M - 107   SOLUTION   *(SEMESTER I, 1432-1433) FIRST MID-TERM*

Question: 1 (a) Let

$$A = \begin{bmatrix} x+3y+z & 2x+3y+4z \\ 4x+3y+5z & 8 \end{bmatrix} \text{ and } B = \begin{bmatrix} -y-z & 8-x-y-2z \\ -x-y-z & 3x+4y+6z \end{bmatrix}$$

Use Gauss Jordan method to find $x,\ y\ and\ z$ such that A and B are equal.   **[8]**

[8]

Solution: As $A = B$,

we can write the system of equations by using equality of matrices

$x+3y+z = -y-z$

$2x+3y+4z = 8-x-y-2z$

$4x+3y+5z = -x-y-z$

$8 = 3x+4y+6z$

rearranging both the sides

$x+4y+2z = 0$

$3x+4y+6z = 8$

$5x+4y+6z = 0$

$3x+4y+6z = 8$

Equation 2 and equation 4 are same so the sytem is reduced to three equations

$x+4y+2z = 0$

$3x+4y+6z = 8$

$5x+4y+6z = 0$

Solving system by Gauss Jorden method, rewritting the system in Augmented form
and reducing the augmented matrix to reduced row echlon form

$$\begin{bmatrix} 1 & 4 & 2 & 0 \\ 3 & 4 & 6 & 8 \\ 5 & 4 & 6 & 0 \end{bmatrix} \xrightarrow{\text{performing row operations}} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Solution of the system is $x = -4,\ y = -1$ and $z = 4$.

1 (b) Find the values of λ for which the system of linear equations

$$2x + y = 5$$
$$x - 3y = -1 \qquad \text{will have solutions and find the solutions.} \qquad [6]$$
$$3x + 4y = \lambda$$

Solution:

$$2x + y = 5$$
$$x - 3y = -1$$
$$3x + 4y = \lambda$$

Augmented form of the system is

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & -3 & -1 \\ 3 & 4 & \lambda \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & -1 \\ 2 & 1 & 5 \\ 3 & 4 & \lambda \end{bmatrix} \xrightarrow{R_2 - 2R_1, R_3 - 3R_1} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 7 & 7 \\ 0 & 13 & \lambda + 3 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{7}R_2, R_3 - 13R_2, R_1 + 3R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & \lambda - 10 \end{bmatrix}$$

The system will have solution if and only if $\lambda - 10 = 0$ or $\lambda = 10$

Sytem is reduced to the form

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \text{ hence the solutionis } x = 2 \text{ and } y = 1$$

Question: 2 (a)

If $A = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 2 & 0 \\ 11 & 3 & 2 \end{bmatrix}$, find the inverse of A by using Elementary matrix method.   [8]

Solution:

$$\left[A | I \right] \approx$$

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 \\ 4 & 2 & 0 & 0 & 1 & 0 \\ 11 & 3 & 2 & 0 & 0 & 1 \end{bmatrix} \approx \dots \approx \begin{bmatrix} 1 & 0 & 0 & -\dfrac{1}{6} & -\dfrac{1}{6} & \dfrac{1}{6} \\ 0 & 1 & 0 & \dfrac{1}{3} & \dfrac{5}{6} & -\dfrac{1}{3} \\ 0 & 0 & 1 & \dfrac{5}{12} & -\dfrac{1}{3} & \dfrac{1}{12} \end{bmatrix} \approx \left[I : A^{-1} \right]$$

$$A^{-1} = \frac{1}{12} \begin{bmatrix} -2 & -2 & 2 \\ 4 & 10 & -4 \\ 5 & -4 & 1 \end{bmatrix}$$

2(b) **Let** $A=\begin{bmatrix} a^2 & 1 \\ 4a & 16 \end{bmatrix}$ **, find all values of a for which A is not invertible.**

[4]

Solution:

If matrix A is not invertible then detA=0

$\det A = 16a^2 - 4a = 0$

$4a(4a-1) = 0 \Rightarrow a = 0 \ or \ a = \dfrac{1}{4}$

Question: 3 (a) Evaluate the determinant by using elementary row operations

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \end{vmatrix}$$

[6]

Solution:

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \end{vmatrix}$$

using row opertions,it is reduced to upper triagular form

$$|A| = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{vmatrix} = 5$$

3(b) Solve the system of linear equations by Cramer's Rule

$$4x + 5y \qquad = 2$$
$$11x + y + 2z = 3$$
$$x + 5y + 2z = 1$$

[8]

Solution:

$$\begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \Rightarrow AX = B$$

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, \det(A) = -132,$$

$$A_1 = \begin{bmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, \det(A_1) = -36$$

$$A_2 = \begin{bmatrix} 4 & 2 & 0 \\ 11 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}, \det(A_2) = -24,$$

$$A_3 = \begin{bmatrix} 4 & 5 & 2 \\ 11 & 1 & 3 \\ 1 & 5 & 1 \end{bmatrix}, \det(A_3) = 12$$

By using Crammer's Rule

$$x = \frac{\det(A_1)}{\det(A)} = \frac{3}{11}, \quad y = \frac{\det(A_2)}{\det(A)} = \frac{2}{11}, \quad z = \frac{\det(A_3)}{\det(A)} = -\frac{1}{11}$$