Chapter 5

Classification of the Manufacturing Processes

A classification of the technological manufacturing processes may be based on many different criteria, depending on its purpose. This classification will, as mentioned, be based on the morphological model discussed in Chapter 1 to obtain a structure oriented toward generation of possible manufacturing methods to produce specific components.

The structure of the classification is thus:

- **Material flow**
  Type of material
  State of material
  Type of process
  Basic process

- **Energy flow**
  Type of energy
  Medium of transfer

- **Information flow**
  Surface creation principle
  Pattern of motion

This structure is shown schematically in Fig. 5.1.
It should be mentioned that, in this context, only processes aiming primarily at geometrical changes (obtaining specific geometries) will be discussed, but, as described in Chapter 2, the geometrical changes are normally accompanied by changes in other properties (mainly mechanical properties) depending on the process.

These process-dependent changes in properties must be taken into consideration when selecting processes, as they may be decisive. An example of a process-dependent change would be the increase in mechanical strength of a metal due to strain hardening by deformation.

As discussed previously, many processes aiming primarily at changes in material properties without changing the geometry are available. A major group, and a very important one, constitutes the heat-treatment processes discussed in Chapter 3.

Types of Material.

In Chapter 3 the materials were divided into metallic, non-metallic, and composite materials. The classification structure (Fig. 5.1) should cover all the different materials, but only metals and plastics are shown and discussed further in this text.

State of Material. A given type of material can be shaped in the solid, granular, or liquid state. The state of the material describes the situation in the shaping phase.

Type of Process. Considering materials in the solid state, shaping can be carried out by:

- Mass-conserving processes \((dM = 0)\): the mass of component is equal to (or closely equal to) the mass of the original material. The basic process is plastic deformation.
- Mass-reducing processes \((dM < 0)\): the final component shape can be circumscribed by the shape of the original material, and the excess is removed by mechanical, thermal, or chemical basic processes.
- Joining processes: the final geometry is obtained by joining the subgeometries. The subgeometries are produced by one or both of the above-mentioned types of processes.

Concerning materials in the granular and liquid states, shaping is in general carried out only by mass-conserving processes.

In the blocks showing the type of process in Fig. 5.1, the chapters dealing with the specific types of processes for metallic materials are noted.
Basic Processes. Three types of basic processes exist: mechanical, thermal, and chemical. These basic processes are not specified in Fig. 5.1, since different basic processes may be utilized for each combination of material, state of material, and actual process. It can be mentioned that, for solid materials, the basic processes within the mass-conserving processes are mechanical, the basic processes within the mass-reducing processes are predominantly mechanical but some are thermal or chemical, and the basic processes within joining processes are predominantly mechanical. For material in the granular and liquid states, the basic processes are predominantly mechanical.

Type of Energy. The main types of energy that can be utilized to create the specific type of energy necessary to carry out a given basic process are mechanical, electrical, thermal, and chemical. The type of energy is not specified in Fig. 5.1, as more than one type can often be utilized for each combination of the previous parameters, depending on the conditions.

Medium of Transfer. The requirements of the media of transfer are determined by the type of basic process, the type of energy, and the way in which the surface creation is brought about. The media of transfer can be classified according to their state as follows: rigid, elastic, plastic, granular, gaseous, liquid, and none (unspecified).

Surface Creation and Pattern of Motion. It was seen in Chapter 1 that a surface can be produced as a result of
   • Total forming (TF)
   • One-dimensional forming (ODF)
   • Two-dimensional forming (TDF)
   • Free forming (FF)

For each of these, the pattern of motions for the work material and the medium of transfer must now be selected, so that the desired component is obtained. The surface creation and the pattern of motions (the information system) describe the geometrical possibilities of the processes.

It is especially important that at this point, the systems are imaginatively utilized. The specification of the information system must be carried out as an iteration by detailing the information system and the energy system.

It should be mentioned that the joining processes are exceptions, as they do not themselves generate geometries. Based on the morphological structure (or model) and the description in the following chapters, the engineer will be able to judge the material properties important in the production, the changes of properties by the processes, the geometrical Possibilities, and the tolerances and surfaces obtainable.
Chapter 6

Solid Materials: Mass-Conserving Processes

Introduction

In this major group of manufacturing processes often called metal-forming processes—the desired geometry is produced by the mechanical basic process, plastic deformation. Within the last couple of decades, this field has developed rapidly, resulting in an increasing number of applications. This is mainly due to the fact that mass-conserving processes used with solid materials provide good material utilization (low waste of expensive material) and excellent final material properties. The mass-conserving processes can according to their location in the series of processes necessary to produce a component be classified as (1) primary processes and (2) secondary processes.

The purposes of the primary processes are twofold: first, to breakdown the initial structure of the materials in the form of ingots produced by casting in order to improve the material properties (in particular the mechanical properties) and second, to provide products (e.g., rods, bars, plates, sheets, tubes, etc.) that can be processed by secondary processes.

The primary processes include rolling, forging, extrusion, and so on. The secondary processes aim at the production of semi-final or final components based on the products of the primary processes. The secondary processes include mass conserving processes, such as forging, sheet metal forming (including bending, deep drawing, stretching, spinning, etc.); mass-reducing processes, such as cutting, electro discharge, and electrochemical machining; and assembly processes.

However, the classification is useful because for each group, it is possible to establish some overriding characteristics, allowing general judgments to be made of the processes, their possibilities, and their limitations. Thus the primary processes are in general hot-working processes that are based on plastic deformations applied to materials heated above the recrystallization temperature. Under these conditions, the metals can be considered to be perfectly plastic, allowing large deformations without fracture in compression. Hot-working processes normally have the following advantages:

- The coarse (dendritic) crystal structure from the casting is broken down to form a refined structure with small and equiaxial grains.
- Impurities are broken up and distributed more evenly throughout the material.
- Pores or voids are closed up.
- Mechanical properties are improved considerably (especially ductility and impact strength), due to the refined structure.
- Forces and energy necessary to carry out the processes are relatively small, due to the lower yield strength of the material at elevated temperatures.
- Shape can be changed quite drastically.
Some of the **disadvantages** associated with hot working are:

- Rapid oxidation (i.e., formation of scales, resulting in rough surfaces). Relatively wide tolerances (2-5%), due to the rough surfaces.
- Hot-working machinery is expensive and requires considerable maintenance.

Basically, the same principles of processing are utilized in both the hot and cold working of metals. Since no distinction between these two categories is made in the following description, the main advantages and disadvantages for cold working are mentioned first. In general, compared to hot working, cold working of metals will give:

- Better surfaces and tolerances
- Better mechanical properties (strength)
- Better reproducibility
- Anisotropy (i.e., directional properties of the material—this is only an advantage when it is possible to utilize the effect)

Some of the **disadvantages of cold working** compared to hot working are:

- Increased force and energy requirements, due to strain hardening (i.e., heavier and more powerful equipment is required).
- Less ductility in the work material.
- Anisotropy is produced in the workpiece (an advantage in many sheet forming processes).
- Clean and scale-free surfaces are required on the original workpiece.
- The distinction made here between primary and secondary processes must not be confused with the classification into primary and secondary basic processes described in Chapter 1.

CHARACTERISTICS OF MASS CONSERVING PROCESSES

In this section some of the general characteristics of mass-conserving processes for solid materials are discussed. As mentioned previously, a **close relationship exists between the information system and the material system** [i.e., among the geometry, the basic process, and the material (see Fig. 6.1)]. These systems cannot be selected independently. Here the process is mass conserving and the material is in the solid state. The conditions under which the process is carried out (i.e., the pressure, temperature, velocity, etc.) play an important role, since they can influence the possibilities and limitations of the process to a high degree.
Geometrical Possibilities

In Chapter 1 the information system was described by principles of surface creation and the pattern of motions for work material and tooling (media of transfer). In the following description the information system will not be related directly to specific materials, types of energy, and media of transfer, as only the overriding characteristics are considered. More details are given in later sections. The principles of surface creation are:
1. Total forming (TF)
2. One-dimensional forming (ODF)
3. Two-dimensional forming (TDF)
4. Free forming (FF)

The pattern of motions must be one of the following: translations \((T)\), rotations \((R)\), combinations of translations and rotations \((T/R)\), and no motion.

In general, it can be stated that the more shape information that is built into the media of transfer (the tooling), the less freedom there is in the selection of the pattern of motions. Tables 6.1 through 6.4 show examples of processes within the four surface creation principles; the corresponding pattern of motions is listed for each process. These examples serve to illustrate the many possibilities for shape generation. Here it must be emphasized that an imaginative utilization of the surface creation principles and pattern of motions is very important in evaluating the geometrical possibilities.
### TABLE 6.1 Examples of Total Forming

<table>
<thead>
<tr>
<th>Pattern of motions</th>
<th>Workpiece</th>
<th>Tool</th>
<th>Total forming</th>
<th>Examples of processes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td></td>
<td></td>
<td>Forging</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bending</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Impact forging</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Tube expansion</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td></td>
<td></td>
<td>Pressing</td>
</tr>
</tbody>
</table>

### TABLE 6.2 Examples of One-Dimensional Forming (ODF)

<table>
<thead>
<tr>
<th>Pattern of motions</th>
<th>Workpiece</th>
<th>Tool</th>
<th>One-dimensional forming</th>
<th>Examples of processes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td></td>
<td></td>
<td>Direct extrusion</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Wire drawing</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td></td>
<td></td>
<td>Indirect extrusion</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Deep drawing</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td></td>
<td></td>
<td>Sheet and tube bending</td>
</tr>
<tr>
<td></td>
<td>T R</td>
<td></td>
<td></td>
<td>Rolling</td>
</tr>
<tr>
<td></td>
<td>T T</td>
<td></td>
<td></td>
<td>Forging</td>
</tr>
<tr>
<td></td>
<td>R T</td>
<td></td>
<td></td>
<td>Ring forging</td>
</tr>
<tr>
<td></td>
<td>R R</td>
<td></td>
<td></td>
<td>Roll bending</td>
</tr>
</tbody>
</table>


### TABLE 6.3 Examples of Two-Dimensional Forming (TDF)

<table>
<thead>
<tr>
<th>Workpiece</th>
<th>Tool</th>
<th>Two-dimensional forming</th>
<th>Examples of processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/T</td>
<td>T</td>
<td>Bar forming</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Swaging</td>
<td></td>
</tr>
<tr>
<td>R/T</td>
<td>R</td>
<td>Tube rolling</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>T</td>
<td>Spinning</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 6.4 Examples of Free-Forming Processes

<table>
<thead>
<tr>
<th>Workpiece</th>
<th>Tool</th>
<th>Free forming</th>
<th>Examples of processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td></td>
<td>Uprighting</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>Torsion</td>
<td></td>
</tr>
</tbody>
</table>
Process Conditions

To be able to judge if a desired geometry and final material properties can be produced, the conditions under which the process is carried out must be known. The major influencing factors are the state of stress in the deformation zone, the temperature, and the velocity.

The State of Stress

The deformation zone in a process can be characterized by the magnitudes and state of the stresses. The size of the deformation zone for a fixed state of stress determines the forces and energy necessary to carry out the process. This information is needed both for the design of the equipment and for the determination of maximum size or yield strength of the components that can be processed using given equipment or machinery.

The size of deformation zone is determined primarily by the contact area between the work piece and the medium of transfer [the tool or die]. Here a distinction must be made between total deformation and partial deformation. In total deformation, the contact area covers the whole or most of the desired surface [i.e. the deformation zone simultaneously extended through the whole component].

In partial deformation, the contact area covers only a fraction of the surface; that is, at given instant, deformation is occurring within only a fraction of the total volume of the component, thus requiring a particular pattern of motions to describe the whole volume. Fig. 6.2
Fig. 6.3 shows how an *incremental process* can be developed. The purpose is to reduce the wall thickness of a tube without changing the internal diameter.

In the basic process, the tube is pushed or pulled through a conical die, and the constancy of the internal diameter is maintained by an internal mandrel.

The **die is subjected to high radial forces**, and **large forces are required** to pull or push the tube through the die.

The deformation zone is circular, extending from the contact area between the die and the tube, through the tube, to the mandrel.
The contact area can be reduced by replacing the die with a number of balls or conical rollers supported by an outer ring, which rotates during deformation.

A reduction of the forces to push or pull the tube is accomplished, but the ring is still subjected to high radial forces.

Furthermore, both these processes require special die system for each tube diameter.

If the number of rollers is reduced to one, and similar forces necessary are supplied by the machine structure, a spinning process [fig. 6.3. c] is the result.

This is a much more versatile process, since different wall thicknesses and diameters can be obtained by simply changing the position of the roller.
The size of the deformation zone has now been discussed, and the next important question is: What state of stress exists in this zone? The state of stress is important, because it determines the deformation obtainable before instability and fracture occur and the forces required.

Most manufacturing processes take place under complex states of stress, and it is, in general, difficult to characterize a process by a single state of stress, since the state of stress varies throughout the deformation zone. The processes can be approximately classified into four groups, according to the dominant state of stress:

1. Tension (one-, two-, or three-dimensional)
2. Compression (one-, two-, or three dimensional)
3. Shearing
4. Bending (non-homogeneous)

Often, two or more of these states can be found in the same process. Considering the deep-drawing process (p. 147), the state of stress in the cylindrical wall is tension, and in the flange, compression. But as described previously (Chapters 2 and 4), the classification of the state of stress is very important in evaluating the maximum deformation that the material can sustain before instability occurs.

When planning a particular process or developing a new process concept, study of the deformation zone (size and state of stress) is fundamental. In Section 6.3, where some important processes are described, the states of stress are listed, allowing a basic evaluation of the deformation characteristics.

Temperature

The temperature in the deformation zone is an important parameter. Above the recrystallization temperature very large deformations in compression can be obtained without fracture. In tension, instability occurs at very low strains. Below the recrystallization temperature strain hardening in tension increases the possible strains up to instability, and in compression it reduces the strains up to fracture.

Velocity

The velocity with which the process is carried out can influence the maximum deformation quite strongly, as shown in Fig. 2. 5.

Different materials react differently to the deformation velocity (strain rate). Some will exhibit increased ductility, and some decreased ductility. In actual situations, the strain rate
must be estimated, and the influence on the properties of the particular material evaluated.
In general, it is found that the strain rate at room temperature has no significant influence on the stress-strain curve, but elevated temperatures normally increase the strain rate sensitivity. In hot-working processes it is therefore necessary to analyze carefully the strain rate situation and its consequences.

Other Important Factors

The geometry and surface of the tools, friction, lubrication, and the state of stress determine the final surface finish. If the desired surface quality is considered, good lubrication sometimes leads to poor surfaces, as the lubrication can be entrapped in the small cavities on the surface.

A substitution of the tool/die material from a metal to an elastic material such as rubber can result in better surfaces with a given work material without destruction occurring.

The tolerances obtainable are difficult to describe in general, since they are dependent on the size of the deformation zone, the state of stress, the workpiece geometry, the tool/die system, and the equipment.

If the deformations are small, elastic recovery must be considered. It may be difficult to obtain fine tolerances when the elastic deformation is of the same order of magnitude as the plastic deformation.

In summary, it can be stated that the major factors affecting processing by mass-conserving processes are workpiece geometry, deformation zone (size and state of stress), temperature, deformation, velocity, lubrication, the properties of the workpiece material, and the tool/die material.

Important Material Properties

The amount of deformation that a material can sustain without instability or fracture depends, as described earlier, on the state of stress, the temperature, and the strain rate.
In Chapter 4 it was found that instability occurs when a strain equal to the strain-hardening exponent was reached in a material following the relation \( \sigma^* = c \varepsilon^* n \).

If the same material is deformed under compressive stresses, considerably higher deformations are obtainable, limited only by fracture at locations where high tensile stresses are generated. In a drawing operation, a material may be elongated 40%, whereas in rolling it can be elongated 400%.

**A major advantage of mass-conserving processes of the cold-working type is the strength improvement of the material, due to strain hardening.** Compared to casting or hot working, the strength improvement is often so high that a cheaper work material can be selected.

The final properties of the material can be evaluated with reasonable accuracy from the stress-strain curve and a knowledge of the strains and the conditions under which the process is carried out. Force and energy calculations are discussed in Section 6.4.

**TYPICAL EXAMPLES OF MASS-CONSERVING PROCESSES**

In the following pages short descriptions are given of a number of mass-conserving processes. **The processes are classified according to the fundamental elements, basic process, energy to carry out the process, media of transfer, principle of surface creation, and the predominant state of stress.**

All the processes are mass conserving and solid materials are used. The following abbreviations are used in the classification:

**Basic process**
- M, mechanical  - T, thermal  - C, chemical

**Energy**
- Me, mechanical  - El, electrical (including magnetic)  - Th, thermal  - Ch, chemical

**Media of transfer**
- Hi, rigid  - Ea, elastic  - Pl, plastic  - Ga, gaseous  - Gr, granular  - Fl, fluid

**Principles of surface creation**
- TF, total forming  - ODF, one-dimensional forming  - TDF, two-dimensional forming  - FF, free forming

**State of stress**
- Te, tension  - Co, compression  - Sh, shearing  - Be, bending

The description of the processes covers:-
In the figures, the deformation zone is characterized by the size of the zone (the whole workpiece or portions of it) and the state of deformation (steady, nonsteady).

A steady state of deformation occurs when the deformation picture is fixed in time and position (e.g., rolling, extrusion) during the process.

A non-steady state of deformation occurs when the deformation picture changes continuously with time and position during the process.

The information sheets are to be considered only as an introduction to the processes; more details may be found in the literature.
**PROCESS 2: Forward extrusion (M, Mo, Ni, ODF, Cu)**

**Description.** The forward extrusion process is, in general, characterized by a solid material, one-dimensional forming, and a compressive state of stress. The workpiece (W) is placed in the die/container (I), and the punch (P) extrudes the material through the die orifice in the direction of the applied force.

Applications. As a hot-working process, extrusion is used extensively for the production of a wide variety of regular and irregular structural profiles, such as window moldings, single sections, I- and U-beams, and circular and non-circular tubing. As a cold working process, it is a variant of cold forging, used alone or combined with cold heading, backward extrusion, and so on.

Material Requirements. In hot working, ferrous and non-ferrous materials must possess sufficient ductility at elevated temperatures. In cold working, non-ferrous metals and low-alloy steels are used, possessing sufficient ductility at room temperature.

Tolerances/Surfaces. Hot extrusion gives good tolerances and surfaces, and is best for ferrous metals. Cold extrusion gives excellent tolerances (0.1-1.0) and surfaces.

Machinery/Energy. For hot extrusion, special hydraulic presses are used; cold extrusion is carried out on general-purpose mechanical and

**PROCESS 3: Hot drawing (M, Mo, Ni, ODF, Cu)**

**Description.** The hot-drawing process is, in general, characterized by a solid material, one-dimensional forming, and a tensile state of stress. The workpiece (W) is placed in a die (2), and the punch (P) pushes the material through the die, forming a cup. The cup can be drawn through several dies with a single punch.

Applications. The hot-drawing process is generally used to produce relatively thick-walled cylindrical parts, such as oxygen tanks, artillery shells, tank heads, and short tubes.

Material Requirements. High ductility (low yield strength) at elevated temperature. Both ferrous and non-ferrous metals are hot drawn.

Tolerances/Surfaces. Reasonably good tolerances are generally obtained (often below 0.5% of the diameter). The surface quality is good.

Machinery/Energy. Hydraulic presses (draw benches) for single or
Determination of Forces and Energies

Basic Principles in Force and Energy Determination

In this context only approximate methods to estimate the forces and energies necessary are discussed. More accurate and advanced methods are available, but they normally require an advanced theoretical knowledge of plasticity theory and solid mechanics.

The simplest possible method to estimate the forces necessary is based on yield in homogeneous deformation. This method can be applied only when the load or force is acting directly on the whole deforming cross section of the workpiece. For example, the maximum force necessary to compress a cylindrical workpiece in the direction of its axis is given by \( P = \sigma_0 A_{\text{max}} \) where \( \sigma_0 \) is the yield strength at maximum strain and \( A_{\text{max}} \) the maximum cross-sectional area.

For most processes this approach is not applicable, as the force is not acting on the whole deforming cross section of the workpiece. A more general approach is to consider the work necessary to deform an element of the workpiece and integrate this over the whole deforming region.

This total work of deformation is then related to the work carried out by the external force, allowing a determination of the latter. In Chapter 4 it was shown that work \( W \) necessary to carry out a deformation is given by

\[
W = \int \int [\varepsilon_1^- \text{ to } \varepsilon_2^-] \sigma^- \, d\varepsilon^- \, dV \quad (6.1)
\]

where \( W \) is the work of deformation, \( V \) the volume of the deforming region, \( \varepsilon_1^- \) and \( \varepsilon_2^- \) the effective strains before and after the deformation, and \( \sigma^- \) the effective stress (which can be expressed as a function of \( \varepsilon^- \)).

If all elements in the workpiece volume (the deforming region) are supplied with the same amount of work, Equation (6.1) can be written as

\[
W = V \int [\varepsilon_1^- \text{ to } \varepsilon_2^-] \sigma^- \, d\varepsilon^- \quad (6.2)
\]
If the stress-strain curve for the material is given by \( \sigma = c \varepsilon \),

Eq. (6.2) becomes

\[
W = V c / n+1 (\varepsilon_2^{n+1} - \varepsilon_1^{n+1})
\]

This work can be characterized as the internal work \( W_i \). The external work is supplied by external forces (\( P_e \)) or pressures (\( p_e \)), which are acting over a certain travel distance (\( \lambda_e \)); that is, the externally supplied work can be written as

\[
W_e = P_e \lambda_e = p_e A_e \lambda_e
\]

where the suffix \( e \) refers to external and \( A_e \) is the cross-sectional area over which the external forces / pressures act.

By equating (6.1) and (6.4), the external forces or pressures can be estimated:

\[
P_e \lambda_e = p_e A_e \lambda_e = \int [v] \int [\varepsilon_1^{-} \text{ to } \varepsilon_2^{-}] \sigma^- d\varepsilon^- dV \quad (6.5)
\]

If the power necessary is to be determined, the velocity \( v_e \) with which the force or pressure is supplying the work or the time \( t_e \) during which it is supplied must be known.

Consequently, the power required is given by:

\[
N = P_e v_e = W_e / t_e
\]

where \( N \) is the power.

If an external moment \( M_e \) is acting with angular velocity \( \omega_e \), the power is correspondingly given by \( N_e = M_e \omega_e \).

*These methods for estimating forces and energies are only approximate*, as they are based on the assumption of homogeneous deformation.

A homogeneous deformation is the most efficient way to carry out a deformation and requires the smallest possible load or force (because of homogeneous def.), since frictional work and redundant work (caused by friction and geometrical constraints) are neglected.

This means that the work [Eqs. (6.1), (6.2), and (6.3)] and the force \( P_{\text{max}} = \sigma_0 A_{\text{max}} \) and Eq. (6.5) are the lower limits for any process, producing the same final deformations. The agreements with the actual work and force are, in general, reasonable for very low coefficients of friction and simple
geometries, producing minimum internal distortion (note that geometrical constraints provide redundant deformation).

This is, however, not the case for many processes, and consequently, to obtain reasonably accurate results, it is often appropriate to introduce empirical correction factors, as discussed later.

**Sheet Rolling: Determination of Rolling Force, Moment, and Power**

The sheet-rolling process (Fig. 6.4) can be approximately considered as a bar-forging process (Table 6.2), where a bar (the sheet) is produced by forging actions succeeding each other along the bar. The main problem here is that the yield strength of the material increases from the entry (thickness \(h_1\)) to the exit (thickness \(h_2\)) of the roll gap.

It is therefore necessary to use a mean yield stress \(\sigma_{3,m}\), where the suffix \(m\) refers to a mean value and the suffix \(3\) refers to the direction of the principal stress (see Fig. 6.4).

Assuming that the curvature of the rolls can be ignored, the rolling load or force can be determined by

\[
P = -\sigma_{3,m} A = -\sigma_{3,m} w L \quad (P \text{ defined positive}) \quad (6.7),
\]

where \(\sigma_{3,m}\) is the mean yield strength in direction 3 of the principal stresses, \(A\) the area of deformation, \(w\) the width of the sheet, and \(L\) the longitudinal projection of the arc of contact (i.e., the chord of contact).
Since for a close approximation the width of the sheet can be considered constant during the deformation ($\Delta w = 0$), the state of strain is plane ($\varepsilon_1 = -\varepsilon_3$, $\varepsilon_2 = 0$).

The state of stress is given by (see Chapter 4, Example 5): $\sigma_1 \approx 0$ (no external longitudinal forces); $\sigma_2 = (\sigma_1 + \sigma_3) / 2 = \sigma_3 / 2$; $\sigma_3$.
Consequently von Mises' yield criterion for the plane state of strain gives
$\sigma_1 - \sigma_3 = -\sigma_3 = (2/\sqrt{3}) \sigma_o$, where $\sigma_o$ is the uniaxial yield strength.
If mean values are used, this becomes:
$-\sigma_{3,m} = 2/\sqrt{3} \sigma_{o,m} = 1.15 \sigma_{o,m}$ \hspace{1cm} (6.8)