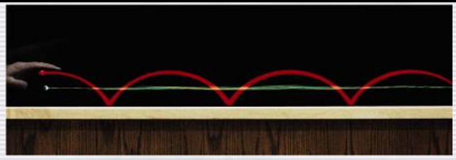
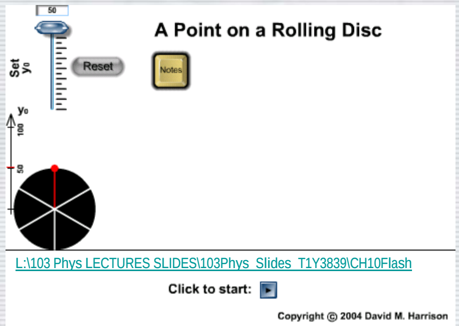


CHAPTER 10

Rotation Motion



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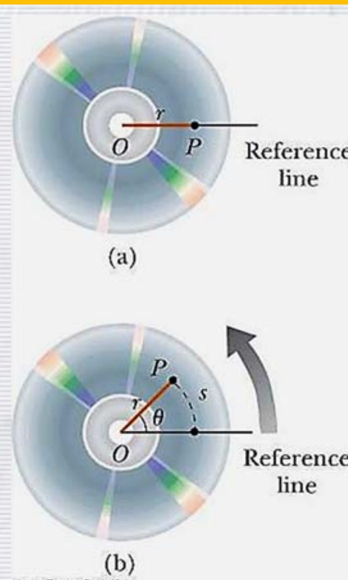
10-1. Angular velocity and angular acceleration

A compact disc rotating about a fixed axis through ***O*** perpendicular to the plane of the figure. (a) In order to define angular position for the disc, a fixed reference line is chosen. A particle at ***P*** is located at a distance ***r*** from the rotation axis at ***O***. (b) As the disc rotates, point ***P*** moves through an arc length *s* on a circular path of radius *r*.



Angle $\boxed{\theta = \dfrac{s}{r}}$

The SI unit of θ is radian (rad), which is a pure number because it is a ratio.

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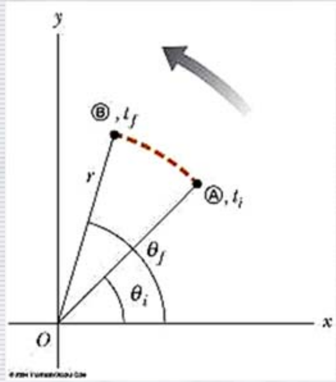
ONE radian is the angle subtended by an arc length equal to the raduis of the arc

$$1\,rev = 360^\circ = \frac{2\pi r}{r} = 2\pi \;\; rad$$
$$1\,rad = 57.3^\circ = 0.159\,rev$$

$$\theta(rad) = \frac{\pi}{180^\circ}\theta(\deg)$$

The angle in rad is positive if it is counterclockwise with respect to the positive x axis.

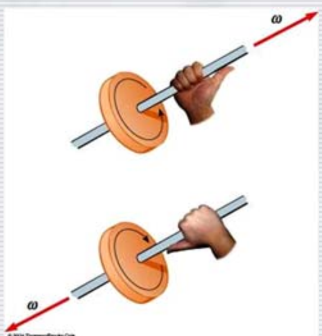


The angular displacement is :

$$\Delta\theta = \theta_f - \theta_i$$

The average angular speed is :

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$



The instantaneous angular velocity is :

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- The SI unit of ω is rad/s
- The angular velocity ω is positive if the rotation is counterclockwise (when θ is increasing).

The average angular acceleration is :

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous angular velocity is :

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\,\omega}{d\,t}$$

The SI unit of α is rad/s^2

10-2. rotational kinematics

Rotational Motion with Constant acceleration

Analogy between linear and angular quantities:

$\vec{x} \Rightarrow \vec{\theta}$

$\vec{v} \Rightarrow \vec{\omega}$

$\vec{a} \Rightarrow \vec{\alpha}$

Table 10.1

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Example 10.1

A wheel rotates with a constant angular acceleration of 3.50 rad/s^2.

(a) If the angular speed of the wheel is 2.00 rad/s at t_i=0, through what angle does the wheel rotate in 2.00s?and (b) how many rev has done during this interval?

$$\theta_f - \theta_i = \omega t + \frac{1}{2}\alpha t^2$$

$$= 2.00 \times 2.00 + \frac{1}{2} 3.50 \times (2.00)^2$$

$$= \frac{11.0}{2\pi} rev = 1.75\ rev$$

(c) What is the angular speed at t=2.00 s?

$$\omega_f = \omega_i + \alpha t$$

$$= 2.00 + 3.50 \times 2.00 = 9.00\ rad/s$$

READ the rest of example

10-3. Relationship Between Angular and Linear Quantities

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- The position :

$$s = \theta r \quad (radian\ measure)$$

- Note that for all linear-angular relations, we must use the radian unit.
- The speed :

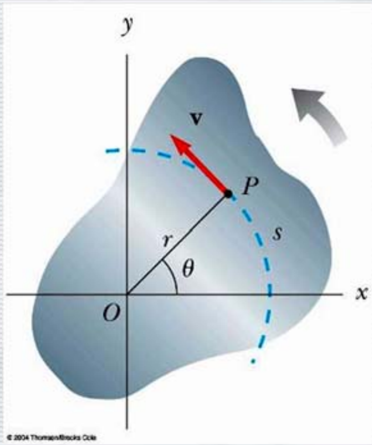
$$\frac{ds}{dt} = \frac{d\theta}{dt} r$$

$$\boxed{v = r\omega \quad (radian\ measure)}$$

- The period of revolution T is

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi}{\omega} \quad (radian\ measure)$$



- The acceleration

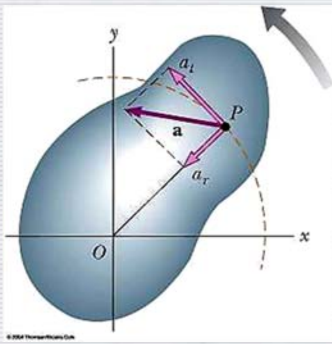
$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

$$\boxed{a_t = r\alpha \quad (radian\ measure)}$$

- For a particle in a circular path, the centripetal acceleration is :

$$\boxed{a_r = \frac{v^2}{r} = r\omega^2 \quad (radian\ measure)}$$

- a_r points radially inward



As a rigid object rotates about a fixed axis through *O*, the point *P* experiences a tangential component of linear acceleration *at* and a radial component of linear acceleration *ar*. The total linear acceleration of this point is $a = a_t + a_r$

Differences between a_r and a_t

- a_r is known as the radial component of linear acceleration, a_t is known as the tangential component of the linear acceleration.

- $a_r = r\omega^2$ is pointed radially inward. It is non-zero even if there is no angular acceleration.

- $a_t = r\alpha$ is tangential to the rotational path of the particle, it is zero if the angular velocity is constant.

Total linear acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

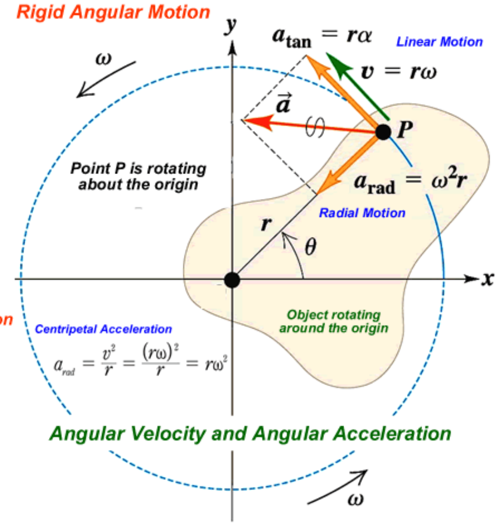
Derivation of Angular quantities from Linear quantities

Linear Velocity / Angular Velocity

$$v = \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} = r\omega$$

Linear Acceleration / Angular Acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{r\Delta\omega}{\Delta t} = r\alpha$$



Example 10.2

Audio information on compact discs are transmitted digitally through the readout system consisting of laser and lenses. The digital information on the disc are stored by the pits and flat areas on the track. Since the speed of readout system is constant, it reads out the same number of pits and flats in the same time interval. In other words, the linear speed is the same no matter which track is played.

(a) Assuming the linear speed is 1.3 m/s, find the angular speed of the disc in revolutions per minute when the inner most (r=23 mm) and outer most tracks (r=58mm) are read.

Using the relationship between angular and tangential speed $v = r\omega$

$r = 23\ mm \Rightarrow \omega = \frac{v}{r} = \frac{1.3\ m/s}{23\ mm} = \frac{1.3}{23\times10^{-3}} = 56.5\ rad/s = 9.00\ rev/s = 5.4\times10^{2}\ rev/\min$

$r = 58\ mm \Rightarrow \omega = \frac{1.3\ m/s}{58\ mm} = \frac{1.3}{58\times10^{-3}} = 22.4\ rad/s = 2.1\times10^{2}\ rev/\min$

(b) The maximum playing time of a standard music CD is 74 minutes and 33 seconds. How many revolutions does the disk make during that time?

$$\bar{\omega} = \frac{(\omega_i + \omega_f)}{2} = \frac{(540+210)\,rev/\min}{2} = 375\ rev/\min$$

$$\theta_f = \theta_i + \bar{\omega}\,t = 0 + \frac{375}{60}\,rev/s \times 4473\ s = 2.8\times10^{4}\ rev$$

(c) What is the total length of the track past through the readout mechanism?

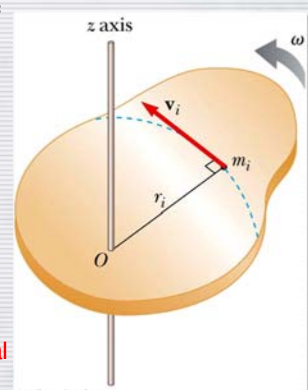
$$l = v_t \Delta t = 1.3\ m/s \times 4473\ s = 5.8\times10^{3}\ m$$

(d) What is the angular acceleration of the CD over the 4473 s time interval, assuming constant α?

$$\alpha = \frac{(\omega_f - \omega_i)}{\Delta t} = \frac{(22.4-56.5)\,rad/s}{4473s} = 7.6\times10^{-3}\ rad/s^{2}$$

10-4. Rotational Kinetic Energy

- We treat the rigid body as a collection of particles with different speeds, m_i is the mass of the i^{th} particle and v_i is its speed.
- The particles move with different v_i but the same ω.



Kinetic energy of a masslet, m_i, moving at a tangential speed, v_i, is

$$K_i = \frac{1}{2} m_i v_i^2$$

Since a rigid body is a collection of masslets, the total kinetic energy of the rigid object is

$$K = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \tfrac{1}{2} m_3 v_3^2 + \cdots$$
$$= \sum \tfrac{1}{2} m_i v_i^2$$

$$K_R = \sum_i K_i = \sum \tfrac{1}{2} m_i v_i^2 = \frac{1}{2}\sum_i m_i r_i^2 \omega^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

$$K_R == \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

Since moment of Inertia, I, is defined as

$$\boxed{I \equiv \sum_i m_i r_i^2}$$

Moment of Inertia

The above expression is simplified as

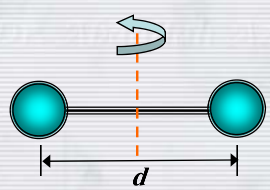
$$\boxed{K_R = \frac{1}{2} I\omega^2}$$

Rotational Kinetic Energy:

Example 10.3

Oxygen Molecule

$d = 1.21 \times 10^{-10}\ m$
$m_i = 2.66 \times 10^{-26}$ kg



$$I = \sum_i m_i r_i^2 = m_i\left(\tfrac{1}{2}d\right)^2 + m_i\left(\tfrac{1}{2}d\right)^2 = 1.95 \times 10^{-46}\ kg \cdot m^2$$

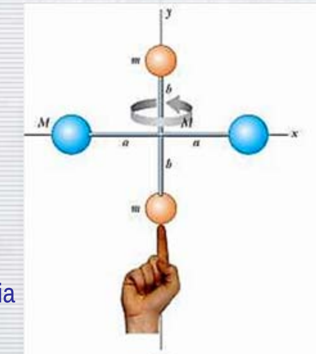
$\omega = 4.6 \times 10^{12}\ rad/sec.$

$$K_R = \tfrac{1}{2} I\omega^2 = 2.06 \times 10^{-21} J$$

Cf) Average linear kinetic energy at RT: $K_L = \tfrac{1}{2} M v^2 = 6.02 \times 10^{-21} J$
$= 3K_R$

Example 10.4

In a system consists of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the particles are massless, compute the moment of inertia and the rotational kinetic energy when the system rotates about the y-axis at ω.


(a)

Since the rotation is about y axis, the moment of inertia about y axis, I_y, is

$$I_y = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + m\cdot 0^2 + m\cdot 0^2 = 2Ma^2$$

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(2Ma^2\right)\omega^2 = Ma^2\omega^2$$

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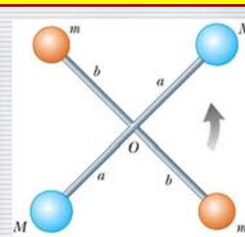
Find the moment of inertia and rotational kinetic energy when the system rotates on the x-y plane about the z-axis that goes through the origin O.

$$I_z = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + mb^2 + mb^2 = 2\left(Ma^2 + mb^2\right)$$

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(2Ma^2 + 2mb^2\right)\omega^2 = \left(Ma^2 + mb^2\right)\omega^2$$


(b)

READ THE REST OF THE EXAMPLE

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10-5. Calculation of Moment of Inertia

We defined the moment of inertia as

$$I \equiv \sum_i m_i r_i^2$$

UNIT; kg.m^2

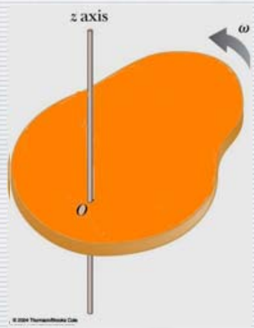
Moments of inertia for large objects can be computed, if we assume the object consists of small volume elements with mass, Δm_i.

The moment of inertia for the large rigid object is

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

Using the volume density, ρ, replace dm in the above equation with dV.

$$\rho = \frac{dm}{dV} \qquad dm = \rho dV$$



The moments of inertia becomes

$$\boxed{I = \int \rho r^2 dV}$$

Example 10.5

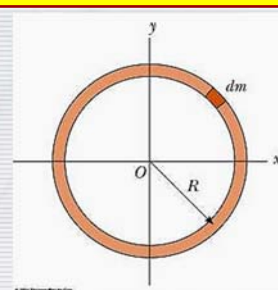
Find the moment of inertia of a uniform hoop of mass M and radius R about an axis perpendicular to the plane of the hoop and passing through its center.

The moment of inertia is

$$I = \int r^2 dm = R^2 \int dm = MR^2$$

What do you notice from this result?

The moment of inertia for this object is the same as that of a point of mass M at the distance R.



Example 10.6

Calculate the moment of inertia of a uniform rigid rod of length L and mass M about an axis perpendicular to the rod and passing through its center of mass.

The line density of the rod is $\lambda = \frac{M}{L}$

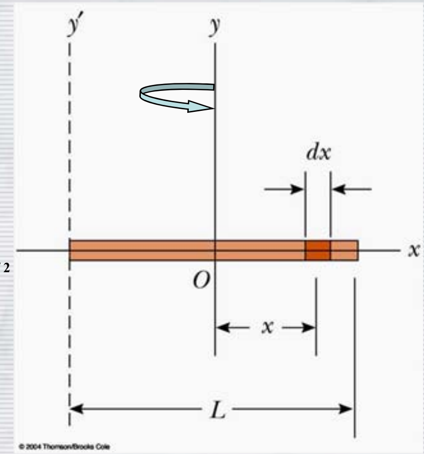
so the masslet is $dm = \lambda dx = \frac{M}{L} dx$

The moment of inertia is

$$I_y = \int r^2 dm = \int_{-L/2}^{L/2} \frac{x^2 M}{L} dx = \frac{M}{L}\left[\frac{1}{3}x^3\right]_{-L/2}^{L/2}$$

$$= \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$= \frac{M}{3L}\left(\frac{L^3}{4}\right) = \frac{ML^2}{12}$$



What is the moment of inertia when the rotational axis is at one end of the rod.

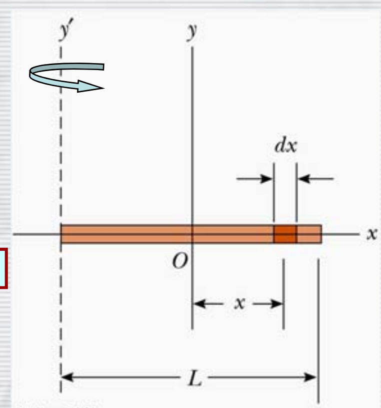
$$I_{y'} = \int r^2 dm = \int_0^L \frac{x^2 M}{L} dx = \frac{M}{L}\left[\frac{1}{3}x^3\right]_0^L$$

$$= \frac{M}{3L}\left[(L)^3 - 0\right] = \frac{M}{3L}\left(L^3\right)$$

$$= \frac{ML^2}{3}$$

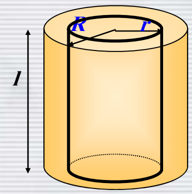
Will this be the same as the above. Why or why not?

Since the moment of inertia is resistance to motion, it makes perfect sense for it to be harder to move when it is rotating about the axis at one end.



Example 10.7

Inertia of Moment of a uniform solid cylinder



Density ρ

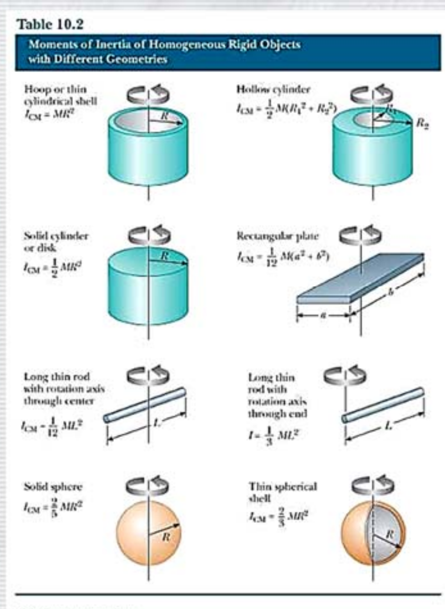
$$\rho \cdot \pi R^2 \cdot l = M \Rightarrow \rho = \frac{M}{\pi R^2 l}$$

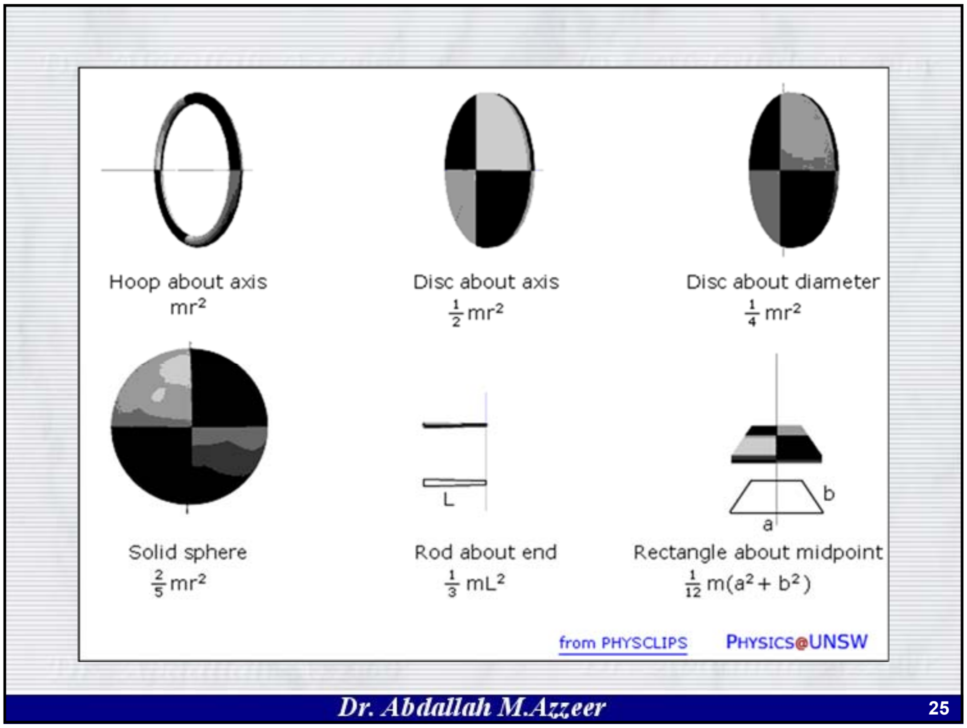
$$dI = dm \cdot r^2 = \rho \cdot 2\pi r \cdot dr \cdot l \cdot r^2 = 2\pi\rho l r^3 dr$$

$$I = 2\pi\rho l \int_0^R r^3 dr = \tfrac{1}{2}\rho\pi l R^4 = \tfrac{1}{2} M R^2$$

$$I = \tfrac{1}{2} M R^2$$

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries




Hoop about axis
mr^2
Disc about axis
$\frac{1}{2}mr^2$
Disc about diameter
$\frac{1}{4}mr^2$
Solid sphere
$\frac{2}{5}mr^2$
L
Rod about end
$\frac{1}{3}mL^2$
b
a
Rectangle about midpoint
$\frac{1}{12}m(a^2+b^2)$
from PHYSCLIPS
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