

## Problem sheet 2

### Section 3.1:

Q1: Let  $X = (0, \infty)$  and  $\tau = \{U \subseteq X; U = \emptyset, \text{ or } \{7, 10\} \in U\}$

- Show that  $\tau$  is a topology on  $X$ .
- Find a base for this topology.
- Describe closed sets in this topology.
- Find,  $Int(\mathbb{N}), cl(\mathbb{N}), Bd(\mathbb{N})$ . Where  $\mathbb{N}$  is the set of natural numbers.
- Let  $A = (1, 5) \subset X$ , show that the subspace topology  $(A, \tau_A)$  is the discrete topology. Is  $\tau_A \subset \tau$ ? Verify.
- Let  $B = (2, 15)$ , describe the subspace topology  $\tau_B$ , then show that  $\tau_B \subset \tau$ .

Q2: Let  $(X, \tau)$  be a topological space,  $A \subset X$ , show that:

- If  $F \subseteq A$ , and  $F$  is  $\tau$  closed, then  $F$  is  $\tau_A$  closed.
- If  $B \subseteq A$ , then  $y$  is a limit point of  $B$  in  $(X, \tau)$  iff  $y$  is a limit point of  $B$  in  $(A, \tau_A)$ .

Q3: Let  $X \neq \emptyset$ , and  $A \subseteq X$ . Define a topology on  $X$  by  $\tau = \{X \text{ or } B \subseteq X : B \cap A = \emptyset\}$ . Describe  $\tau_A$ .

Q4: Let  $(X, \tau)$  be a topological space, and let  $B \subseteq A \subseteq X$ . Show that  $x_0$  is a limit point of  $B$  in  $(X, \tau)$  iff  $x_0$  is a limit point of  $B$  in  $(A, \tau_A)$ .

### Section 3.2:

Q1: Let  $\tau = \{\mathbb{R}, \emptyset, (-\infty, 7) \cup (7, \infty), \{7\}\}$  be a topology on  $\mathbb{R}$ . Let

$f : (\mathbb{R}, \tau) \rightarrow (\mathbb{R}, \mathcal{U})$  be the identity map. Is  $f$  a continuous map?

Q2: Let  $(X, \tau), (Y, \eta)$  be topological spaces. Let  $f : (X, \tau) \rightarrow (Y, \eta)$ . Show that:

- $f$  is closed iff  $cl(f(A)) \subseteq f(cl(A)), \forall A \subseteq X$ .
- $f$  is continuous iff  $(f^{-1}(B)) \subseteq f^{-1}(cl(B)), \forall B \subseteq Y$ .

Q3: Let  $f : (X, \tau) \rightarrow (Y, \eta)$ . Prove that  $f$  is continuous  $\Leftrightarrow$  (If  $N$  is a nbh of  $f(x)$  where  $x \in X$ , then  $f^{-1}(N)$  is a nbh of  $x$  in  $X$ )  $\Leftrightarrow$  for  $B \subseteq Y, f^{-1}(Int B) \subseteq Int(f^{-1}(B))$

Q3: Let  $\tau = \{A \subseteq \mathbb{R}; A = \mathbb{R} \text{ or } 2 \notin A\}$ . Let  $B \subset \mathbb{R}$  such that  $2 \notin B$ . Show that  $i : (B, \tau_B) \rightarrow (\mathbb{R}, \tau)$  is open.

**Section 3.3:**

Q1: Let  $(\mathbb{R}, \tau)$  be a topological space, where  $\tau = \{A \subseteq \mathbb{R} : A = \mathbb{R} \text{ or } \{1, 2, 3\} \cap A = \emptyset\}$ .  
Prove that  $(\mathbb{R}, \tau)$  is homeomorphic to  $(\mathbb{R}, \mathcal{U})$ , where  $\mathcal{U}$  is the usual topology on  $\mathbb{R}$ .

is not topology, why?

is  $(\mathbb{R}, \tau)$  homeomorphic to  $(\mathbb{R}, \mathcal{U})$