King Saud University
Math Department
Women Section

Second Midterm Exam
373 Math
Second term 1427 H

Question I

a) If $X \neq \phi$, and $A \subseteq X$ .Define a topology on X by $\tau = \{X \text{ or } B \subseteq X : B \cap A = \phi\}$. Describe τ_A ?

b) Let (X,τ) be a topological space, and let $B \subseteq A \subseteq X$ .Show that x_0 is a limit point of B in (X,τ) if and only if, x_0 is a limit point for B in (A,τ_A).

Question II

a) Let $f:(X,\tau) \to (Y,\eta)$.Prove that f is continuous $\Leftrightarrow$ If N is a nhd of $f(x)$ where $x \in X$, then $f^{-1}(N)$ is a nhd of $x \in X$. $\Leftrightarrow$ for $B \subseteq Y, f^{-1}(Int\ B) \subseteq Int(f^{-1}(B))$.

b) Let (X,τ) be a topological space. Show that X is Hff if and only if ,for any fixed $a \in X$ and each $x \in X$ with $x \neq a$,there is an open set $V \in \tau$ such that $a \in V$ and $x \notin cl(V)$.

Question III

a) Consider R with the following topologies The usual topology $\mathcal{U}$,the discrete topology $\mathcal{D}$ and the cofinite topology $\mathcal{F}$. Show that:

1) $\mathcal{F} \subseteq \mathcal{U} \subseteq \mathcal{D}$.
2) $i:(R,\mathcal{U}) \to (R,\mathcal{F})$ is continuous.
3) $i:(R,\mathcal{F}) \to (R,\mathcal{U})$ is open.
4) $i:(R,\mathcal{F}) \to (R,\mathcal{D})$ is not homeomorphism.

b) Let (X,τ) be a topological space such that X has the following property " For every closed set $F \subseteq X$ and each point $p \in X - F$ there are open sets $O,U \in \tau$ such that $p \in O$ and $F \subseteq U$ with $O \cap U = \phi$ ".Show that the above property is a topological property.

Good luck.