First Term 1411.
Second Mid Term Exam
Math 282

Answer the following Questions

1) a- Prove that $S \subseteq R$ is bounded iff $\exists$ some closed bounded interval $I \subseteq R$ s.t $S \subseteq I$

b- Let $f: R \to R$ be continuous on R show that $A = \{x \in R, f(x) = 0\}$ is closed in R.

2) a- Let $f: R \to R$ be defined as follows
$$f(x) = \begin{cases} x & x \text{ is rational} \\ 0 & x \text{ is irrational} \end{cases}$$
show that f has a limit at zero only.

b- Let $f: R \to R$ be continuous at $c \in R$ where $f(c) < 0$ show that $\exists$ a n.b.d U of c s.t for $x \in U$ $f(x) < 0$.

3) a- let $I = [a, b]$, $f: I \to R$ be continuous on I let $a < c < b$ such that f has an absolute maximum at c. Show that f is not injective.

b- Let $f: R \to R$ s.t
$$f(x) = \begin{cases} x^3 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$
show that f is differentiable on R, where is f' continuous and where is differentiable.

4 Let f, g be continuous on R s.t $f(r) = g(r)$ for $r \in Q$ show that $f(x) = g(x)$ for $x \in R$.

b) let $f: (a, b) \to R$ be continuous on (a, b) and differentiable at $x_0 \in (a, b)$ let
$$g(x) = \frac{f(x) - f(x_0)}{x - x_0} \quad x \in (a, b) / \{x_0\}$$
$$g(x_0) = f' x_0$$
Show that g is continuous on (a, b).