KING SAUD UNIVERSITY
First Midterm Exam Of Math **282**, Spring of 1422-1423
Duration two hours

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1. Define each of the following :

   - The Archimedean property.
   - A convergent sequence.
   - A monotone sequence.
   - A subsequence of a sequence.
   - a nested sequence of intervals.

2. A- Prove or disprove the following statements :

   (a) the sequence $(\frac{n}{3^n})$ converges to zero.

   (b) if $y > 0$ then there exists $n \in \mathbb{N}$ such that $1/2^n < y$.

   (c) the sequence $((-1)^n + 1/n)$ is convergent.

   (d) $inf\{(\frac{1}{n})^{1/3} | n \in \mathbb{N}\} = 0$.

   B- Let S be a nonempty subset of $\mathbb{R}$ and let u be an upper bound of S. Then $u = sup(S)$ if and only if for each $\epsilon > 0$ there exists an $s_0 \in S$ such that $u - \epsilon < s_0$.

   C- Prove that if $lim_{n\to\infty} x_n = x$ and if $x > 0$, then there exists $M \in \mathbb{N}$ such that $x_n > 0$ for all $n \geq M$.

3. A- Let S be a bounded nonempty subset of $\mathbb{R}$. Show that

$$sup(-4S) = -4inf(S).$$

   B- Show that $lim_{n\to\infty}(C^{1/n}) = 1, \quad \text{if } C > 1.$

   C- Let $x_1 > 1$ and $x_{n+1} = 2 - 1/x_n$ for $n \geq 2$. Show that the sequence (x_n) is bounded and monotone. Find the limit.

4. A- **State** and **prove** the Bolzano-Weierstrass theorem.

   B- Use the ϵ definition of the limit to show that

$$lim(\frac{2n+1}{n+4}) = 2.$$