

بعض الدوال الخاصة

Gamma Functions

$$1. \frac{\Gamma(n)}{a^n} = \int_0^{\infty} x^{n-1} e^{-ax} dx = \int_0^{\infty} x^{-(n+1)} e^{-\frac{a}{x}} dx .$$

$$2. \Gamma(n+1) = n\Gamma(n) = n! .$$

$$3. \Gamma(1/2) = \sqrt{\pi} .$$

$$4. \Gamma\left(n + \frac{1}{2}\right) = \frac{(2n-1)!}{2^n} \sqrt{\pi} \quad ; n = 1, 2, 3, \dots$$

$$5. \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi} .$$

Beta Functions

$$1. \beta(n, m) = \int_0^1 x^{n-1} (1-x)^{m-1} dx = \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)} .$$

$$2. \beta(1-n, 1+n) = \frac{n\pi}{\sin(n\pi)} .$$

Other Functions

$$1. e^a = \sum_{i=0}^{\infty} \frac{a^i}{i!} .$$