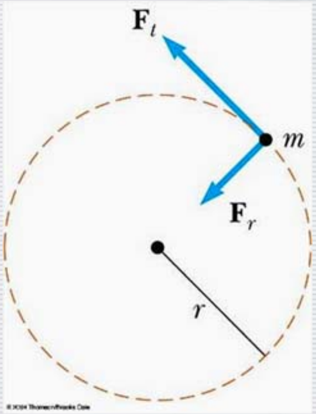
consider a point object with mass *m* rotating in a circle under the influence of a tangential force F_t. A force F_r in the radial direction also must be present to maintain the circular motion.

$$F_t = ma_t = mr\alpha$$

The torque due to tangential force F_t is

$$\tau = F_t r = ma_t r = mr^2\alpha = I\alpha$$

$$\boxed{\tau = I\alpha}$$



Torque acting on a particle is proportional to the angular acceleration.

How about a rigid object?

The external tangential force dF_t is

$$dF_t = dma_t = dmr\alpha$$

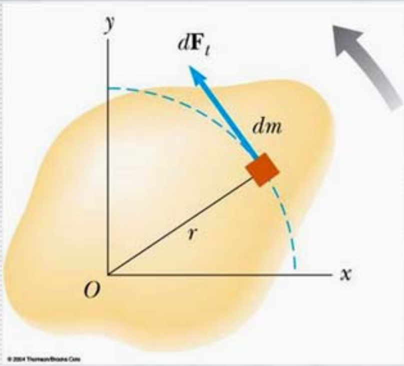
The torque due to tangential force F_t is

$$d\tau = dF_t r = \left(r^2 dm\right)\alpha$$

The total torque is

$$\sum \tau = \alpha \int r^2 dm = I\alpha$$

$$\boxed{\sum \tau = I\alpha}$$



A rigid object rotating about an axis through *O*. Each mass element *dm* rotates about *O* with the same angular acceleration α, and the net torque on the object is proportional to α.

Example 10.10

A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane. The rod is released from rest in the horizontal position. What are the initial angular acceleration of the rod and the initial linear acceleration of its right end?

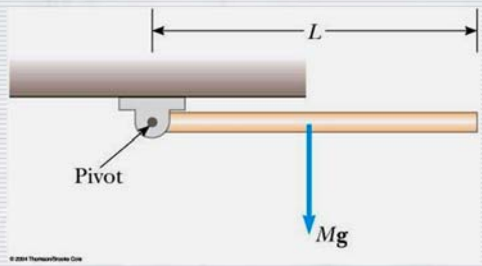
The only force generating torque is the gravitational force Mg

$$\tau = Fd = F\frac{L}{2} = Mg\frac{L}{2} = I\alpha$$

Since the moment of inertia of the rod when it rotates about one end

$$I = \frac{ML^2}{3}$$

$$\alpha = \frac{MgL}{2I} = \frac{MgL}{2\frac{ML^2}{3}} = \frac{3g}{2L}$$



Using the relationship between tangential and angular acceleration

$$a_t = L\alpha = \frac{3g}{2}$$

The tip of the rod falls faster than an object undergoing a free fall.

READ THE REST OF THE EXAMPLE

READ EXAMPLE 10.11

Example 10.12

A wheel of radius *R*, mass *M*, and moment of inertia *I* is mounted on a frictionless horizontal axle, as in Figure 10.20. A light cord wrapped around the wheel supports an object of mass *m*. Calculate the angular acceleration of the wheel, the linear acceleration of the object, and the tension in the cord.

$$\tau = I\alpha = TR \quad \Rightarrow \alpha = \frac{TR}{I}$$

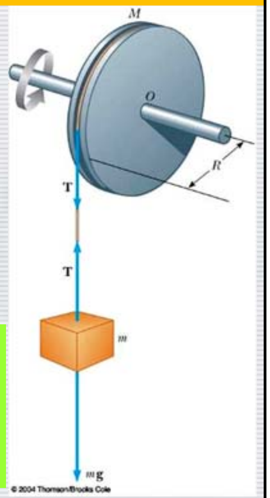
$$F_{net} = mg - T = ma \quad \Rightarrow a = \frac{mg - T}{m}$$

$$a = R\alpha = \frac{TR^2}{I} = \frac{mg - T}{m}$$

$$\Rightarrow T = \frac{mg}{1 + (mR^2/I)}$$

$$a = \frac{g}{1 + (I/mR^2)}$$

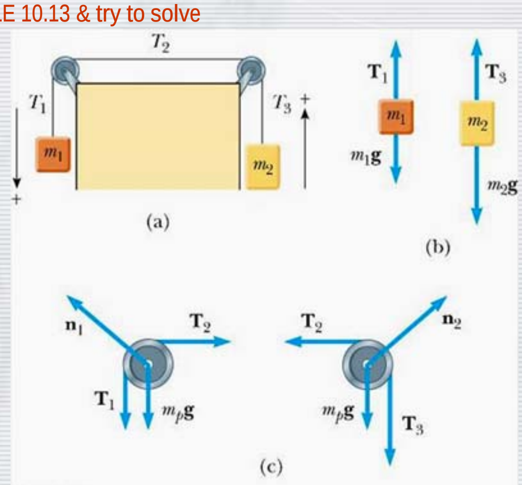
$$\alpha = \frac{g}{R + (I/mR^2)}$$

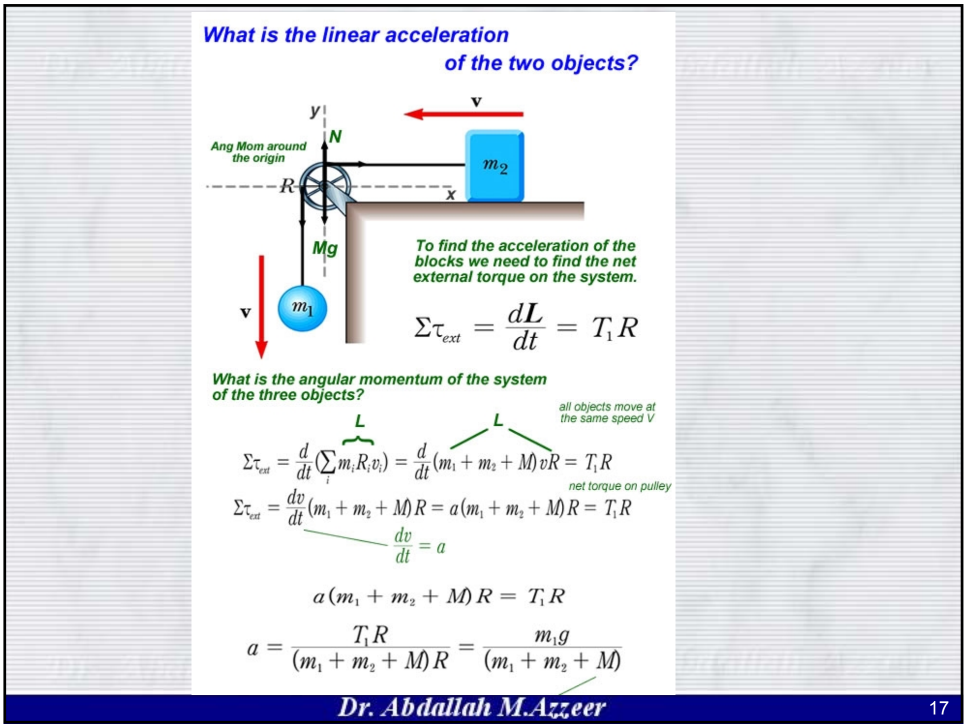


What If? What if the wheel were to become very massive so that I becomes very large? What happens to the acceleration *a* of the object and the tension T? –Read in the book

Example 10.13

READ EXAMPLE 10.13 & try to solve




What is the linear acceleration
of the two objects?
y
N
v
Ang Mom around the origin
m_2
R
x
Mg
v
m_1
To find the acceleration of the blocks we need to find the net external torque on the system.
$\Sigma\tau_{ext} = \frac{dL}{dt} = T_1 R$
What is the angular momentum of the system of the three objects?
all objects move at the same speed V
$\Sigma\tau_{ext} = \frac{d}{dt}(\underbrace{\sum_i m_i R_i v_i}_{L}) = \frac{d}{dt}\underbrace{(m_1 + m_2 + M) v R}_{L} = T_1 R$
net torque on pulley
$\Sigma\tau_{ext} = \frac{dv}{dt}(m_1 + m_2 + M) R = a(m_1 + m_2 + M) R = T_1 R$
$\frac{dv}{dt} = a$
$a(m_1 + m_2 + M) R = T_1 R$
$a = \frac{T_1 R}{(m_1 + m_2 + M) R} = \frac{m_1 g}{(m_1 + m_2 + M)}$
Dr. Abdallah M.Azzeer
17