Root Locus Example - III

Asymptote

Given open loop transfer function: $G_0(s) = \dfrac{k(s+1)}{s(s+2)(s^2+12s+40)}$

Poles at: 0, -2, -6+*j*2, -6-*j*2

Zero at: -1

Asymptote (using only real values): $\dfrac{(0-2-6-6)-(-1)}{3} = -4.33$

Slopes of the asymptotes: $\dfrac{\pm 180°(2k+1)}{3} = \pm 60°(2k+1) \quad k = 0,1,2,\ldots$

$= 60°, 120°, -60°$

Root Locus Example - II

Breakaway – break in points

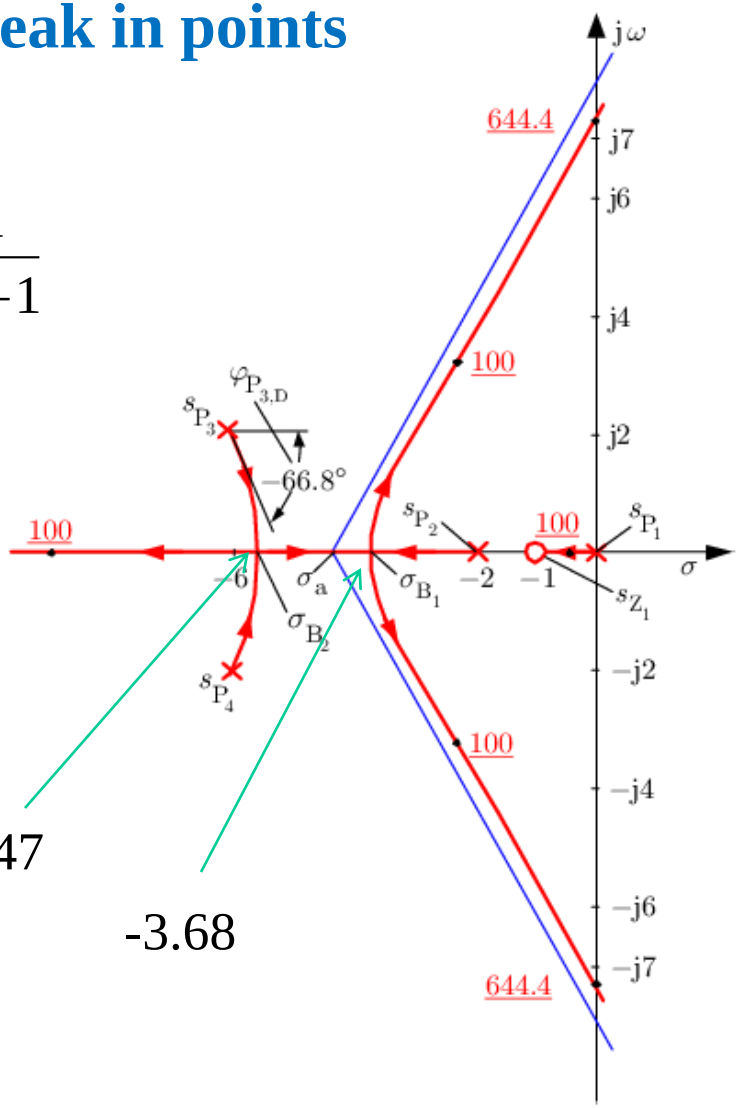
$$\frac{1}{s}+\frac{1}{s+2}+\frac{1}{s+6-j2}+\frac{1}{s+6+j2}=\frac{1}{s+1}$$

$$\Rightarrow 3s^4+32s^3+106s^2+128s+80=0$$

$$\Rightarrow s=-3.68,-5.47,-0.76\pm j0.866$$

Breakaway and break in points cannot be complex numbers.

Breakaway point = -3.68
Break in point = -5.47

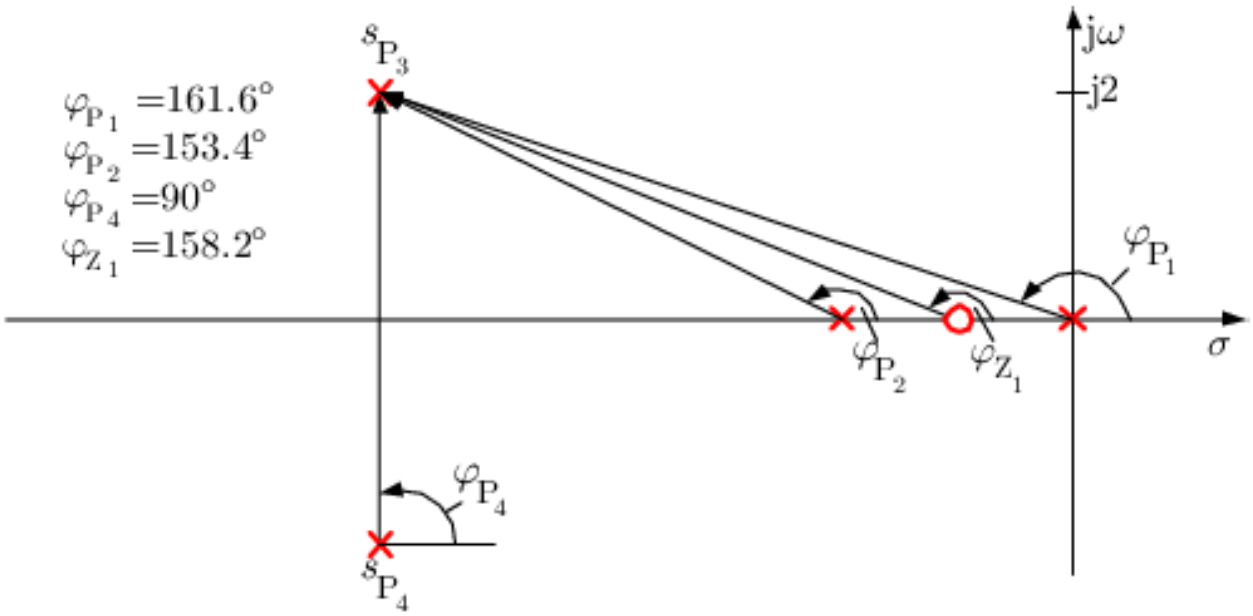


Root Locus Example - II

Angle of Departure

From -6+j2:

$$\varphi_D = -90° - 153.4° - 161.6° + 158.2° \pm 180°(2k+1)$$
$$= -246.8° + 180° = -66.8°$$



Root Locus Example - II

Imaginary Axis Crossing

$$s(s+2)(s^2+12s+40)+k(s+1)$$
$$=s^4+14s^3+64s^2+(80+k)s+k$$

S^4	1	64	k
S^3	14	80+k	0
S^2	(816-k)/14	k	0
S^1	$(65084+540k-k^2)$ /(816-k)	0	0
S^0		0	0

$$k^2-540k-65084=0 \Rightarrow k=644.4$$

$$\frac{(816-k)}{14}s^2+k=0$$
$$\Rightarrow s=\pm j7.2$$