A New Bounding Procedure for the Bin Packing Problem

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1 Introduction

Given a set J of n items where each item j $(1 \leq j \leq n)$ has a positive integer weight w_j, and a set of identical bins of capacity C, the *Bin Packing Problem* (BPP) is to assign each item to exactly one bin in such a way that the cumulated item weight in each bin does not exceed C and the number of loaded bins is minimal. This problem is $\mathcal{NP}$-hard in the strong sense. During the last decade, much efforts have been devoted to the development of effective local search methods, as well as more or less sophisticated polynomial-time approximation algorithms. However, there might be several practical situations in which one is interested in getting an accurate and practical estimate of the empirical performance of BPP heuristics by resorting to computational experiments. Therefore, there is a need for computing fast and tight lower bounds. Moreover, a second important motivation for the investigation of tight bounding strategies is the well-known fact that the quality of the lower bound is of primary importance for the design of an efficient exact branch-and-bound algorithm.

In this paper, we present an effective procedure which aims at strengthening previously developed lower bounds. We restrict our attention to lower bounds $L(.)$ that can be computed in linear time and that are said to be *regular* (i.e. $L(S \cup \{i\}) \leq L(S \cup \{j\})$ for all $S \subseteq J$ and $i, j \in J \backslash S$ such that $w_i \leq w_j$). Preliminary computational results provide empirical evidence of the effectiveness of our procedure and show that it yields new fast lower bounds that outperform lower bounds from the literature.

2 The bounding procedure

Haouari and Gharbi [3] proved (in a slightly different form within the context of multiprocessor scheduling) that if a feasible solution of a BPP with n items fills up exactly b bins, then there is at least a set of k bins $(1 \leq k \leq b)$ which must contain at least $k \lfloor n/b \rfloor + \min(k, n - \lfloor n/b \rfloor b)$ items. This result offers a practical alternative for improving a given lower bound $L(.)$ in the following way. Given a BPP instance defined on a set J of n items, let $m = L(J)$ and define for each $k = 1, ..., m-1$, $\lambda(k, m, n) = k \lfloor n/m \rfloor + \min(k, n - \lfloor n/m \rfloor m)$. Let $S_n^{m,k} \subseteq J$ denote the set of the $\lambda(k, m, n)$ lightest items in J. If $L(S_n^{m,k}) > k$ for some $k = 1, ..., m-1$, then $m+1$

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is a valid lower bound for J. It is worth noting that a further improvement can be obtained if all the subsets of J are considered. However, by virtue of the regularity property, the subset of the l heaviest items clearly dominates any subset of J of cardinality l. Let $S_l^{m,k}$ denote the set of the $\lambda(k, m, l)$ lightest items chosen among the l heaviest ones of J ($1 \leq l \leq n$). Clearly, if $L(S_l^{m,k}) > k$ for some $k = 1, ..., m-1$ and $l = 1, ..., n$ then $m+1$ is a valid lower bound for J.

Based upon these results, an improved lower bound $\bar{L}(.)$ is equal to the minimal value of m satisfying $L(J) \leq m$ and $L(S_l^{m,k}) \leq k$ for $k = 1, ..., m-1; l = 1, ..., n$. A straightforward computation of $\bar{L}(J)$ (which requires $O(n^3)$ computations of $L(.)$) consists in computing $L(S_l^{m,k})$ for all (k, m, l). However, we prove that the only values of l that have to be considered in the computation of $\bar{L}(.)$ are those such that $l = \alpha m + k$, where $1 \leq \alpha \leq \lfloor (n-k)/m \rfloor$. Also, we show that if for given k, m and α, we have $L(S_{\alpha m+k}^{m,k}) \leq k$, then α has not to be considered for larger values of m. Similarly, if for given k and m, we have $L(S_{\alpha m+k}^{m,k}) \leq k$ for all $1 \leq \alpha \leq \lfloor (n-k)/m \rfloor$, then k has not to be considered for larger values of m. The above results yield only $O(n)$ required computations of $L(.)$.

3 Preliminary computational results

Two classes of instances were randomly generated in a similar way as the *hardest* instances described in [2] and [1], respectively. We compared lower bounds from the literature (namely the trivial lower bound $L_1 = \left\lceil (\sum_{j \in J} w_j)/C \right\rceil$, the lower bound L_2 of Martello and Toth [4], and the lower bound $L_*^{(20)}$ of Fekete and Schepers [2]) to their improved versions. We observed that the improved bounds often strictly dominate the original ones. More interestingly, although the bounding procedure of Fekete and Schepers is known to perform remarkably well, the proposed procedure was able to yield even tighter bounds in 28 out of 700 instances of Class 1, and 173 out of 400 instances of Class 2. Also, even when the original bound is rather poor, the improved one often equals the best one. For instance, we observed that the trivial lower bound L_1 yields a remarkably tight improved lower bound which was found equal to the best one for about 94% of the generated instances. Finally, the improved lower bounds require very short computing times. For $n = 1000$, the largest average computation time was only 0.04 sec. on a Pentium IV 2.8 GHz personal computer.

References

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