Electrical Machines

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Chapter 1  
Magnetic Circuits  

Chapter 2  
DC Machines  

Chapter 3  
The Transformer  

Chapter Four  
Three Phase Induction Machine  

Chapter Five  
Synchronous Generator
Chapter 1

Magnetic Circuits

1.1 Introduction

Practically all transformers and electric machinery use magnetic material for shaping and directing the magnetic fields which act as the medium for transferring and converting energy. Thus it is important to analyze and describe magnetic field quantities for understanding these devices. Magnetic materials play a big role in determining the properties of a piece of electromagnetic equipment or the electric machine and affect its size and efficiency.

In electrical machines, ferromagnetic materials may form the magnetic circuits only (as in transformers) or by ferromagnetic materials in conjunction with an air medium (as in rotating machines). In most electrical machines, except permanent magnet machines, the magnetic field (or flux) is produced by passing an electrical current through coils wound on ferromagnetic materials.

This chapter will develop some basic tools for the analysis of magnetic field systems and will provide a brief introduction to the
properties of practical magnetic materials. These results will then be applied to the analysis of transformers and rotating machines. So a carfull study for this chapter is recommended to fully understand the next chapters.

1.2 Magnetic Field Intensity, H And Flux Density, B

When a conductor carries current a magnetic field is produced around it, as shown in Fig.1.1. The direction of flux lines or magnetic field intensity \( H \) (A/m) can be determined by what is known as the thumb rule.

**thumb rule**

*“If the conductor is held with the right hand with the thumb indicating the direction of current in the conductor then, the fingertips will indicate the direction of magnetic field intensity.”*

Fig.1.1 can explains Thumb rule.
**Magnetic Circuit**

Fig.1.1 Field around an infinitely long, straight conductor carrying a current.

**Ampere’s law:**

The magnetic field intensity $H$ around a closed contour $C$ is equal to the total current passing through any surface $S$ linking that contour which is known as Ampere’s law as shown in equation (1.1)

$$\oint H \cdot dl = \sum i$$  \hspace{1cm} (1.1)

where $H$ is the magnetic field intensity at a point on the contour and $dl$ is the incremental length at that point.

Suppose that the field strength at point $C$ distant $r$ meters from the center of the conductor is $H$. Then it means that if a unit N-pole is placed at $C$, it will experience a force of $H$ Newton. The direction of this force would be tangential to the circular line of force passing through $C$. If the unit N-pole is moved once round the conductor against this force, then work done, this work can be obtained from the following relation:

$$Work = Force \cdot dis\tan ce = H \cdot 2\pi \cdot r$$  \hspace{1cm} (1.2)

The relationship between the magnetic field intensity $H$ and the magnetic flux density $B$ is a property of the material in which the
field exists which is known as the permeability of the material; Thus,

\[ B = uH \]  \hspace{1cm} (1.3)

where \( u \) is the permeability.

In SI units \( B \) is in webers per square meter, known as tesla (T), and

In SI units the permeability of free space, Vacuum or nonmagnetic materials is \( u_0 = 4\pi \ast 10^{-7} \). The permeability of ferromagnetic material can be expressed in terms of its value relative to that of free space, or \( u = u_0 \ast u_r \). Where \( u_r \) is known as relative permeability of the material. Typical values of \( u_r \) range from 2000 to 80,000 for ferromagnetic materials used in transformers and rotating machines. For the present we assume that \( u_r \) is a known constant for specific material, although it actually varies appreciably with the magnitude of the magnetic flux density.

Fig.1.2 shows a simple magnetic circuit having a ring-shaped magnetic core, called toroid, and a coil that extends around it. When current \( i \) flows through the coil of \( N \) turns, magnetic flux is mostly confined in the core material. The flux outside the toroid, called leakage flux, is so small that for all practical purposes it can be neglected. Consider a path at main radius \( r \). The magnetic intensity
on this path is $H$ and, from Ampere's circuit law, the following relation can be obtained:

$$\int H \cdot dl = N_i$$  \hspace{1cm} (1.4)

Then $Hl = H \cdot 2\pi r = N_i$  \hspace{1cm} (1.5)

Where $r = \frac{1}{2} \left( \frac{ID + OD}{2} \right)$

Where as shown in Fig.1.2 ID and OD are inner and outer diameter of the core of the triode.

The quantity $N_i$ is called the magnetomotive force (mmf), and its unit is Ampere-turn ($\text{At}$).

$$H = \frac{N_i}{l} \text{ At/m}$$  \hspace{1cm} (1.6)

From Eqs. (1.3) And (1.6)

$$B = \frac{uN_i}{l} \text{ Tesla}$$  \hspace{1cm} (1.7)
If we assume that all the fluxes are confined in the toroid, that is, there is no magnetic leakage, the flux crossing the cross section of the toroid is:

\[ \phi = \int B \, dA \quad (1.8) \]

Then \( \phi = B \, A \, \text{Web.} \) \quad (1.9)

Where \( B \) is the average flux density in the core and \( A \) is the area of cross section of the toroid. The average flux density may correspond to the path at the mean radius of the toroid. If \( H \) is the magnetic intensity for this path, then from Eqs. (1.7) and (1.9),

\[ \phi = \frac{uNi}{l} \, A = \frac{Ni}{l/uA} = \frac{Ni}{\mathcal{R}} = \frac{mmf}{\mathcal{R}} \quad (1.10) \]

Where \( \mathcal{R} = \frac{l}{uA} \) \quad (1.11)

\( \mathcal{R} \) is called the reluctance of the magnetic. Equation (1.10) suggests that the driving force in the magnetic circuit of Fig.1.2 is the magnetomotive force \( mmf \), which produces a flux \( \phi \) against a magnetic reluctance \( \mathcal{R} \). The magnetic circuit of the toroid can therefore be represented by a magnetic equivalent circuit as shown in Fig.1.3. Also note that Equation (1.10) has the form of Ohm's law for an electric circuit \( (i = E/R) \). The
**Magnetic Circuit**

analogous electrical circuit is shown in Fig.1.3. A magnetic circuit is often looked upon as analogous to an electric circuit. The analogy is illustrated in Table 1.1.

![Diagram of magnetic and electric circuits](image)

Fig.1.3 Analogy between (a) magnetic circuit and (b) electric circuit

Table 1.1 Magnetic versus Electrical circuits.

<table>
<thead>
<tr>
<th>Items</th>
<th>Magnetic circuit</th>
<th>Electric circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving force</td>
<td>( \text{mmf} (F=NI) )</td>
<td>( E\text{mf}=E )</td>
</tr>
<tr>
<td>Produces</td>
<td>Flux ( \phi = \frac{F}{\mathcal{R}} )</td>
<td>Current ( i = \frac{E}{R} )</td>
</tr>
<tr>
<td>Limited By</td>
<td>Reluctance ( \mathcal{R} = \frac{1}{uA} )</td>
<td>Resistance ( R = \frac{l}{\sigma A} )</td>
</tr>
</tbody>
</table>

Where, \( u \) is the permeability and \( \sigma \) is the conductivity.

**1.3 Magnetization Curve**

A typical ferromagnetic material is silicon steel, which is widely used for the cores of transformers and rotating machines. When
such a material is magnetized by slowly increasing the applied magnetizing force $H$, the resulting flux density $B$ follows a curve of the form shown in Fig. 1.4. This is known as the magnetization curve for the material.

![Magnetization curves for different magnetic materials.](image)

Fig. 1.4 Magnetization curves for different magnetic materials.

The part of the magnetization curve where the slope begins to change rapidly is termed the **knee**. Below the knee it is often
possible to use a linear approximation to the actual characteristic, with a corresponding constant value for the relative permeability. But the onset of saturation above the knee marks a dramatic change in the properties of the material, which must be recognized in the design and analysis of magnetic structures.

Transformers are wound on closed cores like that of Fig.1.5. Energy conversion devices which incorporate a moving element must have air gaps in their magnetic circuits. A magnetic circuit with an air gap is shown in Fig.1.5. When the air gap length $g$ is much smaller than the dimensions of the adjacent core faces, the magnetic flux $\phi$ is constrained essentially to reside in the core and the air gap and is continuous throughout the magnetic circuit. Thus, the configuration of Fig.1.5 can be analyzed as a magnetic circuit with two series components:

- a magnetic core of permeability $u_o * u_r$ and mean length $l$, and
- an air gap of permeability $u_0$, cross-sectional area $A_g$, and length $g$.

In the core the flux density is uniform, and the cross-sectional area is $A_c$ thus in the core,

$$\frac{B_c}{A_c} = \frac{\phi}{A_c} \quad (1.12)$$
The magnetic field lines bulge outward somewhat as they cross the air gap. This phenomena known as **fringing of magnetic field**.

The effect of the fringing fields is to increase the effective cross-sectional area $A_c$ of the air gap as illustrated in Fig.1.6. Various empirical methods have been developed to account for this effect. A correction for such fringing fields in short air gaps can be made by adding the gap length to each of the two dimensions making up its cross-sectional area. If fringing is neglected,

$$A_g = A_c \text{ and } B_g = B_c = \frac{\phi}{A_c}$$  \hspace{1cm} (1.14)
Example 1.1 In the magnetic system of Fig.1.7 two sides are thicker than the other two sides. The depth of the core is 10 cm, the relative permeability of the core, $\mu_r = 2000$, the number of turns $N = 500$, and the current flowing through the coil is $i = 1 \text{ A}$.

(a) Determine the flux in the core.

(b) Determine the flux densities in the parts of the core.

(c) Find the current $i$ in the coil to produce a flux ($\phi = 0.012\text{ Wb}$).
Solution:

(a) 

\[ R_{\text{Thick}} = \frac{70 \times 10^{-2}}{2000 \times 4 \times \pi \times 10^{-7} \times 15 \times 10^{-4}} = 18568.03 \text{ At/} \text{Web} \]

\[ R_{\text{Thin}} = \frac{80 \times 10^{-2}}{2000 \times 4 \times \pi \times 10^{-7} \times 10 \times 10 \times 10^{-4}} = 31830.91 \text{ At/} \text{Web} \]

Then, \( R_{\text{Thick}} + R_{\text{Thin}} = 50398.94 \text{ At/} \text{Web} \)

Then, \( \phi = \frac{500 \times 1}{50398.94} = 0.009921 \text{ Wb} \)

(b) \( B_{\text{Thick}} = \frac{0.009921}{150 \times 10^{-4}} = 0.6614 \text{ T} \)

\( B_{\text{Thin}} = \frac{0.009921}{100 \times 10^{-4}} = 0.9921 \text{ T} \)

(c) \( B_{\text{Thick}} = \frac{0.012}{15 \times 10 \times 10^{-4}} = 0.8 \text{ T} \)
**Magnetic Circuit**

\[
B_{Thin} = \frac{0.012}{10 \times 10 \times 10^{-4}} = 1.2 \, T
\]

\[
H_{Thick} = \frac{0.8}{2000 \times 4\pi \times 10^{-7}} = 318.31 \, At/m
\]

\[
H_{Thin} = \frac{1.2}{2000 \times 4\pi \times 10^{-7}} = 477.46 \, At/m
\]

\[
F = 318.31 \times 2 \times 35 \times 10^{-2} + 477.46 \times 2 \times 40 \times 10^{-2} = 604.79 \, At
\]

Then, \( i = \frac{604.79}{500} = 1.2096 \, A \)

**Example 1.2** A circular ring of magnetic material has a mean length of 1.0 m and a cross-sectional area of 0.001 m². A saw cut of 5 mm width is made in the ring. Calculate the magnetizing current required to produce a flux of 1.0 mWb in the air-gap if the ring is wound uniformly with a coil of 200 turns. Take relative permeability of the ring material =500 and neglect leakage and fringing.
**Solution:**

\[ B = \frac{1 \times 10^{-3}}{0.001} = 1 \text{Wb/m}^2 \]

The ampere-turn for air gap is:

\[ AT_a = \frac{B}{\mu_0} \times l = \frac{1}{4\pi \times 10^{-7}} \times 5 \times 10^{-3} = 3.978 \text{AT} \]

The ampere-turn for core material is:

\[ AT_c = \frac{B}{\mu_0 \mu_r} \times l = \frac{1}{4\pi \times 10^{-7} \times 500} \times 1 = 1.591 \text{AT} \]

Total \( AT = 3.978 + 1.591 = 5.569 \text{AT} \);

Exciting current \( = \frac{5.569}{200} = 27.35 \text{A} \)

Another Solution of Example 1.2

Assume \( \mathcal{R}_g \) and \( \mathcal{R}_c \) are the reluctance of air gap and core respectively. So;

\[ \mathcal{R}_g = \frac{1}{u_o A_g} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.001} = 3.98 \times 10^6 \text{At/Wb}. \]
The total reluctance is
\[ \mathcal{R} = \mathcal{R}_g + \mathcal{R}_c = 3.98 \times 10^6 + 1.59 \times 10^6 = 5.57 \times 10^6 \text{ At/WEB}. \]

Then Amper turn required is \( \phi \mathcal{R} = 0.001 \times 5.57 \times 10^6 = 5.570 \text{ At} \)

Then exciting current is:
\[ \frac{\text{Total At}}{\text{number of turns}} = \frac{5570}{200} = 27.35 \text{ A} \]

**Example 1.3.** A ring of mean diameter 21 cm and cross-section 10 cm\(^2\) is made up of semi-circular sections of cast steel and cast iron. If each joint has reluctance equal to an air gap of 0.2 mm as shown in Fig.1.8, find the Amp. turn required to produce a flux of \(5 \times 10^{-4}\) weber in the magnetic circuit. Take \(u_r\) for steel and iron as 825 and 165 respectively: Neglect leakage and fringing.
Fig. 1.8

Solution:

\[ \phi = 5 \times 10^{-4} \, \text{Wb} ; A = 10 \, \text{cm}^2 = 10^{-3} \, \text{m}^2 \]

Then, \[ B = 5 \times 10^{-4}/10^{-3} = 0.5 \, \text{Wb/m}^2 \]

\[ H = \frac{B}{u_o} = \frac{0.5}{4\pi \times 10^{-7}} = 3.977 \times 10^5 \, \text{A}t / \text{m} \]

Air gap length is \[ = 0.2 \times 2 = 0.4 \, \text{mm} = 4 \times 10^{-4} \, \text{m} \]

Then, Ampere-turn required \[ = 3.977 \times 10^5 \times 4 \times 10^{-4} = 159 \, \text{At} \]

Cast steel path \[ H = \frac{B}{u_o u_r} = \frac{0.5}{4\pi \times 10^{-7} \times 825} = 482 \, \text{A}t / \text{m} \]

Cast steel path length = \( \pi \times D / 2 \, \text{cm} \)

\[ = 21\pi / 2 = 33 \, \text{cm} = 0.33 \, \text{m} \]

\[ \text{AT required} = Hi = 482 \times 0.33 = 159 \, \text{At} \]

Cast iron path \[ H = 0.5 / 4\pi \times 10^{-7} \times 165 = 2411 \, \text{A}t / \text{m} \]

Cast iron path length is:

\[ \pi \times D / 2 = \pi \times 21 / 2 = 0.33 \, \text{m} \]

Ampere turn required \[ = 2411 \times 0.33 = 795.6 \, \text{At} \]

Then the total Ampere turn required \[ = 159 + 159 + 795.6 = 1113.6 \, \text{At} \]

Example 1.4 Two coils are wound on a toroidal core as shown in Fig. 1.9. The core is made of silicon sheet steel and has a square cross section. The coil currents are \( i_1 = 0.28 \, \text{A} \) and \( i_2 = 0.56 \, \text{A} \).
(a) Determine the flux density at the mean radius of the core.

(b) Assuming constant flux density (same as at the mean radius) over the cross section of the core, determine the flux in the core.

(c) Determine the relative permeability,

Solution:

The Two mmfs aid each other. Then,

\[ mmf = 600 \times 0.28 + 300 \times 0.56 = 336 \, \text{At} \]

\[ H = \frac{336}{2\pi \times 20 \times 10^{-2}} = 267.38 \, \text{At/m} \]

\[ B = 1.14 \, \text{T} \quad \text{From the curve shown in Fig.1.4} \]

(b) \[ \phi = 1.14 \times 2 \times 2 \times 10^{-4} = 0.000456 \, \text{Wb} \]

(c) \[ u_r = \frac{B}{u_0 H} = \frac{1.14}{4\pi \times 10^{-7} \times 267.38} = 3393 \]
Example 1.5 The magnetic circuit of Fig. 1.10 provides flux in the two air gaps. The coils \( N_1 = 700, N_2 = 200 \) are connected in series and carry a current of 0.5 ampere. Neglect leakage flux, reluctance of the iron (i.e., infinite permeability), and fringing at the air gaps. Determine the flux and flux density in the air gaps.

\[ l_{g1} = 0.05 \text{ cm}, \quad l_{g2} = 0.1 \text{ cm} \]
\[ l_1 = l_2 = l_4 = l_5 = 2.5 \text{ cm} \]
\[ l_3 = 5 \text{ cm} \]
\[ \text{depth of core} = 2.5 \text{ cm} \]

Fig. 1.10

Solution:

mmafs of the two coils oppose each other

Then,
\[ \begin{align*}
A_{g1} &= A_{g2} = 2.5 \times 2.5 \times 10^{-4} = 6.25 \times 10^{-4} \text{ m}^2 \\
R_{g1} &= \frac{0.05 \times 10^{-2}}{4\pi \times 10^{-7} \times 6.25 \times 10^{-4}} = 0.637 \times 10^6 \text{ At/Wb} \\
R_{g2} &= \frac{0.1 \times 10^{-2}}{4\pi \times 10^{-7} \times 6.25 \times 10^{-4}} = 1.274 \times 10^6 \text{ At/Wb} \\
\phi_1 &= \frac{(700 - 200) \times 0.5}{0.637 \times 10^6} = 0.392 \times 10^{-3} \text{ Wb} \\
\phi_2 &= \frac{500 \times 0.5}{1.274 \times 10^6} = 0.196 \times 10^{-3} \text{ Wb} \\
\phi &= \phi_1 + \phi_2 = 0.588 \times 10^{-3} \text{ Wb} \\
\text{Then, } B_{g1} &= \frac{0.392 \times 10^{-3}}{6.25 \times 10^{-4}} = 0.627 \text{ Wb/m}^2 \\
\text{And, } B_{g2} &= \frac{0.196 \times 10^{-3}}{6.25 \times 10^{-4}} = 0.3135 \text{ Wb/m}^2
\end{align*} \]
Example 1.6 The electromagnet shown in Fig. 1.13 can be used to lift a length of steel strip. The coil has 500 turns and can carry a current of 20 amps without overheating. The magnetic material has negligible reluctance at flux densities up to 1.4 tesla. Determine the maximum air gap for which a flux density of 1.4 tesla can be established with a coil current of 20 amps. Neglect magnetic leakage and fringing of flux at the air gap.
Solution:

\[ B = 1.4 \text{ T Throughout } H_c = 0 \]

\[ Ni = H_{g_1} \cdot g_1 + H_{g_2} \cdot g_2 \]

\[ H_{g_1} = H_{g_2} = \frac{B}{u_0} \]

\[ g_1 = g_2 = g \]

\[ Ni = 2 \cdot \frac{B}{u_0} \cdot g \]

\[ g = \frac{u_0 \cdot Ni}{2B} = \frac{4\pi \cdot 10^{-7} \cdot 500 \cdot 20}{2 \cdot 1.4} = 4.5 \text{ mm} \]
For the magnetic circuit shown in Fig.1.15 all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 3.75 cm throughout. The turns are arranged on the center limb as shown. Calculate the mmf required to produce a flux of 1.7 mWb in the center limb. Neglect the leakage and fringing.

The magnetization data for the material is as follows

\[
\begin{array}{ccccc}
H (\text{At/m}) : & 400 & 440 & 500 & 600 & 800 \\
B (\text{Wb/m2}) : & 0.8 & 0.9 & 1.0 & 1.1 & 1.2 \\
\end{array}
\]
From the symmetry of the magnetic circuit we can say that

\[ R_e = R_L \quad \text{and} \quad R_{g1} = R_{g2} \]

Also from the magnetization data we can draw the following figure:-
The average value of $u_o \approx \frac{1}{500} = 0.002$

$$R_r = \frac{30 \times 10^{-2}}{0.002 \times 2 \times 3.75 \times 10^{-4}} = 200000 \text{At/Web.}$$

$$R_{g1} = \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 2 \times 3.75 \times 10^{-4}} = 530516.5 \text{At/Web.}$$

$$R_c = \frac{10 \times 10^{-2}}{0.002 \times 5 \times 3.75 \times 10^{-4}} = 26666 \text{At/Web.}$$

$$R_{gc} = \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 2 \times 3.75 \times 10^{-4}} = 212206 \text{At/Web.}$$
Magnetic Circuit

\[ NI = \phi \left[ R_c + R_{gc} + \frac{R_r + R_{g1}}{2} \right] \]

\[ NI = 1.7 \times 10^{-3} \times \left[ 26666 + 212206 + \frac{2100000 + 530516.5}{2} \right] = 576 \text{ At} \]

1.4 Electromagnetic Induction

In 1820 Oersted discovered the magnetic effect of an electric current, and the first primitive electric motor was built in the following year. Faraday's discovery of electromagnetic induction in 1831 completed the foundations of electromagnetism, and the principles were vigorously exploited in the rapidly growing field of electrical engineering. By 1890 the main types of rotating electrical machine had been invented, and the next forty years saw the development of many ingenious variations, along with refinement of the basic types. This was the golden age of machine development. Many machines are now obsolete which were once made in large numbers. Thus the cross-field DC machines, or rotary amplifiers, have been replaced by solid-state power amplifiers; while the Schrage motor and other ingenious variable-speed AC machines have given way to the thyristorcontrolled DC motor and the inverter-fed induction motor.
Chapter One

When a conductor moves in a magnetic field, an \textit{EMF} is generated; when it carries current in a magnetic field, a force is produced. Both of these effects may be deduced from one of the most fundamental principles of electromagnetism, and they provide the basis for a number of devices in which conductors move freely in a magnetic field. It has already been mentioned that most electrical machines employ a different form of construction.

Faraday summed up the above facts into two laws known as Faraday's Laws of Electromagnetic Induction, it revealed a fundamental relationship between the voltage and flux in a circuit.

\textbf{First Law} states: -

\textit{Whenever the magnetic flux linked with a circuit changes, an EMF is always induced in it. Whenever a conductor cuts magnetic flux, an EMF is induced in that conductor.}

\textbf{Second Law} states: -

\textit{The magnitude of the induced EMF is equal to the rate of change of flux-linkages.}

\textbf{Explanation.} Suppose a coil has \(N\) turns and flux through it changes from an initial value of \(\phi_1\) webers to the final value of \(\phi_2\), webers in time \(t\) seconds. Then remembering that by flux-linkages
**Magnetic Circuit**

is meant the product of number of turns by the flux linked with the coil, we have the following relation:

Initial flux linkages = $N\phi_1$. And final flux linkages = $N\phi_2$

Then the induced **EMF** is

$$e = \frac{N\phi_2 - N\phi_1}{t} = N\frac{\phi_2 - \phi_1}{t} = N\frac{d\phi}{dt} \text{ Volts}$$

Usually a minus sign is given to the right-hand side expression to signify the fact that the induced **EMF** sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it.

$$e = -N\frac{d\phi}{dt} \text{ Volts}$$

**Example 1.7** The field coils of a 6-pole DC generator each having 500 turns, are connected in series. When the field is excited, there is magnetic flax of 0.02 Wb/pole. If the field circuit is opened in 0.02 second and the residual magnetism is 0.002 Wb/pole, calculate the average voltage that is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current.

**Solution**

Total number of turns, $N=6*500=3000$ turns

Total initial flux = $6 \times 0.02 = 0.12 \text{ Wb}$;

Total residual flux = $6 \times 0.002 = 0.012 \text{ Wb}$,
Chapter One

Change in flux, \(d\phi = 0.12 - 0.012 = 0.108 \text{ Wb}\);

Time of opening the circuit, \(dt = 0.02 \text{ second}\)

Then the induced EMF = \(\frac{d\phi}{dt} = 3000 \times \frac{0.108}{0.02} = 16200 \text{ V}\)

Example 1.8 A coil of resistance 100 \(\Omega\) is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 \(\Omega\) resistance is connected in series with it: Find the average EMF and the current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.

Solution:

Induced Emf, \(e = \frac{d\phi}{dt} \text{ Volts volt}\)

Here \(d\phi = 1 - 0.2 = 0.8 \text{ mWb} = 8 \times 10^{-3} \text{ Wb}\)

\(dt = 1/10 = 0.1 \text{ second} \); \(N = 100\)

\(e = 100 \times 0.8 \times 10^{-3} / 0.1 = 0.8V\)

Total circuit resistance = 100 + 400 = 500 \(\Omega\)

Current induced = 0.8/500 = 1.6 \times 10^{-3} = 1.6 mA

1.5 Direction Of Induced EMF And Current

There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either Fleming's Right-hand Rule or
**Magnetic Circuit**

*Lenz's Law.* Fleming's rule is used where induced EMF is due to, flux cutting (i.e. dynamically induced. EMF) and Lenz's when it is due to change by flux linkages (i.e. statically induced Emf).

**Fleming's Right-Hand Rule**

“Hold out your right hand with forefinger, second finngure, and thumb at right angles to one another. If the forefinger represents the direction of the field, and the thumb represents the direction of the motion then, the second finger represents the direction of the induced emf in the coil”.

Fleming's Right-hand Rule can be explained as shown in Figure

---

**Lenz’s Law**
The direction of the induced current may also be found by this law which was formulated by Lenz. 1835.

Lenz Law states, in effect, that electromagnetically induced current always flows in such a direction that the action of the magnetic field set up by it tends to oppose the very cause, which produces it.

This statement will be clarified with reference to Figs. 1.15 and Fig. 1.16. It is found that when $N$-pole of the bar magnet approaches the coil, the induced current setup by the induced \textit{EMF} flaws in the anti-clockwise direction in the coil as seen from the magnet side. The result is that the face of the coil becomes a $N$-pole and so tends to retard the onward approach of the $N$ pole' of the magnet (tike poles repel each other). The mechanical energy spent in overcoming this repulsive force is converted into electrical energy, which appears in the coil.

![Fig. 1.15](image_url)
When the magnet is withdrawn as in Fig.1.16, the induced current flows in the clockwise direction, thus making the face of the coil (facing the magnet) a S-pole. Therefore, the N-pole of the magnet has to be withdrawn against the attractive force of the S-pole of the coil. Again the mechanical energy required to overcome this force of attraction is converted into electric energy.

It can be shown that the *Lenz's law* is a direct consequence of *law of conservation of energy*. Imagine for a moment that when N pole of the magnet (Fig.1.16) approaches the coil, induced current flows, in such a direction as to make the coil face a S-pole. Then due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy. It means that we would be able to create electric energy out of nothing, which is denied by the inviolable
Law of Conservation of Energy. In fact, to maintain the sanctity of this law, it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to, oppose the very cause, which produces it. In the present case it is the relative motion of the magnet with respect to the coil which is the cause of the production of the induced current. Hence, the induced current always flows in such a direct as to oppose this relative motion (i.e., the approach or withdrawal of the magnet).

**Electromagnetic Force**

The basic principle of motor action is the so called electromagnetic force or Lorentz force production. Lorentz force states that "when a current carrying conductor is placed in a magnetic field, it is subject to a force which we call Lorentz force".

The magnitude of the force depends upon the orientation of the conductor with respect to the direction of the field. The force is greatest when the conductor is perpendicular to the field and zero when it is parallel to it. Between these two extremes, the force has intermediate values.

The maximum force acting on a straight conductor is given by
Magnetic Circuit

\[ F = Bli \]

Where \( F \): Is the force acting on the conductor (N),

\( B \): Is the flux density of the field (T), and,

\( l \): Is the length of the conductor facing the magnetic field (m).

\( i \): the current in the conductor (A).

The direction of the magnetic force can be determined by using the **Fleming left hand rule**. Before going to show this rule, it is better to explain the physical meaning of the **lorentz force**. This can be easily explained by with the help of the following two figures (Fig. 2.32a). For a current flowing into the page of this book, the circular lines of force have the direction shown in Figure 2.32a. The same figure shows the magnetic field created between the N, S poles of a powerful permanent magnet.

The magnetic field does not, of course, have the shape shown in the figure because lines of force never cross each other. What, then, is the shape of the resulting field? To answer the question, we observe that the lines of force created respectively by the conductor and the permanent magnet act in the same direction above the conductor and in opposite directions below it. Consequently, the number of lines above the conductor must be greater than the number below. The resulting magnetic field therefore has the shape given in Figure 2.32b.
Recalling that lines of flux act like stretched elastic bands, it is easy to visualize that a force acts upon the conductor, tending to push it downward.

**Figure 2.32**

a. Magnetic field due to magnet and conductor.

b. Resulting magnetic field pushes the conductor downward.
Now let us Define *Felmeng left hand rule* It is illustrated in Fig. 6-9.

Felmeng left hand rule:
“Hold out your left hand with forefinger, second finger and thumb at right angles to one another. If the forefinger represents the direction of the field, and the second finger that of the current, then thumb gives the direction of the motion or force.”
The direction of the force can also be determined by using the right-hand screw rule, illustrated in Fig.2.2(b).

*Turn the current vector \( i \) toward the flux vector \( B \). If a screw is turned in the same way, the direction in which the screw will move represents the direction of the force \( f \).*

Note that in both cases (i.e., determining the polarity of the induced voltage and determining the direction of the force) the moving quantities (\( v \) and \( i \)) are turned toward \( B \) to obtain the screw movement.

Equations (2.1) and (2.2) can be used to determine the induced voltage and the electromagnetic force or torque in an electric machine. There are, of course, other methods by which these quantities (\( e \) and \( f \)) can be determined.

---

Fig.2.2 Electromagnetic force. (a) Current-carrying conductor moving in a magnetic field. (b) Force direction.
1.6 Coefficient of Self-Induction ($L$)

It may be defined in any one of the three ways given below.

First Method for $L$ The coefficient of self-induction of a coil is defined as the weber turns per ampere in the coil.

By weber-turns is meant the product of flux in webers and the number of turns with which the flux is linked. In other words, it is the flux-linkages of the coil.

Consider a solenoid having $N$ turns and carrying a current of 1 ampere. If the flux produced is $\phi$ webers, then weber-turns are $N\phi$. Hence, weber-turns per ampere are $N\phi/I$. Then by definition, $L = N\phi/I$ Henry \hspace{1cm} (1.14)

Hence, a coil is said to have a self-inductance of one Henry if a current of 1 ampere when flowing through it produces flux linkages of 1 Wb turn in it.

Example 1.9. The field winding of a DC electromagnet is wound with 960 turns and has resistance of 50$\Omega$. When the exciting voltage is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.

Solution:
Current through the coil is $230/50 = 4.6$ A

$$L = \frac{N\phi}{I} = \frac{960 \times 0.005}{4.6} = 1.0435 H$$

Energy stored is

$$\frac{1}{2} LI^2 = \frac{1}{2} \times 1.0435 \times 4.6^2 = 11.84 \text{ Jouls}$$

**In second method** for $L$ as we know before from equation

$$\phi = \frac{uNi}{l} A = \frac{Ni}{l/uA}$$

Then $\frac{\phi}{i} = \frac{N}{l/uA}$, but,

$$L = N \frac{\phi}{I} = 960 \times \frac{0.005}{4.6} = 1.0435 \text{ } H$$

$$= N \frac{N}{l/uA} = \frac{N^2}{l/uA} = \frac{N^2}{\mathcal{R}}$$

**Example 1.10** An air-cored solenoid 1 cm in diameter and 1 meter long has an inductance of 0.1 mH. Find the number of effective turns on the coil.

**Solution:**

$$A = \pi \times r^2 = \pi \times 0.005^2 = 7.85 \times 10^{-5} \text{ } m^2$$
\[ L = \frac{N^2}{l/uA} \quad (1.16) \]

Then

\[ N = \sqrt{L \cdot l/uA} = \sqrt{\frac{0.1 \times 10^{-3} \times 1}{(4\pi \times 10^{-7} \times 7.85 \times 10^{-5})}} = 1007 \text{ turns} \]

**Third method for L,**

As we know before that \( L = \frac{N\phi}{I} \) then \( LI = N\phi \)

Differentiating both sides, we get:

\[-N \frac{d}{dt} \phi = -L \frac{d}{dt} I\]

But we know from Faraday law that \(-N \frac{d}{dt} \phi = \) the induced EMF. Then if \( \frac{dI}{dt} = 1 \text{ Ampere/second} \), and \( \text{EMF}=1 \text{ Volt} \) then \( L=1 \text{H} \).

Then, \( e = L \frac{dI}{dt} \quad (1.17) \)

Hence, a coil has a self-inductance of one hennery if one volt is induced in it when current through it changes at the rate of one ampere/second.
Chapter One

1.7 Mutual Inductance

Mutual inductance may be defined as *the ability of one coil (or circuit) to produce an EMF in a nearby coil by induction when the current in the first coil changes*. This action being reciprocal, the second coil can also induce an EMF in the first when current in the second coil changes. *This ability of reciprocal induction is measured in terms of the coefficient of mutual induction* $M$.

Mutual inductance can also be defined in two ways as given below:

**First method for $M$**

Let there be two magnetically-coupled coils having $N_1$ and $N_2$ turns as shown in Fig.7.6. Coefficient of mutual inductance between two coils is defined as the Weber turns in one coil due to one Ampere current in the other.

---

$L_{12}$ MUTUAL INDUCTANCE [11]

The mutual inductance between two coils,

$$L_{12} = \frac{N_2 \Lambda_{12}}{I_1} = \frac{N_2 N_1 \Psi_{12}}{I_1}$$

$N = \text{number of turns of the coil}$

$\Lambda = \text{flux linkage [Wb]}$

$I = \text{current [A]}$

$\Psi = \text{magnetic flux [Wb]}$

$r = \text{vector to the point of observation}$

$r' = \text{vector to source}$

Neumann formula:

$$L_{12} = \frac{\mu_0 N_1 N_2}{4\pi} \int \int \frac{\mathbf{a}_1 \cdot \mathbf{a}_2}{|r - r'|}$$

$\mu_0 = \text{magnetic permeability of vacuum}$
Let a current of $I_1$ ampere when flowing in the first coil produces a flux $\phi_1$ webers in it. It is supposed that whole of this flux links with the turns of the second coil. Then flux linkages in the second coil for unit current in the first coil are $\frac{N_2}{I_1} \phi_1$. Hence by definition

$$M = \frac{N_2 \phi_1}{I_1}$$

If Weber-turns in second coil due to one ampere current in the first coil i.e. $\frac{N_2 \phi_1}{I_1} = 1$, then, as seen from above, $M=1$ H

Hence, two coils are said to have a mutual inductance of 1 henry if one ampere current flowing in one coil produces flux linkages of one Weber-turn in the other coil.
Example 7-13. Two identical coils X and Y of 1,000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5 A flowing in X produces a flux of 0.5 mWb in it, find the mutual inductance between X and Y.

Solution. Formula used \( M = \frac{N_2 \phi_1}{I_1} \)

Flux produced in X = 0.05 mWb = 0.05 * 10^{-3} Wb

Flux linked with Y = 0.05 * 10^{-3} * 0.8 = 0.04 * 10^{-3}

Then,

\[
M = \frac{1000 \times 0.04 \times 10^{-3}}{5} = 8 \times 10^{-3} \text{ H} = 8 \text{ mH}
\]

Second Method for M

We will now deduce an expression for the coefficient of mutual inductance in terms of the dimensions of the two coils.

Flux in the first coil \( \phi_1 = \frac{N_1 I_1}{l / u_o u_r A} \) Wb

Flux per ampere = \( \frac{\phi_1}{I_1} = \frac{N_1}{l / u_o u_r A} \)

Assuming that whole of this flux (it usually is some percent of it) is linked with the other coil having \( N_2 \) turns, then Weber-turns in it due to the flux/ampere in the first coil is:

\[
M = \frac{N_2 \phi_1}{I_1} = \frac{N_2 \times N_1}{l / u_o u_r A}
\]
Magnetic Circuit

Then \( M = \frac{u_o u_r A N_1 N_2}{l} H \)

Also, \( M = \frac{N_1 N_2}{l/u_o u_r A} H = \frac{N_1 N_2}{Re luc \tan ce} \)

Example 7-15. Calculate the mutual inductance between two toroidal windings which are closely wound on an iron core of \( u_r = 1000 \). The mean radius of the toroid is 8 cm and the radius of the toroid is 8 cm and the radius of its cross-section is 1 cm. Each winding has 1000 turns.

Solution

\( Area = \pi * 1^2 = \pi cm^2 = \pi * 10^{-4} m^2 \)

\( M = \frac{N_1 N_2}{l/u_o u_r A} H = \frac{N_1 N_2}{Re luc \tan ce} \)

\( l = \pi * mean \ diameter = \pi * 16 \ cm = 16\pi * 10^{-2} m \)

Then, \( M = \frac{4\pi * 10^{-7} * 1000 * \pi * 10^{-4} * 1000^2}{16\pi * 10^{-2}} = 0.7855H \)
1.8 Hysteresis loss

When a magnetic material is taken through a cycle of magnetization, energy is dissipated in the material in the form of heat. This is known as the hysteresis loss.

Transformers and most electric motors operate on alternating current. In such devices the flux in the iron changes continuously both in value and direction. The magnetic domains are therefore oriented first in one direction, then the other, at a rate that depends upon the frequency. Thus, if the flux has a frequency of 50 Hz, the domains describe a complete cycle every \(\frac{1}{50}\) of a second, passing successively through peak flux densities \(+B_m\) and \(-B_m\) as the peak magnetic field intensity alternates between \(+H_m\) and \(-H_m\). If we plot the flux density \(B\) as a function of \(H\), we obtain a closed curve called hysteresis loop (Fig.1.18). The residual induction \(B_r\) and coercive force \(H_c\) have the same significance as before.
Figure 1.18 Hysteresis loop. If $B$ is expressed in tesla and $H$ in amperes per meter, the area of the loop is the energy dissipated per cycle, in joules per kilogram.

In describing a hysteresis loop, the flux moves successively from $+B_m$, $+B_r$, 0, $-B_m$, $-B_r$, 0, and $+B_m$, corresponding respectively to points a, b, c, d, e, f, and a, of Fig.1.18. The magnetic material absorbs energy during each cycle and this energy is dissipated as heat. We can prove that the amount of heat released per cycle (expressed in $J/m^3$) is equal to the area (in T-A/m) of the hysteresis
loop. To reduce hysteresis losses, we select magnetic materials that have a narrow hysteresis loop, such as the grain-oriented silicon steel used in the cores of alternating current transformers.

So the net energy losses/cycle/m³ = (hysteresis loop area) Jule

Scale factors of $B$ and $H$ should be taken into consideration while calculating the actual loop area. For example, if the scale are 1 cm = x AT/m for $H$ and 1 cm = y Wb/m² for $B$ Then,

$$W_h = xy * \text{(area of BH loop)} \frac{\text{Joule}}{m^3} / \text{cycle}$$

It may be shown that the energy loss per unit volume for each cycle of magnetization is equal to the area of the hysteresis loop. The area of the loop will depend on the nature of the material and the value of $B_{\text{max}}$ (Fig. 1.18), and an approximate empirical relationship discovered by Steinmetz is:

$$W_h = \lambda h B_{\text{max}}^n \frac{\text{Joules}}{m^3} \quad (1.18)$$

In this expression $W_h$ is the loss per unit volume for each cycle of magnetization; the index $n$ has a value of about 1.6 to 1.8 for many materials; and the coefficient $\lambda_h$ is a property of the material, with typical values of 500 for 4 percent silicon steel and 3000 for cast iron.

When the material is subjected to an alternating magnetic field of constant amplitude there will be a constant energy loss per cycle, and the power absorbed is therefore proportional to the frequency.
Assuming the Steinmetz law, we have the following expression for the hysteresis loss per unit volume

\[ P_h = \lambda_h B_{\text{max}}^{1.6} f \text{ watts/m}^3 \]  \hspace{1cm} (1.19)

Where \( f \) is the frequency in Hertz.

Example

The hysteresis loop of a sample of sheet steel subjected to a maximum flux density of 1.3 Wb/m\(^2\) has an area of 93 cm\(^2\), the scales being 1 cm = 0.1 Wb/m\(^2\) and 1 cm = 50 AT/m. Calculate the hysteresis loss in watts when 1500 cm\(^3\) of the same material is subjected to an alternating flux density of 1.3 Wb/m\(^2\) peak value at a frequency of 65 Hz.

Solution:

\[ \text{loss} = \text{area of loop} \times \text{frequency per cycle} = 0.1 \times 50 \times 93 = 465 \text{ J/m}^3 \text{ / cycle} \]

Then volume = 1500 cm\(^3\)

= 1.5 \times 10^{-3} \text{ m}^3

Then, \( \text{wh} = 465 \times 1.5 \times 10^{-3} \times 65 \text{j/s} = 45.3 \text{ W} \)

Example 8-5

In a transformer core of volume 0.16 m\(^3\) the total iron loss was found to be 2170 W at 50 Hz. The hysteresis loop of the core material, taken to the same maximum flux density, had an area of 9 cm\(^2\) when drawn to scales of 1 cm = 0.1 Wb/m\(^2\) and 1 cm = 250 AT/m. Calculate the total iron loss to the transformer.
core if it is energised to the same maximum flux density, but at a frequency of 60 Hz,

Solution. \( W_h = xy \times \text{(area of hysteresis loop)} \)
where \( x \) and \( y \) are the scale factors.
\[ W_h = 9 \times 0.1 \times 250 = 225 \text{ J/m}^3/\text{cycle} \]
At 50 Hz:
Hystrisis loss = \( 225 \times 0.16 \times 50 = 1800 \text{ W} \)
Eddy current loss = \( 2170 - 1800 = 370 \text{ W} \)
At 60 Hz
Hystris loss = \( 1800 \times 60 / 50 = 2160 \text{ W} \)
Eddy current loss = \( 370 \times (60/50)^2 = 533 \text{ W} \)
Then total loss = \( 2160 + 533 = 2693 \text{ W} \)

Example 8-7. The area of the hysteresis loop obtained with a certain specimen of iron was 9.3 cm\(^2\). The coordinates were such that 1 cm = 1000 AT/m and 1 cm = 0.2 Wb/m\(^2\), calculate (a) Hysteresis loss per m\(^3\) per cycle and (b) the hysteresis loss per m\(^3\) at a frequency of 50 Hz if the maximum flux density were 1.5 Wb/m\(^2\) (c) calculate the hysteresis loss per m\(^3\) for a maximum flux
density of 1.2 wb/m² and a frequency of 30 Hz, assuming the loss to be proportional to $B_{\text{max}}^{1.8}$.

Solution:

(a) $W_h = xy \times (\text{area of BH loop})$

$= 1000 \times 0.2 \times 9.3 = 1860 \text{ J} / \text{m}^3 / \text{cycle}$

(b) $W_h = 1860 \times 50 \text{ J} / \text{s} / \text{m}^3 = 93000 \text{ W} / \text{m}^3$

(C) $W_h = \eta B_{\text{max}}^{1.8} f V W$

For given specimen, $W_h \propto B_{\text{max}}^{1.8} f$

In (b) above, $93000 \times 1.5^{1.8} \times 50$ and $W_h \propto 1.2^{1.8} \times 30$

Then, $\frac{W_h}{93000} = \left(\frac{1.2}{1.5}\right)^{1.8} \times \frac{30}{50}, \text{Wh}$

$= 93000 \times 0.669 \times 0.6 = 37360 \text{ W}$

Example 8.8 (seraga). Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron if the peak flux density reached is 1.3 Wb/m² and the frequency is 25 Hz. Assume Steinmetz iCient as 628 J/m³ and density of iron as $7.8 \times 10^3 \text{ kg/m}^3$. What will be the area of BH curve for this specimen if 1 cm = 12.5 AT/m and 1 cm = 0.1 Wb/m².

Solution:

$W_h = \eta B_{\text{max}}^{1.6} f V W$

$Volume V = \frac{50}{7.8 \times 10^3} = 6.41 \times 10^{-3} \text{ m}^3$
Then, \( W_h = 628 \times 1.3^{1.6} \times 25 \times 6.41 \times 10^{-3} = 153 \, J / s \)

Loss in iron in one hour = 153 * 3600 = 551200J

As per Steinmetz law, hystrisi loss = \( \eta B_{\text{max}}^{1.6} \, J / m^3 \text{cycle} \)

Also, Hystrisi loss = \( xy \times \text{(area of BH loop)} \)

Equating the two we get:

\[ 628 \times 1.3^{1.6} = 12.5 \times 0.1 \times \text{loop area} \]

Then \( \text{loop area} = 764.3 \, cm^2 \)

### 1.10 Eddy current losses

If a closed loop of wire is placed in an alternating magnetic field, the induced \( EMF \) will circulate a current round the loop. A solid block of metal will likewise have circulating currents induced in it by an alternating field, as shown in Fig.1.19. These are termed \textit{eddy currents}, and they are a source of energy loss in the metal. Eddy current losses occur whenever conducting material is placed in a changing magnetic field; the magnitude of the loss is dependent on the properties of the material, its dimensions and the frequency of the alternating field.

Magnetic structures carrying alternating magnetic flux are usually made from a stack of thin plates or laminations, separated from one another by a layer of insulation (Fig.1.20). This
construction breaks up the eddy current paths, with a consequent reduction in the loss; qualitatively, the effect may be explained as follows. With solid metal (Fig.1.19) the currents would flow in approximately square paths; these paths enclose a large area for a given perimeter, and the induced \textit{EMF} is high for a path of given resistance. When the metal is divided into laminations (Fig.1.20), the current paths are long narrow rectangles; the area enclosed by a given perimeter is much smaller, and the induced \textit{EMF} is smaller, giving lower currents and reduced losses.

Fig.1.19                              Fig.1.20
An approximate analysis shows that in plates of thickness \( t \) (where \( t \) is much smaller than the width or length) the eddy current loss per unit volume is given by

\[
P_e = \frac{\pi^2 B_{\text{max}}^2 f^2 t^2}{6 \rho}
\]  

(1.20)
where the flux density is an alternating quantity of the form,

\[ B = B_{\text{max}} \sin 2\pi ft \]  

(1.21)

and \( \rho \) is the resistivity of the material. Thus if the lamination thickness is reduced by a factor \( x \), the loss is reduced by a factor \( x^2 \). As might be expected, the loss varies inversely with the resistivity \( \rho \). The addition of 3-4 percent of silicon to iron increases the resistivity by about four times, as well as reducing the hysteresis loss; this is the main reason for the widespread use of silicon steel in electrical machines. The thickness of the laminations is typically 0.3-0.5 mm, which ensures that the eddy current loss will be less than the hysteresis loss at a frequency of 50 Hz.

### 1.11 Skin Effect

The eddy currents in a bar such as the one shown in Fig.1.19 will produce a magnetic field within the bar, which by *Lenz's law* will oppose the applied field. Thus the magnetic flux density will fall from a value \( B_o \) at the surface to some lower value in the interior. The effect depends on the properties of the material, the frequency of the alternating field and the dimensions of the bar. It is possible for the magnitude of the flux density to fall very rapidly in the interior of the bar, so that most of the flux is confined to a thin layer or skin near the surface. The phenomenon is termed *skin*...
effect, and it implies very inefficient use of the magnetic material (quite apart from any eddy current losses). A similar effect occurs in conductors carrying alternating current, where the current density falls from some value $J_0$ at the surface to a lower value in the interior.

The phenomenon of skin effect gives a second reason for using laminated magnetic circuits. If the thickness of a plate is much more than twice the depth of penetration $b$, the central region will carry very little flux. The material will be fully utilized if it is divided into laminations less than $b$ in thickness, for the flux density will then be fairly uniform across the lamination. The depth of penetration in silicon steel is about 1 mm at a frequency of 50 Hz, so the typical lamination thickness of 0.5 mm ensures that skin effect will not be significant.

**Permanent magnet materials**

Permanent magnets find wide application in electrical measuring instruments, magnetos, magnetic chucks and moving-coil loudspeakers etc. In permanent magnets, high retentivity as well as high coercivity are most desirable in order to resist demagnetisation. In fact, the product $B_H$ is the best criterion for the merit of a permanent magnet. The material
commonly used for such purposes is carbon-free iron-nickel-aluminium-copper-cobalt alloys which are made anisotropic by heating to a very high temperature and then cooling in a strong magnetic field. This alloy possesses BrF1a value of about 40,000 J/m' as compared with 2,500 J/m' far chromium-steel.
Problems

1- An iron ring of mean length 10 cm has an air gap of 1 mm and a winding of 200 turns. If the relative permeability of iron is 300 when a current of 1 A flows through the coil, find the flux density.

2- Steel ring has the following particulars: Mean diameter of the ring = 30 cm; flux density of 1 Wb/m² is produced by 4000 At/m. The cross-section of the ring is 10 cm², number of turns wound = 600 of If a gap of 1 cm is cut in the ring and the flux density of 1 Wb/m² is maintained in the air gap, determine the inductance.

3- The magnetic circuit made of wrought iron is arranged as shown in Fig.1.21 below. The central limb has a cross-sectional area of 8 cm² and each of the side limb has a cross-sectional area of 5 cm². Calculate the ampere-turns required to produce a flux of 1 milli-weber in the central limb, neglecting magnetic leakage and fringing. The magnetization of wrought iron is given by

\[
\begin{array}{ccc}
B \text{- flux density (Wb/m²)} & 100 & 1.25 \\
H \text{- in At/m} & 200 & 500 \\
\end{array}
\]
A cast-steel DC electromagnet shown in Fig.1.22 has a coil of 1000 turns on its central limb. Determine the current that the coil should carry to produce a flux of 2.5 mWb in the air-gap. Neglect leakage. Dimensions are given in cm: The magnetization curve for cast steel is as under:

<table>
<thead>
<tr>
<th>Flux density (Wb/my)</th>
<th>0.2</th>
<th>0.5</th>
<th>0.7</th>
<th>1.0</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amp-turns/meter</td>
<td>300</td>
<td>540</td>
<td>650</td>
<td>900</td>
<td>1150</td>
</tr>
</tbody>
</table>
6- A cast-steel magnetic structure made of a bar of section 2 cm² is shown in Fig.1.23. Determine the current that the 500-turn magnetizing coil on the left limb should carry so that a flux of 2 mWb is produced in the right limb. Take $u_r=600$ and neglect leakage.

![Fig.1.23](image)

7- A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area 12 cm². A current of 2 A in the magnetizing coil of 200 turns produces a total flux of 12 mWb in the iron. Calculate:

(a) the flux density in the iron

(b) the absolute and relative permeability of iron.

(c) the reluctance of the circuit.
A series magnetic circuit has an iron path of length 50 cm and an air-gap of length 1 mm. The cross-sectional area of the iron is 6 cm\(^2\) and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetization characteristic:

<table>
<thead>
<tr>
<th>Flux density (Wb/m(^2))</th>
<th>1.2</th>
<th>1.35</th>
<th>1.45</th>
<th>1.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetizing force (At/m)</td>
<td>500</td>
<td>1,000</td>
<td>2,000</td>
<td>4,500</td>
</tr>
</tbody>
</table>

9- The magnetic circuit of Fig. 1.24 has a core of relative permeability \(\mu_r = 2000\). The depth of the core is 5 cm. The coil has 400 turns and carries a current of 1.5 A.

(a) Draw the magnetic equivalent circuit. (b) Find the flux and the flux density in the core. (c) Determine the inductance of the coil.

(d) Repeat (a), (b) and (c) for a 1.0 cm wide air gap in the core. Assume a 10\% increase in the effective cross-sectional area of the air gap due to fringing in the air gap.
Fig. 1.24

10 For the magnetic circuit shown in Fig. 1.25 all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 375 cm throughout. The turns are arranged on the center limb as shown.
Calculate the mmf required to produce a flux of 1.7 mWb in the center limb. Neglect the leakage and fringing. The magnetization data for the material is as follows:

\[
\begin{align*}
H \text{ (At/m)} : & \quad 400 \quad 440 \quad 500 \quad 600 \quad 800 \\
B \text{ (Wb/m*)} : & \quad 0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2
\end{align*}
\]


Chapter 2

DC Machines

2.1 Introduction

Converters that are used to continuously translate electrical input to mechanical output or vice versa are called electric machines. The process of translation is known as electromechanical energy conversion. An electric machine is therefore a link between an electrical system and a mechanical system. In these machines the conversion is reversible. If the conversion is from mechanical to electrical energy, the machine is said to act as a generator. If the conversion is from electrical to mechanical energy, the machine is said to act as a motor. These two effects are shown in Fig.2.21. In these machines, conversion of energy from electrical to mechanical form or vice versa results from the following two electromagnetic phenomena:

1. When a conductor moves in a magnetic field, voltage is induced in the conductor. (Generator action)
2. When a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. (Motor action)
These two effects occur simultaneously whenever energy conversion takes place from electrical to mechanical or vice versa. In motoring action, the electrical system makes current flow through conductors that are placed in the magnetic field. A force is produced on each conductor. If the conductors are placed on a structure free to rotate, an electromagnetic torque will be produced, tending to make the rotating structure rotate at some speed. If the conductors rotate in a magnetic field, a voltage will also be induced in each conductor. In generating action, the process is reversed. In this case, the rotating structure, the rotor, is driven by a prime mover (such as a steam turbine or a diesel engine). A voltage will be induced in the conductors that are rotating with the rotor. If an electrical load is connected to the winding formed by these conductors, a current $i$ will flow, delivering electrical power to the load. Moreover, the current flowing through the conductor will interact with the magnetic field to produce a reaction torque, which will tend to oppose the torque applied by the prime mover.
Note that in both motor and generator actions, the coupling magnetic field is involved in producing a torque and an induced voltage.

### 2.2 Induced Voltage

An expression can be derived for the voltage induced in a conductor moving in a magnetic field. As shown in Fig.2.2a, if a conductor of length $l$ moves at a linear speed $v$ in a magnetic field $B$, the induced voltage in the conductor can be obtained with the help of fraday’s law as shown in the following equation:

$$ e = Blv $$

(2.1)

where $B$, $l$, and $v$ are mutually perpendicular. The polarity (Direction) of the induced voltage can be determined from the so-called *Fleming's Right-Hand Rule* as explained in the previous chapter. The direction of this force is shown in Fig.2.2(b).
Chapter Two

Fig. 2.2 Motional voltage, (a) Conductor moving in the magnetic field. (b) Right hand screw rule.

Fleming's Right-Hand Rule

“Hold out your right hand with forefinger, second finger, and thumb at right angles to one another. If the forefinger represents the direction of the field, and the thumb represents the direction of the motion then, the second finger represents the direction of the induced emf in the coil”.

2.3 Electromagnetic Force, $f$

For the current-carrying conductor shown in Fig.2.3(a), the force (known as Lorentz force) produced on the conductor can be determined from the following equation:
where $B$, $l$, and $i$ are mutually perpendicular. The direction of the force can be determined by using the *Fleming’s Left Hand Rule or right-hand screw rule* as explained in the previous chapter and are stated in the following. The direction of the force is illustrated in Fig.2.3(b).

**Fleming’s Left Hand Rule:**

“Hold out your left hand with forefinger, second finger and thumb at right angles to one another. If the forefinger represents the direction of the field, and the second finger that of the current, then thumb gives the direction of the motion or force.”

**Right-Hand Screw Rule:**

*Turn the current vector $i$ toward the flux vector $B$. If a screw is turned in the same way, the direction in which the screw will move represents the direction of the force $f$."

Note that in both cases (i.e., determining the polarity of the induced voltage and determining the direction of the force) the moving quantities ($v$ and $i$) are turned toward $B$ to obtain the screw movement.
Equations (2.1) and (2.2) can be used to determine the induced voltage and the electromagnetic force or torque in an electric machine. There are, of course, other methods by which these quantities \( e \) and \( f \) can be determined.

Fig.2.3 Electromagnetic force. (a) Current-carrying conductor moving in a magnetic field. (b) Force direction.

### 2.4 Simple Loop Generator

In Fig.2.4 is shown a single turn rectangular copper coil ABCD moving about its own axis, a magnetic field provided by either permanent magnets or electromagnets. The two ends of the coil are joined to two slip-rings or discs \( a \) and \( b \) which are insulated from each other and from the central shaft. Two collecting brushes (of carbon or copper) press against the slip rings. Their function is to collect the current induced in the coil and to convey it to the external load resistance \( R \).
**2.4.1 Working Theory:**

Imagine the coil to be rotating in clockwise direction (Fig:2.5). As the coil assumes successive positions in the field, the flux linked with it changes. Hence, an *EMF* is induced in it which is proportional to the rate of change of flux linkages \( e = -N \frac{d\phi}{dt} \).

When the plane of the coil is at right angles to the lines of flux as shown in Fig.2.5 (a), then, the direction of velocity of both sides of the coil is parallel to the direction of the field lines so, there is no cutting for the field lines. Then flux linkages with the coil is
minimum. Hence; there is no induced \textit{EMF} in the coil. Let us take this no \textit{emf} on vertical position of the coil as the starting position. The angle of rotation or time will be measured from this position so we can determine the first point (angle 0°) of Fig.2.6.
Fig. 2.5
As the coil continues rotating further, the rate of change of flux linkages (and hence induced EMF in it) increases, till position 2, Fig.2.5 (b) is reached at 90 degrees. Here the coil plane is horizontal i.e. parallel to the lines of flux. As seen, the rate of change of flux linkages is maximum. Hence, maximum EMF is induced in the coil as shown in Fig.2.6 at 90 degrees and the direction of current flow is ABXYCD as shown in Fig.2.5 (b).

In the next quarter revolution i.e. from 90° to 180°, the rate of change of flux linkages decreases. Hence, the induced EMF decreases gradually till in position 4 Fig.2.5(c) of the coil, it is reduced to zero value and the direction of current flow is ABXYCD as shown in Fig.2.5 (b).

Note that:
The direction of this induced EMF can be found by applying Fleming’s Right hand rule. So, the current through the load resistance R flows from X to Y during the first half revolution of the coil.

In the next half revolution i.e. from 180 to 360 degrees, the variations in the magnitude of EMF are similar to those in the first half revolution but it will be found that the direction of the induced current is from D to C and B to A. Hence, the path of current flow is along DCYXBA which is just the reverse of the previous direction of flow. Therefore, we find that the current the resistive load, R which we obtain from such a simple generator reverses its direction after every half revolution as shown in Fig.2.5 and Fig.2.6. Such a current undergoing periodic reversals is known as alternating current. It is, obviously, different from a direct current which continuously flows in one and the same direction which we need to generate from the DC generator. So, we should make some modification to get unidirection current in the load which will be explained soon. It should be noted that alternating current not only reverses its direction, it does not even keep its magnitude constant while flowing in any one direction. The two halfcycles may be called positive and negative half cycles respectively (Fig.2.6).
For making the flow of current unidirectional in the external circuit, the slip rings are replaced by split rings which shown in Fig.2.7. The splitrings are made out of a conducting cylinder which is cut into two halves or segments insulated from each other by a thin sheet of mica or some other insulating material. As before, the coil ends are joined to these segments on which rest the carbon or copper brushes. In the practical genertor which has more than two poles and more than one coil the split rings has not just two halves but has many parts as shown in Fig.2.7 (b).

![Fig.2.7](image)

(a) Two segments split rings in simple generator loop.
(b) Many segments split rings in Practical generator loop.
It is seen (Fig.2.8(a) to Fig.2.8 (b)) that in the first half revolution current flows along ABXYCD i.e. the brush No. 1 in contact with segment $a$ acts as the positive end of the supply and $b$ as the negative end. In the next half revolution (Fig.2.8 (c) to Fig.2.8 (d)), the direction of the induced current in the coil has reversed. But at the same time, the positions of segments $a$ and $b$ have also reversed with the result that brush No. 1 comes in touch with that segment which is positive i.e. segment $b$ in this case. Hence, the current in the load resistance again flows from $X$ to $Y$. The waveform of the current through the external circuit is as shown in Fig.2.9. This current is unidirectional but not continuous like pure direct current.

It should be noted that the position of brushes is so arranged that the changeover of segments $a$ and $b$ from one brush to the other takes place when the plane of the rotating coil is at right angles to the plane of the flux lines. It is so because in that position, the induced $EMF$ in the coil is zero.
Fig. 2.8
**Chapter Two**

**Fig.2.9**

**Commutator**

The function of the commutator is to facilitate collection of current from the armature conductors i.e. converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit. It is of cylindrical structure and is built up of wedge-shaped segments of high conductivity hard-drawn or drop-forged.

**2.5 Practical Generator**

The simple loop generator has been considered in detail merely to bring out the basic principle underlying the construction and working of an actual generator illustrated in Fig.2.10 which consists of the following essential parts:

(i) Magnetic Frame or Yoke      (ii) Pole-cores and Pole-shoes
(iii) Pole Coils or Field Coils   (iv) Armature Core
(v) Armature Windings or         (vi) Commutator
(vii) Brushes and Bearings

Of these, the yoke, the pole cores, the armature core and air gaps between the poles and the armature core form the magnetic circuit whereas the rest form the electrical circuit.
Armature Windings

Two types of windings mostly employed for the armatures of DC machines are known as Lap Winding and Wave Winding. The difference between the two is merely due to the different arrangement of the end connections at the front or commutator end of armature. The following rules, however, apply to both types of the windings:
In a lap winding, the number of parallel paths (a) is always equal to the number of poles (b) and also to the number of brushes.

In wave windings, the number of parallel paths (a) is always two and there may be two or more brush positions.

The coils of rotor part or armature are arranged in many options which influence the performance of DC machines. Now, we will briefly discuss the winding of an actual armature. But before doing this, we have to explain many terms as shown in the following items.

**Pole-pitch**

The periphery of the armature divided by the number of poles of the generate.

i.e. the distance between two adjacent poles

It is equal to the number of armature conductors (or armature slots) per pol. If there are 400 conductors and 4 poles, then pole pitch is 400/4 = 100 conductor.

**Coil span or Coil pitch**

It is the distance, measured in terms of armature slots (or armature conductors), between two sides of a coil. It is, in fact, the
periphery of the armature spanned by the two sides of the coil as shown in Fig.2.11.

![Diagram of an armature coil with labels for End connections, Coil sides, and Coil Span.]

**Back Pitch**

The distance, measured in terms of the armature conductors, which a coil advances on the back of the armature is called back pitch and is denoted by $Y_b$. It is equal to the number difference of the conductors connected to a given segment of the commutator.
Front Pitch

The number of armature conductors or elements spanned by a coil on the front (or commutator end of an armature) is called the front pitch and is designated by $Y_f$ as shown in Fig. 2.12.

Alternatively, the front pitch may be defined as the distance (in terms of armature conductors) between the second conductor of one coil and the first conductor of the next coil which are connected together to commutator end of the armature. In other words, it is the number difference of the conductors connected together at the back end of the armature. Both front and back pitches for lap and wave-winding are shown in Fig. 2.12 and Fig. 2.13 respectively.
Resultant Pitch

It is the distance between the beginning of one coil and the beginning of the next coil to which it is connected as shown in Fig.2.12 and Fig.2.13.

As a matter of precaution, it should be kept in mind that all these pitches, though normally stated in terms of armature conductors, are also sometimes given in terms of armature slots or commutator bars.

Commutator Pitch \((Y_c)\)

It is the distance (measured in commutator bars or segments) between the segments to which the two ends of a coil are connected as shown in Fig.2.12 and Fig.2.13. From these figures it is clear that for lap winding, \(Y_c\) is the difference of \(Y_b\) and \(Y_f\) whereas for wavewinding it is the sum of \(Y_b\) and \(Y_f\). 

21-2t, Single-layer Winding

It is that winding in which one conductor or one coil side is placed in each armature slot as shown in Fig.2.14. Such a winding is not much used.

2t-2Z. Two-Layer Winding

In this type of winding, there are two conductors or coil sides-per slot arranged in two layers. Usually, one side of every coil lies in the upper half of one slot and other side lies in the lower
half of some other slot at a distance of approximately one pitch away (Fig.2.15). The transfer of the coil from one slot to another is usually made in a radial plane by means of a peculiar bend or twist at the back.
Lap and wave windings

The winding types of the armature of the DC machines can be divided to two main types, Lap and Wave windings. The main difference between them is the method of connecting the terminals of coils to the commutator segments.

In case of lap wound there are two kinds, the simplex lap winding and multiple lap winding.

For simple lap winding the two terminals of one coil is connected to adjacent two commutator segments as shown in Fig.2.12 and Fig.2.16.

2.7 Generated EMF Equation of a Generator.

Let $\phi =$ flux/pole in weber.
\( Z = \) total number of armature conductors = No. of slots*No. of conductors/slot.

\( P = \) No. of poles.

\( A = \) No. of parallel paths in armature.

\( N = \) armature rotation in rpm

\( E = \) EMF induced in any parallel path in armature.

Generated \( EMF = e.m.f. \) generated in one of the parallel paths.

1. Average EMF generated/conductor = \( \frac{d\phi}{dt} \) volt \( (2.3) \)

2. Now, flux cut/conductor in one revolution,

\( d\phi = \phi * P \) web. \( (2.4) \)

3. Number of revolution per second = N/60;

4. Then, time for one revolution, \( dt = 60/N \) second

5. Hence according to Faraday's laws of electromagnetic induction,

\( EMF \) generated/conductor = \( \frac{d\phi}{dt} = \frac{\phi PN}{60} \) Volt \( (2.5) \)

For a wave-wound generator

No. of parallel paths is 2

No. of conductors (in series) in one path = \( Z/2 \)

Then, \( EMF \) generated/path = \( \frac{\phi PN}{60} * \frac{Z}{2} = \frac{\phi ZPN}{120} \) Volt \( (2.6) \)

For a lap-wound generator

No. of parallel paths = \( P \)

No. of conductors (in series) in one path = \( Z/P \)
Then, \( EMF \) generated/\( \text{path} = \frac{\phi P N}{60} \times \frac{Z}{P} = \frac{\phi Z N}{60} \text{Volt} \) \hspace{1cm} (2.7)

In general, generated \( EMF = \frac{\phi P N}{60} \times \frac{Z}{A} \text{Volt} \) \hspace{1cm} (2.8)

Where \( A=2 \) for wave-winding.
And \( A=P \) for lap-winding.

### 2.8 Classification Of DC Machines

The field circuit and the armature circuit can be interconnected in various ways to provide a wide variety of performance characteristics an outstanding advantage of DC machines. Also, the field poles can be excited by two field windings, a shunt field winding and a series field winding as shown in Fig.2.9. The shunt winding has a large number of turns and takes only a small current (less than 5\% of the rated armature current). This winding can be connected across the armature (i.e., parallel with it), hence the name shunt winding. The series winding has fewer turns but carries a large current. It is connected in series with the armature, hence the name series winding. If both shunt and series windings are present, the series winding is wound on top of the shunt winding, as shown in Fig.2.9.
Fig. 2.9 Series and shunt field winding DC generators.

The various connections of the field circuit and armature circuit are shown in Fig. 2.10. In the *separately excited* DC machine (Fig. 2.10a), the field winding is excited from a separate source. In the *self-excited* DC machine, the field winding can be connected in three different ways. The field winding may be connected in series with the armature (Fig. 2.10b), resulting in a series DC machine; it may be connected across the armature (i.e., in shunt), resulting in a *shunt machine* (Fig. 2.10c); or both shunt and series windings may be used (Fig. 2.10d), resulting in a compound machine. If the shunt winding is connected across the armature, it is known as *short-shunt* machine. In an alternative connection, the shunt winding is connected across the series connection of armature and series winding, and the machine is known as *long-shunt* machine. There is no significant difference between these two connections, which are shown in Fig. 2.10d. In the compound machine, the
series winding *mmf* may aid or oppose the shunt winding *mmf*, resulting in different performance characteristics.

![Different connections of DC machines](image)

Fig. 2.10 Different connections of DC machines. (a) Separately excited DC machine. (b) Series DC machine. (c) Shunt DC machine. (d) Compound DC machine.

A rheostat is normally included in the circuit of the shunt winding to control the field current and thereby to vary the field *mmf*. 
Field excitation may also be provided by permanent magnets. This may be considered as a form of separately excited machine, the permanent magnet providing the separate but constant excitation.

**Example 2.1** A 4 pole, long-shunt, compound generator supplies 100 A at a terminal voltage of 500 V. If armature resistance is 0.02Ω, series field resistance is 0.04 Ω and shunt field resistance 100Ω, find the generated EMF. Take drop per brush as 1 V. Neglect armature reaction.

**Solution:**

Generator circuit is shown in Fig.2.11
Current through armature and series field winding is:

\[ I_{sh} = \frac{500}{100} = 5 \, A \]

Voltage drop on series field windings = \(105 \times 0.04 = 4.2\, \text{V}\)

Armature voltage drop = \(105 \times 0.02 = 2.1\, \text{volt}\)

Drop at brushes = \(2 \times 1 = 2\, \text{V}\)

Now,

\[ EMF = V + I_a R_a + \text{series drop} + \text{brush drop} \quad (2.9) \]

\[ = 500 + 4.2 + 2.1 + 2 = 508.3\, \text{Volt} \]

**Example 2.2** A 20 kW compound generator works on full load with a terminal voltage of 250 V. The armature, series and shunt windings have resistances of 0.05Ω, 0.025Ω and 100 Ω respectively. Calculate the total EMF generated in the armature when the machine is connected as short shunt.

**Solution:**

Generator voltage is shown in Fig. 2.12

Load current = \(20000/250 = 80\, \text{A}\)

Voltage drop in the series windings = \(80 \times 0.025 = 2\, \text{V}\)

Voltage across shunt winding = \(252\, \text{V}\).
Example 2.3 The following information is given for a 300 kW, 
600 V, long-shunt compound generator: Shunt field 
resistance=$75\Omega$, armature resistance including brush resistance=$
0.03\ \Omega$, commutating field winding resistance=$0.011\Omega$, series field 
resistance=$0.012\ \Omega$, divertor resistance =$0.036\ \Omega$. When the 
machine is delivering full load, calculate the voltage and power 
generated by the armature.
Solution:
Generator voltage is shown in Fig. 2.13

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Fig.2.13
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Power output = 300,000 W
Output current = 300,000/600 = 500 A

\[ I_{sh} = 600/75 = 8 \text{ A} \]

\[ I_a = 500 + 8 = 508 \text{ A} \]

Since the series field resistance and divertor resistance are in parallel (Fig.2.13) their combined resistance is:

\[ 0.012 \times 0.036/0.048 = 0.009 \Omega \]

Total armature circuit resistance

\[ = 0.03 + 0.011 + 0.009 = 0.05 \Omega \]
Voltage drop = $508 \times 0.05 = 25.4$ V
Voltage generated by armature = $600 + 25.4 = 625.4$ V
Power generated = $625.4 \times 508 = 317,700$ W = 317.7 kW

**Example 2.4** A 4 pole, lap-wound, DC shunt generator has a useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns each of 0.004 Ω resistance. Calculate the terminal voltage when running at 900 rpm if the armature current is 50 A.

**Solution:**

Since each turn has two sides,

$Z = 220 \times 2 = 440$ ; $N = 900$ rpm ;

$\phi = 0.07$ Wb ; $P = A = 4$

$$E = \frac{\phi Z N}{60} \times \frac{P}{A} = \frac{0.07 \times 440 \times 900}{60} \times \frac{4}{4} = 462$ Volt$$

Total resistance of 220 turns or 440 conductors = $220 \times 0.004 = 0.88$ Ω

Since, there are 4 parallel paths in armature, .

Resistance of each path = $0.88/4 = 0.22$ Ω

There are four such resistances in parallel each of value $0.22$ Ω

Armature resistance, $R_a = 0.22/4 = 0.055$ Ω

Armature drop $= I_a R_a = 50 \times 0.055 = 2.75$ volt

Now, terminal voltage $V = E - I_a R_a = 462 - 2.75 = 459.25$ volt
Example 2.5 A 4 pole, DC shunt generator with a shunt field resistance of 100 $\Omega$ and an armature resistance of 1 $\Omega$ has 378 wave-connected conductors in its armature. The flux per pole is 0.01 Wb. If a load resistance of 10 $\Omega$ is connected across the armature terminals and the generator is driven at 1000 rpm, calculate the power absorbed by the load.

Solution:
Induced EMF in the generator is:

$$E = \frac{\phi Z N}{60} \left( P \frac{A}{A} \right) = \frac{0.02 \times 378 \times 1000}{60} \times \left( \frac{4}{2} \right) = 252 \text{Volt}$$

Now let $V$ be the terminal voltage i.e. the voltage available across the load as well as the shunt resistance (Fig.2.14).
Load current= $V/10$ ampere and shunt current= $V/100$ ampere

Armature current=$\frac{V}{10} + \frac{V}{100} = \frac{11V}{100}$

Now, $V=E$–armature drop

Then, $V = 252 - 1 \times \frac{11V}{100} = 227 \text{Volt}$

Load current =227/10=22.7A;
Power absorbed by the load is 227*22.7 = 5 153 W.

### 2.9 Armature Reaction (AR)

With no current flowing in the armature, the flux in the machine is established by the mmf produced by the field current (Fig.2.14a). However, if the current flows in the armature circuit it produces its own mmf (hence flux) acting along the q-axis. Therefore, the original flux distribution in the machine due to the field current is disturbed. The flux produced by the armature mmf opposes flux in the pole under one half of the pole and aids under the other half of the pole, as shown in Fig.2.14b. Consequently, flux density under the pole increases in one half of the pole and decreases under the other half of the pole. If the increased flux density causes magnetic saturation, the net effect is a reduction of flux per pole. This is illustrated in Fig.2.14c.

At no load \( (I_a = I_r = 0) \) the terminal voltage is the same as the generated voltage \( V_{t0} = E_{a0} \). As the load current flows, if the flux decreases because of armature reaction, the generated voltage will decrease (Equation (2.6) and (2.8)). The terminal voltage will further decrease because of the \( I_a R_a \) drop (Equation (2.9)).
Fig. 2.14 Armature reaction effects.

In Fig. 2.14d, the generated voltage for an actual field current $I_f(\text{actual})$ is $E_{a0}$. When the load current $I_a$, flows the generated voltage is $E_a = V_t + I_a R_a$. If $E_a < E_{a0}$, the flux has decreased (assuming the speed remains unchanged) because of armature reaction, although the actual field current $I_f(\text{actual})$ in the field winding remains unchanged. In Fig. 2.14b, the generated voltage $E_a$ is produced by an effective field current $I_f(\text{eff})$. The net effect of armature reaction can therefore be considered as a reduction in the field current. The difference between the actual field current and effective field current can be considered as armature reaction in equivalent field current. Hence,

$$I_f(\text{eff}) = I_f(\text{actual}) - I_f(\text{AR})$$

(2.10)

where $I_f(\text{AR})$ is the armature reaction in equivalent field current.
2.9.1 Demagnetizing AT per Pole,

Since armature demagnetising ampere-turns are neutralized by adding extra ampereturas to the main field winding, it is essential to calculate their number. But before proceeding further, it should be remembered that the number of turns is equal to half the number of conductors because two conductors constitute one turn.

Let \( Z \) = total number of armature conductors

\[ I = \frac{I_a}{2} \quad \text{for wave winding} \quad (2.11) \]

\[ I = \frac{I_a}{P} \quad \text{for lap winding} \quad (2.12) \]

\( \theta_m \) = forward lead in mechanical or geometrical or angular degrees.

Total number of conductors in these angles = \( \frac{4\theta_m}{360} \times Z \)  \( (2.13) \)
As two conductors constitute one turn,

Then total number of turns in these angels = \( \frac{2\theta_m}{360} \) \( Z \) \hspace{1cm} (2.14)

Demagnetising amp-turns per pair of poles = \( \frac{2\theta_m}{360} \) \( ZI \) \hspace{1cm} (2.15)

Demagnetising amp-turns/pole = \( \frac{\theta_m}{360} \) \( ZI \) \hspace{1cm} (2.16)

\( AT_d \) per pole = \( \frac{\theta_m}{360} \) \( ZI \). \hspace{1cm} (2.17)

### 2.9.2 Cross-magnetizing AT per Pole

The conductors lying between angles \( \text{AOD} \) and \( \text{BOC} \) constitute what are known as distorting or cross conductors. Their number is found as under:

Total armature conductors/pole bath cross and demagnetising = \( \frac{Z}{P} \)

Demagnetising conductors/pole = \( \frac{2\theta_m}{360} \) \( Z \) \hspace{1cm} (found above)

Then, cross-magnetising conductors/pole

\[
= \frac{Z}{P} - \frac{2\theta_m}{360} \cdot \frac{Z}{P} = Z \left( \frac{1}{P} - \frac{2\theta_m}{360} \right) \hspace{1cm} (2.18)
\]

Cross-magnetising amp-conductors/pole = \( ZI \left( \frac{1}{P} - \frac{2\theta_m}{360} \right) \)
Cross-magnetising amp-turns/pole = \( ZI \left( \frac{1}{2P} - \frac{\theta_m}{360} \right) \)

(remembering that two conductors make one turn)

\[
AT_c/pole = ZI \left( \frac{1}{2P} - \frac{\theta_m}{360} \right) \quad (2.19)
\]

Note. (i) For neutralizing the demagnetising effect of armature-reaction, an extra number of turns may be put on each pole.

No. of extra turns/pole = \( \frac{AT_d}{I_{sh}} \) for shunt generator

\[= \frac{AT_d}{I_a} \quad \text{for series generator} \]

If the leakage coefficient \( a \) is given, then multiply each of the above expression by it.

(ii) If lead angle is given in electrical degrees, it should be converted into mechanical degrees by the following relation

\[
\theta_{(\text{mechanical})} = \frac{\theta_{(\text{electrical})}}{\text{pair of pole}} \quad \text{or} \quad \theta_m = \frac{\theta_e}{P/2} = \frac{2\theta_e}{P} \quad (2.20)
\]

2.9.3 Compensating Windings

These are used for large direct current machines which are subjected to large fluctuations in load i.e. rolling mill motors and turbo-generators etc. Their function is to neutralize the
cross-magnetizing effect of armature reaction. In the absence of compensating windings, the flux will be suddenly shifting backward and forward with every change in load. This shifting of flux will induce statically induced *EMF* in the armature coils. The magnitude of this *EMF* will depend upon the rapidity of changes in load and the amount of change. It may be so high as to strike an arc between the consecutive commutator segments across the top of the mica sheets separating them. This may further develop into a flash-over around the whole commutator thereby shortcircuiting the whole armature.

These windings are embedded in slots in the poleshoes and are connected in series with armature in such a way that the current in them flows in opposite direction to that flowing in armature conductors directly below the poleshoes. An elementary scheme of compensating winding is shown in Fig.2.16.

Owing to their cost and the room taken up by them, the compensating windings are used in the case of large machines which are subject to violent fluctuations in load and also for generators which have to deliver their full-load output at considerably low induced voltages.
2.9.4 No. of Compensating Windings

No. of armature conductors/pole = \( \frac{Z}{P} \)  \( (2.21) \)

No. of armature turns/pole = \( \frac{Z}{2P} \)  \( (2.22) \)

Then, No. of armature-turns immediately under one pole

\[
= \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}} \approx 0.7 \times \frac{Z}{2P} \quad (2.23)
\]

Then, No. of armature amp-turns/pole for compensating winding

\[
= 0.7 \times \frac{ZI}{2P} = 0.7 \times \text{armature amper – Turn / pole} \quad (2.24)
\]
**Example 2.6** A 4 pole, DC shunt generator has a simple weve-wound armature with 35 slots and 12 conductors per slot and delivers 35 A on full-load. If the brush lead is 10 space degrees, calculate the demagnetising and cross-magnetising ampere-turns per pole at full-load.

**Solution.** \( Z = 35 \times 12 = 420 \); \( \theta_m = 10^\circ \), \( I = \frac{I_a}{2} = \frac{35}{2} = 17.5 \, A \)

\[
AT_d/\text{pole} = \frac{\theta_m}{360} Z I = 420 \times 17.5 \times \frac{10}{360} = 204 \, At
\]

\[
AT_c/\text{pole} = Z I \left( \frac{1}{2P} - \frac{\theta_m}{360} \right) = 420 \times 17.5 \times \left( \frac{1}{2 \times 4} - \frac{10}{360} \right) = 715 \, At
\]

**Example 2.7** A 250 V, 500 kW, 8 pole DC generator has a lap-wound armature with 480 conductors. Calculate the demagnetising and cross-magnetising ampere-turns per pole at full-load if the brushes are given a forward lead of 7.5 mechanical.

**Solution:** Output current = \( \frac{500,000}{250} = 2,000 \, A \)

\[
I_a = 2000 \, A, \quad I = \frac{2000}{8} = 250 \, A, \quad Z = 480, \quad \theta_m = 7.5^\circ
\]

\[
AT_d/\text{pole} = \frac{\theta_m}{360} Z I = 480 \times 250 \times \frac{7.5}{360} = 2500 \, At
\]
Example 2.8 A 4 pole, wave-wound motor armature has 880 conductors and delivers 120 A. The brushes have been displaced through 3 angular degrees from the geometrical axis. Calculate (a) demagnetising amp-turns/pole (b) cross-magnetising amp-turns/pole (c) the additional field current for neutralizing the demagnetisation if the field winding has 1100 turns/pole.

Solution:

\[ Z = 880; \quad I = \frac{120}{2} = 60A, \quad \theta = 3^\circ \]

(a) \[ AT_d/pole = \frac{\theta_m}{360} Z I = 880 \times 60 \times \frac{3}{360} = 440 \, At \]

(b) \[ AT_c/pole = 880 \times 60 \times \left( \frac{1}{8} - \frac{3}{360} \right) = 6160 \]

Or total \[ AT_d/pole = \frac{880 \times 60}{2} = \frac{Z \times I_a}{2} \quad \frac{P}{2} = 6600 \, At \]

Hence \[ AT_c/pole = \text{Total } AT/pole - AT_d/pole = 6600 - 440 = 6160 \]

(c) Additional field current = \[ \frac{440}{1100} = 0.4 \, A \]

Example 2.9 A 4 pole generator supplies a current of 143 A. It has 492 armature conductors (a) wave-wound (b) lap-connected. When delivering full load, the brushes are given an actual lead of
$10^\circ$. Calculate the demagnetising amp-turns/pole. This field winding is shunt connected and takes 10 A: Find the number of extra shunt field turns necessary to neutralize this demagnetisation.

**Solution.**

\[
\begin{align*}
Z &= 492; \quad \theta = 10^\circ; \quad AT_{d/pole} = Z I \frac{\theta}{360} \\
I_a &= 143 + 10 = 153 \text{ A} \\
I &= 153/2 \text{ ...when wave wound} \\
I &= 153/4 \text{ ...when lap-wound} \\
(a) \quad AT_{d/pole} &= 492 \times \frac{153}{2} \times \frac{10}{360} = 1046 \text{ At} \\
\text{Extra shunt field turns} &= 1046/10 = 105 \text{ approx.} \\
(b) \quad AT_{d/pole} &= 492 \times \frac{153}{4} \times \frac{10}{360} = 523 \\
\text{Extra shunt field turns} &= 523/10 = 52 \text{ (approx.)}
\end{align*}
\]

**Example 2.10.** A 4 pole, 50-kW, 250-V wave-wound shunt generator has 400 armature conductors. Brushes are given a lead of 4 commutator segments. Calculate the demagnetisation amp-turns/pole if shunt field resistance is 50Ω. Also calculate extra shunt field turns/pole to neutralize the demagnetisation.

**Solution.**
Chapter Two

Load current supplied=50,000/250=200 A

\[ I_{sh} = \frac{250}{50} = 5 \, A \]

Then \( I_a = 200 + 5 = 205 \, A \)

Current in each conductor \( I = 205/2 = 102.5 \, A \)

No. of commutator segments = \( Z/A \) where \( A = 2 \). for wave winding

Then, No. of segments = \( 400/2 = 200 \) segments

\[ \theta_m = \frac{\text{no. of segments} \times 360}{200} = \frac{4 \times 360}{200} = 36 \, \text{degrees} \]

\[ AT_d / \text{pole} = 400 \times \frac{205}{2} \times \frac{36}{5 \times 360} = 820 \, At \]

Extra shunt turns/pole = \( \frac{AT_d}{I_{sh}} = \frac{820}{5} = 164 \) turns

Example 2.11 A 30 h.p. (British) 440 V, 4 pole, wave-wound DC shunt motor has 840 armature conductors and 140 commutator segments. Its full load efficiency is 88% and the shunt field current is 1.8 A. If brushes are shifted backwards through 1.5 segment from the geometrical neutral axis, find the demagnetising and distorting amp-turns/pole.
Solution:

The shunt motor is shown diagrammatically in Fig.2.17

Motor output = 30 \times 746 \, W, \quad - \eta = 0.88\% 

Motor input = \frac{30 \times 746}{0.88} = 25432 W 

Motor input current = \frac{25432}{440} = 57.8 A 

I_{sh} = 1.8 A, \quad I_a = 57.8 - 1.8 = 56 A 

Current in each conductor = \frac{56}{2} = 28 A 

\theta_m = 1.5 \times 360 / 140 = 27/7 \, degrees 

AT_{d/pole} = 840 \times 28 \times \frac{27}{7 \times 360} = 252 At 

AT_{c/pole} = 840 \times 28 \times \left( \frac{1}{8} - \frac{27}{7 \times 360} \right) = 2688 \, At
2.10 Shifting The Brushes To Improve Commutation

Due to the shift in the neutral zone (Fig.2.18) when the generator is under load, we could move the brushes to reduce the sparking.

For generators, the brushes are shifted to the new neutral zone by moving them in the direction of rotation. For motors, the brushes are shifted against the direction of rotation.

As soon as the brushes are moved, the commutation improves, meaning there is less sparking. However, if the load fluctuates, the armature mmf rises and falls and so the neutral zone shifts back and forth between the no load and full load positions. We would therefore have to move the brushes back and forth to obtain spark-
less commutation. This procedure is not practical and other means are used to resolve the problem. For small DC machines, however, the brushes are set in an intermediate position to ensure reasonably good commutation at all loads.

Fig. 2.18 (a) Magnetic field produced by the current flowing in the armature conductors.
Fig.2.18 (b) Armature reaction distorts the field produced by the N, S poles.

2.11 Commutating Poles (Interpoles Windings)

To counter the effect of armature reaction in medium and large-power DC machines, we always place a set of commutating poles (Sometimes called interpoles windings) between the main poles (Fig.2.19). These narrow poles carry windings that are connected in series with the armature. The number of turns on the windings is designed so that the poles develop a magnetomotive force $mmf_c$ equal and opposite to the magnetomotive force $mmf_a$ of the armature. As the load current varies, the two magnetomotive forces rise and fall together, exactly bucking each other at all
times. By nullifying the armature \textit{mmf} in this way, the flux in the space between the main poles is always zero and so we no longer have to shift the brushes. In practice, the \textit{mmf} of the commutating poles is made slightly greater than the armature \textit{mmf}. This creates a small flux in the neutral zone, which aids the commutation process.

Fig.2.19 shows how the commutating poles of a 2-pole machine are connected. Clearly, the direction of the current flowing through the windings indicates that the \textit{mmf} of the commutating poles acts opposite to the \textit{mmf} of the armature and, therefore, neutralizes its effect. However, the neutralization is restricted to the narrow brush zone where commutation takes place. The distorted flux distribution under the main poles, unfortunately, remains the same.
Fig. 2.19 Commutating poles produce an $mmf_c$ that opposes the $mmf_a$ of the armature.

### 2.12 Separately Excited Generator

Now that we have learned some basic facts about DC generators, we can study the various types and their properties. Thus, instead of using permanent magnets to create the magnetic field, we can use a pair of electromagnets, called field poles, as shown in Fig.2.20. When the DC field current in such a generator is supplied by an independent source (such as a storage battery or another generator, called an exciter), the generator is said to be separately excited. Thus, in Fig.2.20 the DC source connected to terminals $a$ and $b$ causes an exciting current $I_x$ to flow. If the armature is driven by a motor or a diesel engine, a voltage $E_a$ appears between brush terminals $x$ and $y$. 
2.12.1 No-Load Operation And Saturation Curve

When a separately excited DC generator runs at no load (armature circuit open), a change in the exciting current causes a corresponding change in the induced voltage. We now examine the relationship between the two.

2.12.2 Field flux vs. exciting current.

Let us gradually raise the exciting current $I_X$, so that the $mmf$ of the field increases, which increases the flux ($\phi$ per pole). If we plot $\phi$ as a function of $I_X$, we obtain the saturation curve of Fig.2.21a. This curve is obtained whether or not the generator is turning.
When the exciting current is relatively small, the flux is small and the iron in the machine is unsaturated. Very little mmf is needed to establish the flux through the iron, with the result that the mmf developed by the field coils is almost entirely available to drive the flux through the air gap. Because the permeability of air is constant, the flux increases in direct proportion to the exciting current, as shown by the linear portion 0a of the saturation curve.

However, as we continue to raise the exciting current, the iron in the field and the armature begins to saturate. A large increase in the mmf is now required to produce a small increase in flux, as shown by portion bc of the curve. The machine is now said to be saturated. Saturation of the iron begins to be important when we reach the so-called "knee" ab of the saturation curve.

![Fig. 2.21a Flux per pole versus exciting current.](image-url)
The output voltage varies to the field current for separately excited DC generator is shown in Fig.2.21b.

![Saturation curve of a DC generator](image)

Fig.2.21b Saturation curve of a DC generator.

### 2.13 Shunt Generator

A shunt-excited generator is a machine whose shunt field winding is connected in parallel with the armature terminals, so that the generator can be self-excited (Fig.2.22). The principal advantage of this connection is that it eliminates the need for an external source of excitation.

How is self-excitation achieved? When a shunt generator is started up, a small voltage is induced in the armature, due to the remaining flux in the poles. This voltage produces a small exciting current $I_x$ in the shunt field. The resulting small mmf acts in the same direction as the remaining flux, causing the flux per pole to increase. The increased flux increases $E_o$ which increases $I_x$, which
increases the flux still more, which increases $E_0$ even more, and so forth. This progressive buildup continues until $E_0$ reaches a maximum value determined by the field resistance and the degree of saturation. See next section.

Fig.2.22 a. Self-excited shunt generator.
b. Schematic diagram of a shunt generator. A shunt field is one designed to be connected in shunt (alternate term for parallel) with the armature winding.

2.13.1 Controlling The Voltage Of A Shunt Generator

It is easy to control the induced voltage of a shunt-excited generator. We simply vary the exciting current by means of a rheostat connected in series with the shunt field (Fig.2.23).

Fig.2.23 Controlling the generator voltage with a field rheostat. A rheostat is a resistor with an adjustable sliding contact.

To understand how the output voltage varies, suppose that $E_0$ is 120 V when the movable contact $p$ is in the center of the rheostat.
If we move the contact toward extremity $m$, the resistance $R_t$ between points $p$ and $b$ decreases, which causes the exciting current to increase. This increases the flux and, consequently, the induced voltage $E_0$. On the other hand, if we move the contact toward extremity $n$, $R_t$ increases, the exciting current decreases, the flux decreases, and so $E_0$ will fall.

We can determine the no-load value of $E_0$ if we know the saturation curve of the generator and the total resistance $R_t$ of the shunt field circuit between points $p$ and $b$. We draw a straight line corresponding to the slope of $R_t$ and superimpose it on the saturation curve (Fig. 4.24). This dotted line passes through the origin, and the point where it intersects the curve yields the induced voltage.

For example, if the shunt field has a resistance of 50 $\Omega$ and the rheostat is set at extremity $m$, then $R_t = 50$ $\Omega$. The line corresponding to $R_t$ must pass through the coordinate point $E = 50$ V, $I = 1$ A. This line intersects the saturation curve where the voltage is 150 V (Fig.2.24). That is the maximum voltage the shunt generator can produce.
By changing the setting of the rheostat, the total resistance of the field circuit increases, causing $E_0$ to decrease progressively. For example, if $R_t$ is increased to 120 Ω, the resistance line cuts the saturation curve at a voltage $E_0$ of 120 V.

Fig 2.24 The no-load voltage depends upon the resistance of the shunt-field circuit.
If we continue to raise $R_t$, a critical value will be reached where the slope of the resistance line is equal to that of the saturation curve in its unsaturated region. When this resistance is attained, the induced voltage suddenly drops to zero and will remain so for any $R_t$ greater than this critical value. In Fig.2.24 the critical resistance corresponds to 200Ω.

We have seen that the armature winding contains a set of identical coils, all of which possess a certain resistance. The total armature resistance $R_a$ is that which exists between the armature terminals when the machine is stationary. It is measured on the commutator surface between those segments that lie under the (+) and (-) brushes. The resistance is usually very small, often less than one hundredth of an ohm. Its value depends mainly upon the power and voltage of the generator. To simplify the generator circuit, we can represent $R_a$ as if it were in series with one of the brushes. If the machine has interpoles, the resistance of these windings is included in $R_a$.

The equivalent circuit of a generator is thus composed of a resistance $R_a$ in series with a voltage $E_0$ (Fig.2.25). The latter is the voltage induced in the revolving conductors. Terminals 1, 2 are the external armature terminals of the machine, and $F_1, F_2$ are the field winding terminals. Using this circuit, we will now study the more common types of direct-current generators and their behavior under load.
2.14 Separately Excited Generator Under Load

Let us consider a separately excited generator that is driven at constant speed and whose field is excited by a battery (Fig.2.26). The exciting current is constant and so is the resultant flux. The induced voltage $E_o$ is therefore fixed. When the machine operates at no load, terminal voltage $E_{12}$ is equal to the induced voltage $E_o$ because the voltage drop in the armature resistance is zero. However, if we connect a load across the armature (Fig.2.26), the resulting load current $I$ produces a voltage drop across resistance $R_a$. Terminal voltage $E_{12}$ is now less than the induced voltage $E_o$. As we increase the load, the terminal voltage decreases progressively, as shown in Fig.2.27. The graph of terminal voltage as a function of load current is called the load curve of the generator. In practice, the induced voltage $E_o$ also decreases slightly with increasing load, because pole-tip saturation tends to decrease the field flux. Consequently, the terminal voltage $E_{12}$ falls off more rapidly than can be attributed to armature resistance alone.
2.15 **Shunt generator under load**

The terminal voltage of a self-excited shunt generator falls off more sharply with increasing load than that of a separately excited
generator (Fig.2.27). The reason is that the field current in a separately excited machine remains constant, whereas in a self-excited generator the exciting current falls as the terminal voltage drops. For a self-excited generator, the drop in voltage from no load to full load is about 15 percent of the full-load voltage, whereas for a separately excited generator it is usually less than 10 percent. The voltage regulation is said to be 15% and 10%, respectively.

2.16 Compound Generator

The compound generator was developed to prevent the terminal voltage of a DC generator from decreasing with increasing load. Thus, although we can usually tolerate a reasonable drop in terminal voltage as the load increases, this has a serious effect on lighting circuits. For example, the distribution system of a ship supplies power to both DC machinery and incandescent lamps. The current delivered by the generator fluctuates continually, in response to the varying loads. These current variations produce corresponding changes in the generator terminal voltage, causing the lights to flicker. Compound generators eliminate this problem.

A compound generator (Fig.2.28a) is similar to a shunt generator, except that it has additional field coils connected in series with the armature. These series field coils are composed of a few turns of heavy wire, big enough to carry the armature current.
The total resistance of the series coils is, therefore, small. Fig. 2.28b is a schematic diagram showing the shunt and series field connections.
When the generator runs at no-load, the current in the series coils is zero. The shunt coils, however, carry exciting current $I_X$ which produces the field flux, just as in a standard self-excited shunt generator. As the generator is loaded, the terminal voltage tends to drop, but load current $I_c$ now flows through the series field coils. The \textit{mmf} developed by these coils acts in the same direction as the \textit{mmf} of the shunt field. Consequently, the field flux under load rises above its original no-load value, which raises the value of $E_o$. If the series coils are properly designed, the terminal voltage remains practically constant from no-load to full-load. The rise in the induced voltage compensates for the armature drop.
In some cases we have to compensate not only for the armature voltage drop, but also for the $IR$ drop in the feeder line between the generator and the load. The generator manufacturer then adds one or two extra turns on the series winding so that the terminal voltage increases as the load current rises. Such machines are called over compound generators. If the compounding is too strong, a low resistance can be placed in parallel with the series field (The name of this resistance is diverter resistance). This reduces the current in the series field and has the same effect as reducing the number of turns. For example, if the value of the diverter resistance is equal to that of the series field, the current in the latter is reduced by half.

### 2.17 Differential Compound Generator

In a differential compound generator the $mmf$ of the series field acts opposite to the shunt field. As a result, the terminal voltage falls drastically with increasing load as shown in Fig.2.28c. We can make such a generator by simply reversing the series field of a standard compound generator. Differential compound generators were formerly used in DC arc welders, because they tended to limit the short-circuit current and to stabilize the arc during the welding process.

### 2.18 DC Motor
The DC machine can operate both as a generator and as a motor. This is illustrated in Fig.2.29. When it operates as a generator, the input to the machine is mechanical power and the output is electrical power. A prime mover rotates the armature of the DC machine, and DC power is generated in the machine. The prime mover can be a gas turbine, a diesel engine, or an electrical motor. When the DC machine operates as a motor, the input to the machine is electrical power and the output is mechanical power. If the armature is connected to a DC supply, the motor will develop mechanical torque and power. In fact, the DC machine is used more as a motor than as a generator. DC motors can provide a wide range of accurate speed and torque control.

![Fig.2.29 Reversibility of a DC machine. (a) Generator. (b) Motor.](image)

In both modes of operation (generator and motor) the armature winding rotates in the magnetic field and carries current. Therefore, the same basic equations (2.5), (2.6) and (2.8) hold good for both generator and motor action.
2.18.1 Shunt Motor

A schematic diagram of a shunt field DC motor is shown in Fig.2.30. The armature circuit and the shunt field circuit are connected across a DC source of fixed voltage $V_t$. An external field rheostat ($R_{fc}$) is used in the field circuit to control the speed of the motor. The motor takes power from the DC source, and therefore the current $I_t$ flows into the machine from the positive terminal of the DC source. As both field circuit and armature circuit are connected to a DC source of fixed voltage, the connections for separate and shunt excitation are the same. The behavior of the field circuit is independent of the armature circuit.

The governing equations for steady-state operation of the DC motor are as follows:

\[ V_i = I_a R_a + E_a \]  \hspace{1cm} (2.25a)

\[ I_t = I_a + I_f \]  \hspace{1cm} (2.25b)

\[ E_a = K_a \Phi \omega_m = \frac{K_a N \Phi}{60} \frac{\rho}{A} \]  \hspace{1cm} (2.25c)

\[ E_a = V_i - I_a R_a \]  \hspace{1cm} (2.25d)

where $K_a$ is the machine constant.

The armature current $I_a$ and the motor speed $\omega_m$ depend on the mechanical load connected to the motor shaft.
2.19 Power Flow and Efficiency

The power flow in a DC machine is shown in Fig.2.31. The various losses in the machine are identified and their magnitudes as percentages of input power are shown. A short-shunt compound DC machine is considered as an example (Fig.2.31a).

With the machine operating as a generator (Fig.2.31b), the input power is the mechanical power derived from a prime mover. Part of this input power is lost as rotational losses required to rotate the machine against windage and friction (rotor core loss is also included in the rotational loss). The rest of the power is converted
into electrical power $E_a I_a$. Part of this developed power is lost in $R_a$ (which includes brush contact loss), part is lost in $R_f = R_{fc} + R_{fw}$, and part is lost in $R_{sr}$. The remaining power is available as the output electrical power. Various powers and losses in a motoring operation are shown in Fig.2.31c.

The percentage losses depend on the size of the DC machine. The range of percentage losses shown in Fig.2.31 is for DC machines in the range 1 to 100 kW or 1 to 100 hp. Smaller machines have a larger percentage of losses, whereas larger machines have a smaller percentage of losses.

The efficiency of the machine is:

$$\text{efficiency} = \frac{P_{output}}{P_{input}} \quad (2.26)$$
2.20 Condition for Maximum Power

The mechanical power developed by a motor is:
\[ P_m = V_i * I_a - I_a^2 R_a \] (2.27)

Differentiating both sides with respect to \( I_a \) we get:
\[ \frac{dP_m}{dI_a} = V_i - 2I_a R_a = 0 \] (2.28)

Then, \( I_a R_a = V_i / 2 \) (2.29)

As \( V_i = E_a + I_a R_a \) and \( I_a * R_a = V_i / 2 \)

Then \( E_a = V_i / 2 \) (2.30)

Thus mechanical power developed by a motor is maximum when back EMF is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 percent.

2.21 Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius \( r \) meters acted upon by a circumferential force of \( F \) Newton which causes it to rotate at \( N \ rps \)
Then, torque $T = F \times r$ Newton meter (N-m)

Work done by this force in one revolution

$= \text{Force} \times \text{distance} = F \times 2\pi r$

Power developed $= F \times 2\pi r \times N$ joule/second

$= (F \times r) \times 2\pi N$ joule/second

Now $2\pi N =$ angular velocity $\omega$ in rad/second

power developed $= T \times \omega$ joule/second or watt \hspace{1cm} (2.31)

**Series-wound DC motor**

In series DC motor field & armature circuits are connected in series, as shown in the Figure below; so $i_a = i_f$. Assuming linear dependence of flux on the field current (approximately), we have:
• Note, there is no voltage drop across the inductance in a steady-state regime (because $I_a=\text{const in time}$)

\section*{2.22 Armature Torque of a Motor}

Let $T_a$ be the torque developed by the armature of a motor running at $N$ rps. If $T_a$ is in N-m, then power developed,

$$P_{\text{dev}} = T_a \times 2\pi N \text{ watt} \tag{2.32}$$
We also know that electrical power converted into mechanical power in the armature is \( E_a I_a \) \( (2.33) \)

Equating (2.32) and (2.33), we get \( T_a * 2\pi N = E_a I_a \)

Since \( E_a = \phi ZN \frac{P}{A} \) volt, we have:

\[
T_a * 2\pi N = \phi ZN \frac{P}{A} * I_a \quad (2.34)
\]

Then, \( T_a = \frac{1}{2\pi} \phi ZI_a \frac{P}{A} \) N·m \( (2.35) \)

or \( T_a = 0.159 \phi ZI_a \frac{P}{A} \) N·m \( (2.36) \)

Note. From the above equation for the torque, we find that

\( T_a \propto \phi I_a \)

(a) In the case of a series motor, \( \phi \) is directly proportional to \( I_a \)
(before saturation) because field windings carry full armature current. \( \therefore T_a \propto I_a^2 \).

(b) For shunt motors, \( \phi \) is practically constant, hence \( \therefore T_a \propto I_a \)

It is also seen from (2.34) above that,

\[
T_a = \frac{1}{2\pi} \frac{E_a I_a}{N} \text{ N·m} \quad (2.37)
\]


Chapter Two

2.23 Shaft Torque

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of this is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as *shaft torque* $T_{sh}$. It is so called because it is available at the shaft. The horse-power obtained by using the shaft torque is called *Brake Horse-Power* (*B.H.P.* ) because it is the horse-power available at the brake.

\[
B.H.P.(\text{metric}) = \frac{T_{sh} \times 2\pi N}{735.5} \tag{2.38}
\]

\[
T_{sh} = \frac{735.5 \times B.H.P.(\text{metric})}{2\pi N} \tag{2.39}
\]

The difference $T_a - T_{sh}$ is known as lost torque \( (2.40) \)

**Example 2.12** A 500V, 50 b.h.p. (37.3 kW), 1000 rpm DC shunt motor has on full-load an efficiency of 90 percent. The armature circuit resistance is 0.24 Ω and there is total voltage drop of 2 V at the brushes. The field current is 1.8 A. Determine (i) full-load line current (ii) full-load shaft torque in N-m and (iii) total
resistance in motor, starter to limit the starting current to 1.5 times the full-load current.

\textbf{Solution:} (i) Motor input = \(37,300/0.9 = 41,444\ W\);
Full load line current = \(41,444/500 = 82.9\ A\)

\[(ii) \quad T_{sh} = \frac{output}{\omega} = \frac{37300}{2\pi \left(\frac{1000}{60}\right)} = 356\ N.m\]

(iii) Starting line current = \(1.5 \times 82.9 = 124.3\ A\)
Armature current at starting = \(124.3 - 1.8 = 122.5\ A\)
If \(R\) is the starter resistance (which is in series with armature); then
\[122.5 \times (R + 0.24) + 2 = 500\]
Then, \(R = 3.825\ \Omega\)

\textbf{Example 2.13} A 4-pole, 220-V shunt motor has 340 lap-wound conductors. It takes 32 A from the supply mains and develops 7.5 h. p. (5.595 kW). The field winding takes 1 A. The armature resistance is 0.09 \(\Omega\) and the flux per pole is 30 mWb. Calculate (i) the speed and (ii) the torque developed in newton-metere.

\(I_a = 32 - 1 = 31\ A\)
\(E_b = V - I_a R_a = 220 - (0.09 \times 31) = 217.2\ V\)

Now, \(E_b = \frac{\phi ZN}{60} \left(\frac{P}{A}\right)\)
\[ 217.2 = \frac{30 \times 10^{-2} \times 540 \times N}{60} \left( \frac{4}{4} \right) \]

\( i \) \( \therefore N = 804.4 \text{ rpm} = 13.4 \text{ rps} \)

\( ii \) \( T_{sh} = \frac{\text{output in watt}}{\omega} = \frac{5595}{2\pi \times 13.4} = 66.5 \text{ N.m} \)

**Example 2.14** A DC series motor takes 40 A at 220 V and runs at 800 rpm. If the armature and field resistances are 0.2 Ω and 0.1 Ω respectively and the iron and friction losses are 0.5 kW, find the torque developed in the armature. What will be the output of the motor?

**Solution:**

Armature torque is given by \( T_a = \frac{E_a I_a}{2\pi N} \text{ N.m} \)

Now

\[ E_a = V - I_a (R_a + R_{se}) = 220 - 40 \times (0.2 + 0.1) = 208V \]

\[ N = \frac{800}{60} = \frac{40}{3} \text{ rps} \]

\( \therefore T_a = \frac{1}{2\pi} \frac{E_a I_a}{N} \text{ N.m} = \frac{1}{2\pi} \frac{208 \times 40}{(40/3)} = 99.3 \text{ N.m} \)
Cu loss in armature and series-field resistance:

\[ = 40^2 \times 0.3 = 480 W \]

Iron and friction losses= 500 W,
Total losses=480+500=980 W
Motor power input =220 \times 40=8,800 W
Motor output=8,800-980 =7,820 W =7.82 kW

**Example 2.15** A 4-pole, 240 V, wave-connected shunt motor gives 11.19 kW when running at 1000 rpm and drawing armature and field-currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 \( \Omega \). Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux/pole (d) rotational losses and (e) efficiency.

**Solution:**

\[ E_a = V - I_a R_a - Brush \text{ drop} = 240 - 50 \times 0.1 - 2 = 233 V \]

Also, \( I_a = 50 A \)

(a) Armature Torque:

\[ T_a = \frac{0.159 \times E_a I_a}{N} = \frac{0.159 \times 233 \times 50}{50 / 3} = 111 N.m \]

(b) \[ T_{sh} = \frac{Output}{2 \pi N} = \frac{11190}{2 \pi \times 50 / 3} = 106.9 N.m \]

(c) \[ E_a = \phi ZN \frac{P}{A} \]
Then, \( 233 = \phi 540 \times 50 / 3 \times \frac{4}{2} \)

Then, \( \phi = 12.9 \text{ mWb} \).

(d) Armature Input = \( V \times I_a = 240 \times 50 = 12000 \text{W} \)

Armature Cu loss = \( I_a^2 \times R_a = 50^2 \times 0.1 = 250 \text{W} \)

Brush contact loss = 50 \times 2 = 100 \text{W} 

Power developed = 12,000 - 350 = 11,650 \text{W}; 

Output = 11.19 kW = 11,190 \text{W} 

Rotational losses = 11,650 - 1,190 = 460 \text{W} 

(e) Total motor input = \( VI = 240 \times 51 = 12,240 \text{W}; \)

Motor output = 11,190 \text{W}

\[ \therefore \text{Effeciency} = \frac{\text{output}}{\text{Input}} = \frac{11190}{12240} \times 100 = 91.4\% \]

Example 2.16 A 460 V series motor runs at 500 rpm taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 \( \Omega \). Assume flux is proportional to the field current.

Solution:

Since \( \phi \alpha I_a \) hence \( T\alpha \phi I_a \alpha I_a^2 \)

\( T_1 \alpha 40^2 \) and \( T_2 \alpha 30^2 \)
Then, \( \frac{T_2}{T_1} = \frac{9}{16} \)

Percentage change in torque is

\[
\frac{T_1 - T_2}{T_1} \times 100 = \frac{7}{16} \times 100 = 43.75\% 
\]

Now \( E_{a_1} = 460 - (40 \times 0.8) = 428V \)

\( E_{a_2} = 460 - (30 \times 0.8) = 436V \)

\[
\frac{N_2}{N_1} = \frac{E_{a_2} \times I_{a_1}}{E_{a_1} \times I_{a_2}} 
\]

Then, \( \frac{N_2}{500} = \frac{436 \times 40}{428 \times 30} \)

Then, \( N_2 = 679 \text{ rpm} \)

### 2.24 Speed of a DC Motor

From the voltage equation of a motor we get,

\[
E_a = V - I_a \times R_a \quad \text{Or} \quad \phi Z N \left( \frac{P}{A} \right) = V - I_a R_a \quad (2.41) 
\]

\[
N = \frac{V - I_a R_a}{\phi} \times \left( \frac{A}{Z P} \right) \text{ rps} \quad (2.42) 
\]

Now \( V - I_a R_a = E_a \quad (2.43) \)
Then, \[ N = \frac{E_a}{\phi} \left( \frac{A}{ZP} \right) rps \quad \text{or} \quad N = k \frac{E_a}{\phi} \quad (2.44) \]

It shows that speed is directly proportional to back EMF. \( E_a \) and inversely-to the flux \( \phi \) or \[ N \propto \frac{E_a}{\phi} \quad (2.45) \]

**For Series Motor**

Let \( N \), speed in the 1st case;

\[ I_{a_1} = \text{armature current in the 1st case} \]

\( \phi_1 = \text{flux/pole in the first case} : \)

\[ N_1, I_{a_2}, \phi_2 = \text{corresponding quantities in the 2nd case.} \]

Then using the above relation, we get:

\[ N_1 \alpha \frac{E_{a_1}}{\phi_1}, \quad \text{Where} \quad E_{a_1} = V - I_{a_1} R_a \quad (2.46) \]

\[ N_2 \alpha \frac{E_{a_2}}{\phi_2}, \quad \text{Where} \quad E_{a_2} = V - I_{a_2} R_a \quad (2.47) \]

\[ \therefore \frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} \frac{\phi_1}{\phi_2} \quad (2.48) \]

Prior to saturation of magnetic poles; \( \phi \propto I_a \)

\[ \therefore \frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} \frac{I_{a_1}}{I_{a_2}} \quad (2.49) \]
For Shunt Motor

In this case the same equation applies,

\[ \frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} \times \frac{\phi_1}{\phi_2}, \text{ If } \phi_2 = \phi_1 \text{ Then, } \frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} \]  
(2.50)

2.25 Speed Regulation

The term speed regulation refers to the change in speed of a motor with change in applied load torque, other conditions remaining constant. By change in speed here is meant the change which occurs under these conditions due to inherent properties of the motor itself and not those changes which are affected through manipulation of rheostats or other speed-controlling devices.

The speed regulation is defined as the change in speed when the load on the motor is reduced from rated value to zero, expressed as percent of the rated load speed.

\[ \therefore \% \text{ Speed regulation} = \frac{N.L.\text{speed} - F.L.\text{Speed}}{F.L.\text{Speed}} \times 100 \]  
(2.51)

Example 2.17 A shunt generator delivers 50 kW at 250 V and 400 rpm. The armature and field resistances are 0.02 Ω and 50 Ω respectively. Calculate the speed of the machine running as a
Chapter Two

shunt motor and taking 50 kW input at 250 V. Allow 1 volt per brush for contact drop.

**Solution:**

As Generator [Fig.2.32(a)]

Load current, \( I = \frac{50,000}{250} = 200 \) A;

Shunt current, \( I_{sh} = \frac{250}{50} = 5 \) A

\[
I_a = I + I_{sh} = 205 \text{A,}
\]

\[
I_a R_a = 205 \times 0.02 = 4.1 \text{ V;}
\]

Brush drop=2 * 1 = 2V

Induced *EMF* in armature =250 + 4.1 +2 =256.1 V

Obviously, if this machine were to run as a motor at 400 rpm, it would have a back *EMF* of 256.1 V induced in its armature.

\[
E_{a1} = 256.1V, \ N_1 = 400 \text{ rpm}
\]

*As Motor* [Fig. 2.32 (b)]
Input line current \( I = \frac{50,000}{250} = 200 \, \text{A} \)

\[ I_{sh} = \frac{250}{50} = 5 \, \text{A}; \]

\[ I_a = 200 - 5 = 195 \, \text{A} \]

\[ I_a R_a = 195 \times 0.02 = 3.9 \, \text{V} \]

Brush drop = 2 \times 1 = 2 \, \text{V}

\[ E_{a_2} = 25 - (3.9 + 2) = 244.1 \, \text{V} \]

\[ \frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} \cdot \frac{\phi_1}{\phi_2}, \quad \text{Since} \quad \phi_2 = \phi_1 \]

Then,

\[ \frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} \quad \text{Then}, \quad \frac{N_2}{400} = \frac{244.1}{256.1} \]

Then, \( N_2 = 381.4 \, \text{rpm} \)

**Example 2.18** The input to a 220 V, DC shunt motor is 11 kW. Calculate (a) the torque developed (b) the efficiency (c) the speed at this load. The particulars of the motor are as follows:

- No-load current = 5 A;
- No-load speed = 1150 rpm.
- Arm. resistance = 0.5 \( \Omega \)
- Shunt field resistance = 110 \( \Omega \)

**Solution.**

No-load input = 220 \times 5 = 1,100 \, \text{W} ;
Chapter Two

$I_{sh} = 220/110 = 2 \text{ A}$

$I_{a0} = 5 - 2 = 3 \text{ A}$

No-load armature Cu loss $= 3^2 * 0.5 = 4.5 \text{ W}$

Then, Constant losses $= 1,100 - 4.5 = 1095.5 \text{ W}$

**When input is 11 kW**

Input current $= 11,000/220 = 50 \text{ A}$;

Armature current $= 50 - 2 = 48 \text{ A}$

Arm. Cu loss $= 48^2 * 0.5 = 1,152 \text{ W}$

Total loss $= \text{Arm. Cu loss} + \text{constant losses}$:

$= 1152 - 1095.5 = 2248 \text{ W}$

Output $= 11,000 - 2,248 = 8,752 \text{ W}$

(b) Efficiency $= 8,752 * 100 / 11,000 = 79.6%$

(c) Back EMF at no-load $= 220 - (3 * 0.5) = 218.5 \text{ V}$

Back EMF at given load $= 220 - (48 * 0.5) = 196 \text{ V}$

Then, Speed $N = 1150 * 196 / 218.5 = 1,031 \text{ rpm}$

(d) Power developed in armature $= E_a I_a = 196 * 48 \text{ W}$

$T_a * 2\pi * 1031 / 60 = 196 * 48,$

Then, $T_a = 196 * 48 * 60 / 2\pi * 1031 = 87.1 \text{ N.m}$

**Example 2.19** A 220 V, series motor in which the total armature and field resistance is 0.1\(\Omega\) is working with unsaturated
field, taking 100 A and running at 800 rpm Calculate at what speed the motor will run when developing half the torque?

**Solution:**

\[
\frac{N_2}{N_1} = \frac{E_{a_2} \phi_1}{E_{a_1} \phi_2} = \frac{E_{a_2}}{E_{a_1}} \frac{I_{a_1}}{I_{a_2}}
\]

Since field is unsaturated, \( T_a \propto \phi I_a \propto I_a^2 \)

Then, \( T_1 \propto I_{a_1}^2 \) and \( T_2 \propto I_{a_2}^2 \)

**Or,**

\[
\frac{T_2}{T_1} = \left( \frac{I_{a_2}}{I_{a_1}} \right)^2
\]

Then, \( \frac{1}{2} = \left( \frac{I_{a_2}}{I_{a_1}} \right)^2 \)

Then, \( I_{a_2} = I_{a_1} / \sqrt{2} = 1000 / \sqrt{2} = 70.7 \ A \)

\( E_{a_1} = 220 - 100 * 0.1 = 210 V \)

\( E_{a_2} = 220 - 70.7 * 0.1 = 212.9 \ V \)

\[
\therefore \frac{N_2}{800} = \frac{212.9 * 100}{210 * 70.7} \text{, Then, } N_2 = 1147 \text{ rpm}
\]
Chapter Two

Problems

1- A DC machine (6 kW, 120 V, 1200 rpm) has the following magnetization characteristics at 1200 rpm.

<table>
<thead>
<tr>
<th>$I_f, A$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_a, V$</td>
<td>5</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>79</td>
<td>93</td>
<td>102</td>
<td>114</td>
<td>120</td>
<td>125</td>
</tr>
</tbody>
</table>

The machine parameters are $R_a = 0.2 \Omega$, $R_{fw} = 100 \Omega$. The machine is driven at 1200 rpm and is separately excited. The field current is adjusted at $I_f = 0.8 A$. A load resistance $R_L = 2 \Omega$ is connected to the armature terminals. Neglect armature reaction effect.

(a) Determine the quantity $K_a \phi$ for the machine. (b) Determine $E_a$ and $I_a$. (c) Determine torque $T$ and load power $P_L$.

2- Repeat Problem 1 if the speed is 800 rpm.
3- The DC generator in Problem 1 rotates at 1500 rpm and it delivers rated current at rated terminal voltage. The field winding is connected to a 120 V supply.

(a) Determine the value of the field current.

(b) Determine the value of Rf, required.

4- The DC machine in Problem 1 has a field control resistance whose value can be changed from 0 to 150 Ω. The machine is driven at 1200 rpm. The machine is separately excited and the field winding is supplied from a 120 V supply.

(a) Determine the maximum and minimum values of the no-load terminal voltage.

(b) The field control resistance $R_{fc}$ is adjusted to provide a no-load terminal voltage of 120 V. Determine the value of $R_{fc}$. Determine the terminal voltage at full load for no armature reaction and also if $I_{f(AR)} = 0.1$ A.

5- Repeat Problem 4 if the speed is 1500 rpm.

6- The DC machine in Problem 1 is separately excited. The machine is driven at 1200 rpm and operates as a generator. The
rotational loss is 400 W at 1200 rpm and the rotational loss is proportional to speed.

(a) For a field current of 1.0 A, with the generator delivering rated current, determine the terminal voltage, the output power, and the efficiency. (b) Repeat part (a) if the generator is driven at 1500 rpm.

7-The DC machine in Problem 4 is self-excited.

(a) Determine the maximum and minimum values of the no-load terminal voltage.

(b) $R_{fc}$ is adjusted to provide a no-load terminal voltage of 120 V. Determine the value of $R_{fc}$.

(i) Assume no armature reaction. Determine the terminal voltage at rated armature current. Determine the maximum current the armature can deliver. What is the terminal voltage for this situation?

(ii) Assume that $I_{f_AR(j)} = 0.1$ A at $I_a = 50$ A and consider armature reaction proportional to armature current. Repeat part (i).

8- A DC machine (10 kW, 250 V, 1000 rpm) has $R_a = 0.2 \ \Omega$ and $R_{fw} = 133 \ \Omega$. The machine is self-excited and is driven at 1000 rpm. The data for the magnetization curve are
(a) Determine the generated voltage with no field current.

(b) Determine the critical field circuit resistance.

(c) Determine the value of the field control resistance \( R_f \) if the no-load terminal voltage is 250 V.

(d) Determine the value of the no-load generated voltage if the generator is driven at 800 rpm and \( R_f = 0 \).

(e) Determine the speed at which the generator is to be driven such that no-load voltage is 200 V with \( R_f = 0 \).

9- The self-excited DC machine in Problem 8 delivers rated load when driven at 1000 rpm. The rotational loss is 500 watts.

(a) Determine the generated voltage.

(b) Determine the developed torque.

(c) Determine current in the field circuit. Neglect the armature reaction effect.

(d) Determine the efficiency.

10- A DC shunt machine (24 kW, 240 V, 1000 rpm) has \( R_a = 0.12 \) \( \Omega \), \( N_f = 600 \) turns/pole. The machine is operated as a separately excited DC generator and is driven at 1000 rpm. When \( I_f = 1.8 \) A,
the no-load terminal voltage is 240 V. When the generator delivers full-load current, the terminal voltage drops to 225 V.

(a) Determine the generated voltage and developed torque when the generator delivers full load.

(b) Determine the voltage drop due to armature reaction.

(c) The full-load terminal voltage can be made the same as the no-load terminal voltage by increasing the field current to 2.2 A or by using series winding on each pole. Determine the number of turns per pole of the series winding required if If is kept at 1.8 A.

11- A DC series machine (9.25 kW, 185 V, 1500 rpm) has $R_a + R_{sr} = 0.3 \Omega$. The data for the magnetization curve are:

<table>
<thead>
<tr>
<th>$I_a$ (A)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_a$ (V)</td>
<td>10</td>
<td>50</td>
<td>106</td>
<td>156</td>
<td>184</td>
<td>200</td>
<td>208</td>
</tr>
</tbody>
</table>

Determine the terminal voltage at (a) $I_a = 20$ A. (b) $I_a = 40$ A. (c) $I_a = 60$ A. if the DC series machine operates as a generator.

12- A separately excited DC motor has the following nameplate data: 100 hp, 440 V, 2000 rpm. (a) Determine the rated torque. (b) Determine the current at rated output if the efficiency of the motor is 90% at rated output.

13- The DC machine of Problem 1 is operated at If = 1.0 A. The terminal voltage of the DC machine is 220 V and the developed
torque is 100 N. m. Determine the speed of the DC machine when it operates (a) As a motor. (b) As a generator.

14 The DC machine in Problem 1 is operated as a DC shunt motor. Determine the minimum and maximum no-load speeds if $R_{fc}$ is varied from 0 to 200 Ω.

15- Repeat Problem 4.18 for the full-load condition if $R_{fc} = 0Ω$.
   (a) Assume no armature reaction.
   (b) Assume a 10% reduction of flux at full load.

16- The DC shunt machine in Problem 1 is provided with a series field winding of $N_s = 5$ turns/pole and $R_S = 0.05Ω$. It is connected as along-shunt compound machine. If $R_{fc} = 50 Ω$ and the machine is operated as a cumulative compound motor
   (a) Determine its no-load speed.
   (b) Determine its full-load speed. Assume no armature reaction.

17-Repeat Problem 4.20 if the DC motor is connected as a differentially compounded motor.
18- The compound DC machine in Problem 16 is operated as a series motor by not using the shunt field winding. Determine the speed and torque at (a) 50% rated current. (b) 100% rated current.

19- A DC machine is connected across a 240-volt line. It rotates at 1200 rpm and is generating 230 volts. The armature current is 40 amps. (a) Is the machine functioning as a generator or as a motor? (b) Determine the resistance of the armature circuit. (c) Determine power loss in the armature circuit resistance and the electromagnetic power. (d) Determine the electromagnetic torque in newton-meters. (e) If the load is thrown off, what will the generated voltage and the rpm of the machine be, assuming (i) No armature reaction. (ii) 10% reduction of flux due to armature reaction at 40 amps armature current.

20- The DC shunt machine in Problem 4.13 is used as a motor to drive a load which requires a constant power of 15.36 kW. The motor is connected to a 300 V DC supply. (a) Determine the speed range possible with a field rheostat of 200 Ω (b) Determine the efficiency at the lowest and highest speeds. For this part, assume a constant rotational loss of 300 W over the speed range.
21-A permanent magnet DC motor drives a mechanical load requiring a constant torque of 25 N·m. The motor produces 10 N·m with an armature current of 10 A. The resistance of the armature circuit is 0.2 \( \Omega \). A 200 V DC supply is applied to the armature terminals. Determine the speed of the motor.

22- A DC shunt motor drives an elevator load which requires a constant torque of 300 N.m. The motor is connected to a 600 V DC supply and the motor rotates at 1500 rpm. The armature resistance is 0.5 \( \Omega \).

(a) Determine the armature current.

(b) If the shunt field flux is reduced by 10%, determine the armature current and the speed of the motor.

23- A DC shunt motor (50 hp, 250 V) is connected to a 230 V supply and delivers power to a load drawing an armature current of 200 amperes and running at a speed of 1200 rpm. \( R_a = 0.2 \Omega \).

(a) Determine the value of the generated voltage at this load condition.

(b) Determine the value of the load torque. The rotational losses are 500 watts.
c) Determine the efficiency of the motor if the field circuit resistance is 115 Ω.

24 A DC shunt machine (23 kW, 230 V, 1500 rpm) has R. = 0.1 Ω.

1. The DC machine is connected to a 230 V supply. It runs at 1500 rpm at no-load and 1480 rpm at full-load armature current.
   (a) Determine the generated voltage at full load.
   (b) Determine the percentage reduction of flux in the machine due to armature reaction at full-load condition.

2. The DC machine now operates as a separately excited generator and the field current is kept the same as in part 1. It delivers full load at rated voltage.
   (a) Determine the generated voltage at full load.
   (b) Determine the speed at which the machine is driven.
   (c) Determine the terminal voltage if the load is thrown off.

25 A DC shunt machine (10 kW, 250 V, 1200 rpm) has Ra = 0.25 S.

2. The machine is connected to a 250 V DC supply, draws rated armature current, and rotates at 1200 rpm.
   (a) Determine the generated voltage, the electromagnetic power developed, and the torque developed.
   (b) The mechanical load on the motor shaft is thrown off, and the motor draws 4 A armature current.
(i) Determine the rotational loss. (iii) Determine speed, assuming a 10% change in flux due to armature reaction, for a change of armature current from rated value to 4 A.

26 A 240 V, 2 hp, 1200 rpm DC shunt motor drives a load whose torque varies directly as the speed. The armature resistance of the motor is 0.75 SΩ. With If = 1 A, the motor draws a line current of 7 A and rotates at 1200 rpm.

Assume magnetic linearity and neglect armature reaction effect. (a) The field current is now reduced to 0.7 A. Determine the operating speed of the motor.

(b) Determine the line current, mechanical power developed, and efficiency for the operating condition of part (a). Neglect rotational losses.

27- Repeat Problem 26 if the load torque is constant. Determine the torque.
Chapter 3

The Transformer

3.1 Introduction

The transformer is probably one of the most useful electrical devices ever invented. It can raise or lower the voltage or current in an ac circuit, it can isolate circuits from each other, and it can increase or decrease the apparent value of a capacitor, an inductor, or a resistor. Furthermore, the transformer enables us to transmit electrical energy over great distances and to distribute it safely in factories and homes.

A transformer is a pair of coils coupled magnetically (Fig. 3.1), so that some of the magnetic flux produced by the current in the first coil links the turns of the second, and vice versa. The coupling can be improved by winding the coils on a common magnetic core (Fig. 3.2), and the coils are then known as the windings of the transformer.

Practical transformers are not usually made with the windings widely separated as shown in Fig. 3.1, because the coupling is not very good. Exceptionally, some small power transformers, such as
domestic bell transformers, are sometimes made this way; the physical separation allows the coils to be well insulated for safety reasons. Fig. 3.2 shows the shell type of construction which is widely used for single-phase transformers. The windings are placed on the center limb either side-by-side or one over the other, and the magnetic circuit is completed by the two outer limbs.

Fig. 3.1 Core Type Transformer.

Fig. 3.2 Shell Type Transformer.
Two types of core constructions are normally used, as shown in Fig.3.1. In the *core type* the windings are wound around two legs of a magnetic core of rectangular shape. In the *shell type* (Fig.3.2), the windings are wound around the center leg of a three legged magnetic core. To reduce core losses, the magnetic core is formed of a stack of thin laminations. Silicon-steel laminations of 0.014 inch thickness are commonly used for transformers operating at frequencies below a few hundred cycles. L-shaped laminations are used for *core-type* construction and *E-shaped* laminations are used for *shell-type* construction. To avoid a continuous air gap (which would require a large exciting current),

For small transformers used in communication circuits at high frequencies (kilocycles to megacycles) and low power levels, compressed powdered ferromagnetic alloys, known as permalloy, are used.

A schematic representation of a two-winding transformer is shown in Fig.3.3. The two vertical bars are used to signify tight magnetic coupling between the windings. One winding is connected to an AC supply and is referred to as the *primary winding*. The other winding is connected to an electrical load and is referred to as the *secondary winding*. The winding with the higher number of turns will have a *high voltage* and is called the *high-voltage* (HV) or
high-tension (HT) winding. The winding with the lower number of turns is called the low-voltage (LV) or low-tension (LT) winding. To achieve tighter magnetic coupling between the windings, they may be formed of coils placed one on top of another (Fig.3.2).

![Fig.3.3 A schematic representation of a two-winding transformer](image)

**3.2 Elementary Theory of an Ideal Transformer**

An ideal transformer is one which has no losses i.e. its windings have no ohmic resistance, there is no magnetic leakage and hence which has no $I^2 \cdot R$ and core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core. It may, however, be noted that it is impossible to realize such a transformer in practice, yet for convenience, we will start with such a transformer and step by step approach an actual transformer.
Consider an ideal transformer [Fig.3.3] whose secondary is open and whose primary is connected to sinusoidal alternating voltage $V_1$. This potential difference causes an alternating current to flow in primary. Since the primary coil is purely inductive and there is no output (secondary being open) the primary draws the magnetising current $I_p$ only. The function of this current is merely to magnetise the core, it is small in magnitude and lags $V_1$ by $90^\circ$. This alternating current $I_s$ produces an alternating flux $\phi$ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, is in-phase with it. This changing flux is linked both with the primary and the secondary windings. Therefore, it produces self-induced EMF in the primary. This *self-induced EMF* $E_1$ is, at every instant, equal to and in opposition to $V_1$. It is also known as *counter EMF* or back EMF of the primary.

Similarly, there is produced in the secondary an induced EMF $E_2$ which is known as mutually induced EMF. This EMF is antiphase with $V_1$ and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

Fig.3.3 shows an ideal transformer in which the primary and secondary respectively possess $N_1$ and $N_2$ turns. The primary is connected to a sinusoidal source $V_1$ and the magnetizing current $I_m$
creates a flux $\phi_m$. The flux is completely linked by the primary and secondary windings and, consequently, it is a *mutual flux*. The flux varies sinusoidaly, and reaches a peak value $\phi_{\text{max}}$. Then,

$$E_1 = 4.44 f N_1 \phi_{\text{max}} \quad (3.1)$$

$$E_2 = 4.44 f N_2 \phi_{\text{max}} \quad (3.2)$$

From these equations, we deduce the expression for the voltage ratio and turns ratio $a$ of an ideal transformer:

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \quad (3.3)$$

Where:

$E_1$ = voltage induced in the primary [V].
$E_2$ = voltage induced in the secondary [V].
$N_1$ = numbers of turns on the primary.
$N_2$ = numbers of turns on the secondary.
$a$ = turns ratio.

This equation shows that the ratio of the primary and secondary voltages is equal to the ratio of the number of turns. Furthermore, because the primary and secondary voltages are induced by the same mutual $\phi$ they are necessarily in phase.
The phasor diagram at no load is given in Fig.3.4. Phasor $E_2$ is in phase with phasor $E_1$ (and not $180^\circ$ out of phase) as indicated by the polarity marks. If the transformer has fewer turns on the secondary than on the primary, phasor $E_2$ is shorter than phasor $E_1$. As in any inductor, current $I_m$ lags 90 degrees behind applied voltage $E_1$. The phasor representing flux $\phi$ is obviously in phase with magnetizing current $I_m$ which produces it. However, because this is an ideal transformer, the magnetic circuit is infinitely permeable and so no magnetizing current is required to produce the flux $\phi$. Thus, under no-load conditions, the phasor diagram of such a transformer is identical to Fig.3.4 except that phasor $I_m, I_c, \text{and } I_o$ are infinitesimally small.

![Fig.3.4 transformer vector diagram.](image-url)
Example 3.1 A not quite ideal transformer having 90 turns on the primary and 2250 turns on the secondary is connected to a 120 V, 60 Hz source. The coupling between the primary and secondary is perfect, but the magnetizing current is 4 A.

Calculate:

a. The effective voltage across the secondary terminals

b. The peak voltage across the secondary terminals

c. The instantaneous voltage across the secondary when the instantaneous voltage across the primary is 37 V.

Solution:

The turns ratio is: \[
\frac{N_2}{N_1} = \frac{2250}{90} = 25
\]

The secondary voltage is therefore 25 times greater than the primary voltage because the secondary has 25 times more turns. Consequently:

\[E_2 = 25 \times E_1 = 25 \times 120 = 3000 \text{ V}\]

b. The voltage varies sinusoidaly; consequently, the peak secondary voltage is:

\[E_{2(\text{peak})} = \sqrt{2} E = \sqrt{2} \times 3000 = 4242 \text{ V}\]

c. The secondary voltage is 25 times greater than \(E_1\) at every instant. Consequently, when \(e_1 = 37 \text{ V}\)

\[e_2 = 25 \times 37 = 925 \text{ V}\]
Pursuing our analysis, let us connect a load $Z$ across the secondary of the ideal transformer Fig.3.5. A secondary current $I_2$ will immediately flow, given by:

$$I_2 = \frac{E_2}{Z} \quad (3.4)$$

Does $E_2$ change when we connect the load?

To answer this question, we must recall two facts. First, in an ideal transformer the primary and secondary windings are linked by a mutual flux $\phi_m$, and by no other flux. In other words, an ideal transformer, by definition, has no leakage flux. Consequently, the voltage ratio under load is the same as at no-load, namely:

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \quad (3.5)$$
Second, if the supply voltage $V_1$ is kept fixed, then the primary induced voltage $E_1$ remains fixed. Consequently, mutual flux $\phi$ also remains fixed. It follows that $E_2$ also remains fixed. We conclude that $E_2$ remains fixed whether a load is connected or not. Let us now examine the magnetomotive forces created by the primary and secondary windings. First, current $I_2$ produces a secondary mmf $N_2I_2$. If it acted alone, this mmf would produce a profound change in the mutual flux gym. But we just saw that $\phi_m$ does not change.
under load. We conclude that flux $\phi_m$ can only remain fixed if the primary develops a mmf which exactly counterbalances $N_2I_2$ at every instant. Thus, a primary current $I_1$ must flow so that:

$$N_1I_1 = N_2I_2 \quad (3.6)$$

To obtain the required instant-to-instant bucking effect, currents $I_1$ and $I_2$ must increase and decrease at the same time. Thus, when $I_2$ goes through zero $I_1$ goes through zero, and when $I_2$ is maximum ($+$) $I_1$ is maximum ($+$). In other words, the currents must be in phase. Furthermore, in order to produce the bucking effect, when $I_1$ flows into a polarity mark on the primary side, $I_2$ must flow out of the polarity mark on the secondary side (Fig.3.5a).

Using these facts, we can now draw the phasor diagram of an ideal transformer under load (Fig.3.5b). Assuming a resistive inductive load, current $I_2$ lags behind $E_2$ by an angle $\theta$. Flux $\phi_m$ lags $90^\circ$ behind $V_1$, but no magnetizing current $I_m$ is needed to produce this flux because this is an idea transformer. Finally, the primary and secondary currents are in phase. According to Eq. (3.6), they are related by the equation:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \quad (3.7)$$

$I_1 =$ primary current [A]
$I_2 =$ secondary current [A]
$N_1$ = number of turns on the primary.

$N_2$ = number of turns on the secondary.

Comparing Eq. (3.5) and Eq. (3.7), we see that the transformer current ratio is the inverse of the voltage ratio. In effect, what we gain in voltage, we lose in current and vice versa. This is consistent with the requirement that the apparent power input $E_1 I_1$ to the primary must equal the apparent power output $E_2 I_2$ of the secondary. If the power inputs and outputs were not identical, it would mean that the transformer itself absorbs power. By definition, this is impossible in an ideal transformer.

**Example 3.2** An ideal transformer having 90 turns on the primary and 2250 turns on the secondary is connected to a 200 V, 50 Hz source. The load across the secondary draws a current of 2 A at a power factor of 80 percent lagging.

Calculate:

a. The effective value of the primary current

b. The instantaneous current in the primary when the instantaneous current in the secondary is 100 mA

c. The peak flux linked by the secondary winding

**Solution:**

The turns ratio is $a = \frac{N_1}{N_2} = \frac{90}{2250} = \frac{1}{25}$
The Transformer

The current ratio is therefore 25 and because the primary has fewer turns, the primary current is 25 times greater than the secondary current. Consequently:

\[ I_1 = 25 \times 2 = 50 \ A \]

Instead of reasoning as above, we can calculate the current:

\[
\frac{I_1}{I_2} = \frac{N_2}{N_1}
\]

Then,\[
\frac{I_1}{2} = \frac{2250}{90}
\]

Then \( I_1 = 50 \ A \)

b. The instantaneous current in the primary is always 25 times greater than the instantaneous current in the secondary. Therefore when \( I_2 = 100 \ mA \), \( I_1 \) is:

\[
I_{1, \text{instantaneous}} = 25I_{2, \text{instantaneous}}
\]

\[= 25 \times 0.1 = 2.5 \ A\]

c. In an ideal transformer, the flux linking the secondary is the same as that linking the primary. The peak flux in the secondary is

\[
\phi_{\text{max}} = \frac{V_1}{(4.44 f N_1)}
\]

\[= \frac{200}{(4.44 \times 50 \times 90)} = 0.01 = 10 mWb\]

d. To draw the phasor diagram, we reason as follows: Secondary voltage is:

\[ E_2 = 25 \times E_1 = 25 \times 200 = 5000V \]
E₂ is in phase with E₁ indicated by the polarity marks. For the same reason E₁ is in phase with I₂. Phase angle between E₂ and I₂ is:

\[
\text{Power factor} = \cos \theta \\
0.8 = \cos \theta
\]

Then, \( \theta = 36.9^\circ \)

The phase angle between E₁ and I₁ is also 36.9 degrees. The mutual flux lags 90 degrees behind \( V_1 \).

### 3.3 Impedance Ratio

Although a transformer is generally used to transform a voltage or current, it also has the important ability to transform impedance. Consider, for example, Fig.3.6a in which an ideal transformer T is connected between a source \( V_1 \) and a load \( Z \). The ratio of transformation is \( a \), and so we can write:

\[
\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \\ (3.8)
\]

\[
\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \\ (3.9)
\]

As far as the source is concerned, it sees an impedance \( Z_x \) between the primary terminals given by:

\[
Z_x = \frac{E_1}{I_1} \\ (3.10)
\]
Fig. 3.6 a. Impedance transformation using a transformer. b The impedance seen by the source differs from $Z$.

On the other hand, the secondary sees an impedance $Z$ given by

$$Z = \frac{E_2}{I_2} \quad (3.11)$$

However, can be expressed in another way:

$$Z_x = \frac{E_1}{I_1} = \frac{aE_2}{I_2/a} = a^2 \frac{E_2}{I_2} = a^2 Z \quad (3.12)$$

Consequently,

$$Z_x = a^2 Z \quad (3.13)$$
This means that the impedance seen by the source is $a^2$ times the real impedance (Fig.3.6 b). Thus, an ideal transformer has the amazing ability to increase or decrease the value of impedance. In effect, the impedance seen across the primary terminals is identical to the actual impedance across the secondary terminals multiplied by the square of the turns ratio.

### 3.4 Polarity Of The Transformer

Windings on transformers or other electrical machines are marked to indicate terminals of like polarity. Consider the two windings shown in Fig.3.7a. Terminals 1 and 3 are identical, because currents entering these terminals produce fluxes in the same direction in the core that forms the common magnetic path. For the same reason, terminals 2 and 4 are identical. If these two windings are linked by a common time-varying flux, voltages will be induced in these windings such that, if at a particular instant the potential of terminal 1 is positive with respect to terminal 2, then at the same instant the potential of terminal 3 will be positive with respect to terminal 4. In other words, induced voltages $e_{12}$ and $e_{34}$ are in phase. Identical terminals such as 1 and 3 or 2 and 4 are sometimes marked by dots or ± as shown in Fig.3.7b. These are called the polarity markings of the windings. They indicate how the windings are wound on the core.
If the windings can be visually seen in a machine, the polarities can be determined. However, usually only the terminals of the windings are brought outside the machine. Nevertheless, it is possible to determine the polarities of the windings experimentally. A simple method is illustrated in Fig.3.7c, in which terminals 2 and 4 are connected together and winding 1-2 is connected to an ac supply.

The voltages across 1-2, 3-4, and 1-3 are measured by a voltmeter. Let these voltage readings be called $V_{12}$, $V_{34}$, and $V_{13}$.
respectively. If a voltmeter reading $V_{13}$ is the sum of voltmeter readings $V_{12}$ and $V_{34}$ (i.e., $V_{13} \cong V_{12} + V_{34}$), it means that at any instant when the potential of terminal 1 is positive with respect to terminal 2, the potential of terminal 4 is positive with respect to terminal 3. The induced voltages $e_{12}$ and $e_{43}$ are in phase, as shown in Fig.3.7c, making $e_{13} = e_{12} + e_{43}$. Consequently, terminals 1 and 4 are identical (or same polarity) terminals. If the voltmeter reading $V_{13}$ is the difference between voltmeter readings $V_{12}$ and $V_{34}$ (i.e., $V_{13} \cong V_{12} - V_{34}$), then 1 and 3 are terminals of the same polarity.

Polarities of windings must be known if transformers are connected in parallel to share a common load. Fig.3.8a shows the parallel connection of two single-phase (1φ) transformers. This is the correct connection because secondary voltages $e_{21}$ and $e_{22}$ oppose each other internally. The connection shown in Fig.3.8b is wrong, because $e_{21}$ and $e_{22}$ aid each other internally and a large circulating current $i_{cir}$ will flow in the windings and may damage the transformers. For three-phase connection of transformers, the winding polarities must also be known.
Fig. 3.8 Parallel operation of single-phase transformers. (a) Correct connection. (b) Wrong connection.

3.5 Polarity tests

To determine whether a transformer possesses additive or subtractive polarity, we proceed as follows (Fig. 3.9):

1. Connect the high voltage winding to a low voltage (say 120V) AC source $V_1$.

2. Connect a jumper $J$ between any two adjacent $HV$ and $LV$ terminals.

3. Connect a voltmeter $E_X$ between the other two adjacent $HV$ and $LV$ terminals.

4. Connect another voltmeter $E_P$ across the $HV$ winding. If $E_X$ gives a higher reading than $E_P$, the polarity is additive. This tells us that $H_l$ and $X_l$ are diagonally opposite. On the other hand, if $E_X$ gives a lower reading than $E_P$, the polarity is subtractive, and terminals $H_l$ and $X_l$ are adjacent.
In this polarity text, jumper J effectively connects the secondary voltage $E_S$ in series with the primary voltage $E_P$. Consequently, $E_S$ either adds to or subtracts from $E_P$. In other words, $E_X = E_P + E_S$ or $E_X = E_P - E_S$, depending on the polarity. We can now see how the terms additive and subtractive originated.

In making the polarity test, an ordinary 120 V, 60 Hz source can be connected to the HV winding, even though its nominal voltage may be several hundred kilovolts.

**Example 3.3** During a polarity test on a 500 kVA, 69 kV/600 V transformer (Fig.3.9), the following readings were obtained: $E_P = 118$ V, $E_X = 119$ V. Determine the polarity markings of the terminals.
Solution:

The polarity is additive because $E_X$ is greater than $E_p$. Consequently, the $HV$ and $LV$ terminals connected by the jumper must respectively be labeled $H_1$ and $X_2$ (or $H_2$ and $X_1$).

Fig. 3.10 shows another circuit that may be used to determine the polarity of a transformer. A DC source, in series with an open switch, is connected to the $LV$ winding of the transformer. The transformer terminal connected to the positive side of the source is marked $X_1$. A DC voltmeter is connected across the $HV$ terminals. When the switch is closed, a voltage is momentarily induced in the $HV$ winding. If, at this moment, the pointer of the voltmeter moves upscale, the transformer terminal connected to the (+) terminal of the voltmeter is marked $H_1$ and the other is marked $H_2$.

![Fig. 3.10 Determining the polarity of a transformer using a DC source.](image)
3.6 Practical Transformer

In Section 3.2 the properties of an ideal transformer were discussed. Certain assumptions were made which are not valid in a practical transformer. For example, in a practical transformer the windings have resistances, not all windings link the same flux, permeability of the core material is not infinite, and core losses occur when the core material is subjected to time-varying flux. In the analysis of a practical transformer, all these imperfections must be considered.

Two methods of analysis can be used to account for the departures from the ideal transformer:

1. An equivalent circuit model based on physical reasoning.
2. A mathematical model based on the classical theory of magnetically coupled circuits.

Both methods will provide the same performance characteristics for the practical transformer. However, the equivalent circuit approach provides a better appreciation and understanding of the physical phenomena involved, and this technique will be presented here.

A practical winding has a resistance, and this resistance can be shown as a lumped quantity in series with the winding (Fig. 3.11(a)). When currents flow through windings in the transformer, they establish a resultant mutual (or common) flux $\phi_m$ that is confined
essentially to the magnetic core. However, a small amount of flux known as leakage flux, $\phi_l$ (shown in Fig. 3.11a), links only one winding and does not link the other winding. The leakage path is primarily in air, and therefore the leakage flux varies linearly with current. The effects of leakage flux can be accounted for by an inductance, called leakage inductance:.

If the effects of winding resistance and leakage flux are respectively accounted for by resistance $R$ and leakage reactance $X_L = 2\pi f L$ as shown in Fig. 3.11b, the transformer windings are tightly coupled by a mutual flux.

Fig. 3.11 Development of the transformer equivalent circuits.
\[ L_{l1} = \frac{N_1 \Phi_{l1}}{i_1} = \text{leakage inductance of winding 1} \]

\[ L_{l2} = \frac{N_2 \Phi_{l2}}{i_2} = \text{leakage inductance of winding 2} \]

---

**Fig. 3.11 Continued.**
In a practical magnetic core having finite permeability, a magnetizing current $I_m$ is required to establish a flux in the core. This effect can be represented by a magnetizing inductance $L_m$. Also, the core loss in the magnetic material can be represented by a resistance $R_c$. If these imperfections are also accounted for, then what we are left with is an ideal transformer, as shown in Fig.3.11c. A practical transformer is therefore equivalent to an ideal transformer plus external impedances that represent imperfections of an actual transformer.

The ideal transformer in Fig.3.11c can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. This is almost invariably done. The equivalent circuit with the ideal transformer moved to the right is shown in Fig.3.11d. For convenience, the ideal transformer is usually not shown and the equivalent circuit is drawn, as shown in Fig.3.11e, with all quantities (voltages, currents, and impedances) referred to one side. The referred quantities are indicated with primes. By analyzing this equivalent circuit the referred quantities can be evaluated, and the actual quantities can be determined from them if the turns ratio is known.
### 3.7 Approximate Equivalent Circuits

The voltage drops $I_1R_1$ and $I_1X_1$ (Fig. 3.11e) are normally small and $E_1 \cong V_1$. If this is true then the shunt branch (composed of $R_{c1}$ and $X_m$) can be moved to the supply terminal, as shown in Fig. 3.12a. This approximate equivalent circuit simplifies computation of currents, because both the exciting branch impedance and the load branch impedance are directly connected across the supply voltage. Besides, the winding resistances and leakage reactances can be lumped together. This equivalent circuit (Fig. 3.12a) is frequently used to determine the performance characteristics of a practical transformer.

In a transformer, the exciting current $I_o$ is a small percentage of the rated current of the transformer (less than 5%). A further approximation of the equivalent circuit can be made by removing the excitation branch, as shown in Fig. 3.12b. The equivalent circuit referred to side 2 is also shown in Fig. 3.12c.
Fig. 3.12 Approximate equivalent circuits.

(a) $V_2 = aV_1$, $I_2' = I_2/a$

(b) Referred to side 1, $Z_{eq1} = R_{eq1} + jX_{eq1}$

(c) Referred to side 2, $Z_{eq2} = R_{eq2} + jX_{eq2}$

$$R_{eq2} = \frac{R_{eq1}}{a^2} = R_2 + R'_1$$

$$X_{eq2} = \frac{X_{eq1}}{a^2} = X_{i2} + X'_{i1}$$

$$V'_1 = \frac{V_1}{a}, \quad I'_1 = I_2 = aI_1$$

Fig. 3.12 Approximate equivalent circuits.
3.8 Transformer Rating

The kVA rating and voltage ratings of a transformer are marked on its nameplate. For example, a typical transformer may carry the following information on the nameplate: 10 kVA, 1100/110 volts. What are the meanings of these ratings? The voltage ratings indicate that the transformer has two windings, one rated for 1100 volts and the other for 110 volts. These voltages are proportional to their respective numbers of turns, and therefore the voltage ratio also represents the turns ratio \( a = \frac{1100}{110} = 10 \). The 10 kVA rating means that each winding is designed for 10 kVA. Therefore the current rating for the high-voltage winding is \( \frac{10,000}{1100} = 9.09 \) A and for the lower-voltage winding is \( \frac{10,000}{110} = 90.9 \) A. It may be noted that when the rated current of 90.9 A flows through the low-voltage winding, the rated current of 9.09 A will flow through the high-voltage winding. In an actual case, however, the winding that is connected to the supply (called the primary winding) will carry an additional component of current (excitation current), which is very small compared to the rated current of the winding.
3.9 Determination Of Equivalent Circuit Parameters

The equivalent circuit model (Fig.3.12(a)) for the actual transformer can be used to predict the behavior of the transformer. The parameters $R_{1}, X_{l1}, R_{c1}, X_{m1}, R_{2}, X_{l2}$ and $a = N_{1} / N_{2}$ must be known so that the equivalent circuit model can be used.

If the complete design data of a transformer are available, these parameters can be calculated from the dimensions and properties of the materials used. For example, the winding resistances $(R_{1}, R_{2})$ can be calculated from the resistivity of copper wires, the total length, and the cross-sectional area of the winding. The magnetizing inductances $L_{m}$ can be calculated from the number of turns of the winding and the reluctance of the magnetic path. The calculation of the leakage inductance $(L_{l})$ will involve accounting for partial flux linkages and is therefore complicated. However, formulas are available from which a reliable determination of these quantities can be made.

These parameters can be directly and more easily determined by performing tests that involve little power consumption. Two tests, a no-load test (or open-circuit test) and a short-circuit test, will provide information for determining the parameters of the equivalent circuit of a transformer.
3.9.1 No-Load Test (Or Open-Circuit Test)

This test is performed by applying a voltage to either the high-voltage side or low-voltage side, whichever is convenient. Thus, if a 1100/110 volt transformer were to be tested, the voltage would be applied to the low-voltage winding, because a power supply of 110 volts is more readily available than a supply of 1100 volts.

A wiring diagram for open circuit test of a transformer is shown in Fig.3.13a. Note that the secondary winding is kept open. Therefore, from the transformer equivalent circuit of Fig.3.12a the equivalent circuit under open-circuit conditions is as shown in Fig.3.12b. The primary current is the exciting current and the losses measured by the wattmeter are essentially the core losses. The equivalent circuit of Fig.3.13b shows that the parameters $R_{c1}$ and $X_{m1}$ can be determined from the voltmeter, ammeter, and wattmeter readings.

Note that the core losses will be the same whether 110 volts are applied to the low-voltage winding having the smaller number of turns or 1100 volts are applied to the high-voltage winding having the larger number of turns. The core loss depends on the maximum value of flux in the core.
Fig. 3.13 No-load (or open-circuit) test. (a) Wiring diagram for open-circuit test. (b) Equivalent circuit under open circuit
3.9.2 Short-Circuit Test.

This test is performed by short-circuiting one winding and applying rated current to the other winding, as shown in Fig.3.14a. In the equivalent circuit of Fig.3.12a for the transformer, the impedance of the excitation branch (shunt branch composed of $R_{c1}$ and $X_{m1}$) is much larger than that of the series branch (composed of $R_{eq1}$ and $R_{eq1}$). If the secondary terminals are shorted, the high impedance of the shunt branch can be neglected. The equivalent circuit with the secondary short-circuited can thus be represented by the circuit shown in Fig.3.14b. Note that since $Z_{eq1} = \sqrt{R_{eq1}^2 + X_{eq1}^2}$ is small, only a small supply voltage is required to pass rated current through the windings. It is convenient to perform this test by applying a voltage to the high-voltage winding.

As can be seen from Fig.3.14b, the parameters $R_{eq1}$ and $X_{eq1}$ can be determined from the readings of voltmeter, ammeter, and wattmeter. In a well designed transformer, $R_1 = a^2 R_2 = R'_2$ and $X_{l1} = a^2 X_{l2} = X'_{l2}$.

Note that because the voltage applied under the short-circuit condition is small, the core losses are neglected and the wattmeter reading can be taken entirely to represent the copper losses in the windings, represented by $I_1^2 R_{eq1}$. 
Fig. 3.14 Short-circuit test. (a) Wiring diagram for short-circuit test. (b). Equivalent circuit at short-circuit condition.

The following example illustrates the computation of the parameters of the equivalent circuit of a transformer.
Example 3.4 Tests are performed on a 1φ, 10 kVA, 2200/220 V, 60 Hz transformer and the following results are obtained.

<table>
<thead>
<tr>
<th></th>
<th>Open-Circuit Test (high-voltage side open)</th>
<th>Short-Circuit Test (low-voltage side shorted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltmeter</td>
<td>220 V</td>
<td>150 V</td>
</tr>
<tr>
<td>Ammeter</td>
<td>2.5 A</td>
<td>4.55 A</td>
</tr>
<tr>
<td>Wattmeter</td>
<td>100 W</td>
<td>215 W</td>
</tr>
</tbody>
</table>

(a) Derive the parameters for the approximate equivalent circuits referred to the low-voltage side and the high-voltage side.

(b) Express the excitation current as a percentage of the rated current.

(c) Determine the power factor for the no-load and short-circuit tests.

Solution:

Note that for the no-load test the supply voltage (full-rated voltage of 220V) is applied to the low-voltage winding, and for the short-circuit test the supply voltage is applied to the high-voltage winding with the low-voltage Equivale winding shorted. The ratings of the windings are as follows:

\[ V_{1(rated)} = 2200 \, V \]

\[ V_{2(rated)} = 220 \, V \]
The equivalent circuit and the phasor diagram for the open-circuit test are shown in Fig.3.15a.

Power, $P_{oc} = \frac{V_2^2}{R_{c2}}$

Then $R_{c2} = \frac{220^2}{100} = 484 \ \Omega$

$I_{c2} = \frac{220}{484} = 0.45 \ A$

$I_{m2} = \sqrt{I_2^2 - I_{c2}^2} = \sqrt{(2.5^2 - 0.45^2)} = 2.46 \ A$

$X_{m2} = \frac{V_2}{I_{m2}} = \frac{22}{2.46} = 89.4 \ \Omega$

The corresponding parameters for the high-voltage side are obtained as follows:

Turns ratio $a = \frac{2200}{220} = 10$

$R_{c1} = a^2 R_{c2} = 10^2 \times 484 = 48400 \ \Omega$

$X_{m1} = a^2 X_{m2} = 10^2 \times 89.4 = 8940 \ \Omega$
The equivalent circuit with the low-voltage winding shorted is shown in Fig. 3.15b.

Power \( P_{sc} = I_1^2 R_{eq1} \)

Then, \( R_{eq1} = \frac{215}{4.55^2} = 10.4 \, \Omega \)

\[ Z_{eq1} = \frac{V_{sc1}}{I_{sc1}} = \frac{150}{4.55} = 32.97 \, \Omega \]

Then, \( X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = 31.3 \, \Omega \)

Fig. 3.15

The corresponding parameters for the low-voltage side are as follows:
The approximate equivalent circuits referred to the low-voltage side and the high-voltage side are shown in Fig.3.15c. Note that the impedance of the shunt branch is much larger than that of the series branch.

(b) From the no-load test the excitation current, with rated voltage applied to the low-voltage winding, is:

\[ I_o = 2.5 \text{ A} \]

This is \( \frac{2.5}{45.5} \times 100\% = 5.5\% \) of the rated current of the winding

\[ \text{c power factor at no load} = \frac{\text{Power}}{\text{volt ampere}} = \frac{100}{220 \times 2.5} = 0.182 \]

Power factor at short circuit condition = \( \frac{215}{150 \times 4.55} = 0.315 \)
Example 3.5 Obtain the equivalent circuit of a 200/400-V, 50 Hz, 1 phase transformer from the following test a:--

O.C. test : 200 V, 0.7 A, 70W-on LV side

S.C. test : 15 V, 10 A, 85 W-on HV side

Calculate the secondary voltage when delivering 5 kW at 0.8 power factor lagging, the primary voltage being 200 V.

Solution:

From O.C. Test

\[ P_o = V_o I_o \cdot \cos \varphi_o \]

\[ \cos \varphi_o = \frac{P_o}{V_o I_o} = \frac{70}{200 \times 0.7} = 0.5 \]

Then \( \varphi_o = \cos^{-1} 0.5 = 60^\circ \)

Then \( I_{c1} = I_o \cos \varphi_o = 0.7 \times 0.5 = 0.35A \)

\( I_{m1} = I_o \sin \varphi_o = 0.7 \times 0.866 = 0.606A \)

Then \( R_{c1} = \frac{V_{o1}}{I_{c1}} = \frac{200}{0.35} = 571.4 \ \Omega \)

And \( X_{m1} = \frac{V_{o1}}{I_{m1}} = \frac{200}{0.606} = 330 \ \Omega \)

As shown in Fig.3.16, these values refer to primary i.e. low-voltage side
From Short Circuit test:

It may be noted that in this test instruments have been placed in the secondary i.e. highvoltage winding and the low-voltage winding i.e. primary has been short-circuited.

Now, 

\[ Z_{eq2} = \frac{V_{2sc}}{I_{2sc}} = \frac{15}{10} = 1.5\Omega \]

\[ Z_{eq1} = a^2 \times Z_{eq2} = \left(\frac{1}{2}\right)^2 \times 1.5 = 0.375\Omega \]

Also, 

\[ P_{sc} = I_{2sc}^2 \times R_{eq2} \]

Then, 

\[ R_{eq2} = \frac{85}{100} = 0.85\Omega \]

Then, 

\[ R_{eq1} = a^2 \times R_{eq2} = \left(\frac{1}{2}\right)^2 \times 0.85 = 0.21\ \Omega \]

Then, 

\[ X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = \sqrt{0.375^2 - 0.21^2} = 0.31\ \Omega \]

Fig.3.16
Output KVA = \frac{\text{real power}}{\text{Power factor}} = \frac{5}{0.8} = 6.3 \text{ kVA}

Output current \( I_2 = \frac{5000}{0.8 \times 400} = 15.6 A \)

Now, from the approximate equivalent circuit referred to secondery: \( V_2 \angle 0^\circ = V_1' \angle \delta^\circ - I_2 \angle \phi^\circ \times Z_{eq2} \)

Then, \( V_2 \angle 0^\circ = 400 \angle \delta^\circ - 15.6 \angle -36.87^\circ \times (0.85 + j1.2) \)

\( V_2 \angle 0^\circ = 400 \angle \delta^\circ - 15.6 \angle -36.87^\circ \times 1.5 \angle 54.7^\circ \)

\( V_2 \angle 0^\circ = 400 \angle \delta^\circ - 23.4 \angle 18.17^\circ \)

From the above equation we have two unknown variables \( V_2 \) and \( \delta^\circ \), it need two equations to get both of them. The above equation is a complex one so we can get two equations out of it. If we equate the real parts together and the equate the imaginary parts:

**So from the Imaginary parts:**

\[ |V_2| \sin(0) = 400 \sin(\delta^\circ) - 23.4 \sin(18.17^\circ) \]

\[ 0 = 400 \sin(\delta^\circ) - 7.41^\circ \]

Then, \( \delta^\circ = 7.4^\circ \)

**So from the Real parts:**

\[ |V_2| \cos(0) = 400 \cos(7.41^\circ) - 23.4 \cos(18.17^\circ) \]

Then, \( |V_2| = 374.5 \text{ V} \)
Example 3.6 A 50 Hz, 1–φ transformer has a turns ratio of 6.
The resistances are 0.9 Ω, 0.03 Ω and reactances are 5Ω and 0.13 Ω for high-voltage and low-voltage, windings respectively.
Find (a) the voltage to be applied to the HV side to obtain full-load current of 200 A in the LV winding on short-circuit (b) the power factor on short-circuit.

Solution:
The turns ratio is $a = \frac{N_1}{N_2} = 6$

$R_{eq1} = R_1 + a^2 R_2 = 0.9 + 6^2 * 0.03 = 1.98 \, \Omega$

$X_{eq1} = X_1 + a^2 X_2 = 5 + 6^2 * 0.13 = 9.68 \, \Omega$

$Z_{eq1} = \sqrt{R_{eq1}^2 + X_{eq1}^2} = \sqrt{1.98^2 + 9.68^2} = 9.88 \, \Omega$

$I_1 = \frac{I_2}{a} = \frac{200}{6} = 33.33 \, A$

(a) $V_{sc} = I_1 * Z_{eq1} = 9.88 * 33.33 = 329.3 \, V$

(b) $\cos \phi = \frac{R_{eq1}}{Z_{eq1}} = \frac{1.98}{9.88} = 0.2$
Example 3.7 A 1 phase, 10 kVA, ,500/250-V, 50 Hz transformer has the following constants:

Resistance: Primary 0.2 Ω; Secondary 0.5Ω
Reactance: Primary 0.4Ω; Secondary 0.1Ω

Resistance of equivalent exciting circuit referred to primary, \( R_{cl} = 1500\Omega \)

Reactance of equivalent exciting circuit referred to primary, \( X_{m1} = 750\ \Omega \)

What would be the readings of the instruments when the transformer is connected for the open-circuit and-short-circuit tests?

Solution:

O.C. Test:

\[
I_{m1} = \frac{V_1}{X_m} = \frac{500}{750} = \frac{2}{3} \ A
\]

\[
I_{cl} = \frac{V_1}{R_{cl}} = \frac{500}{1500} = \frac{1}{3} \ A
\]

\[
I_o = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 0.745 \ A
\]

No load primary input \( V_1 \times I_{cl} = 500 \times \frac{1}{3} = 167 \ W \)
Instruments used in primary circuit are: voltmeter, ammeter and wattmeter, their readings being 500 V, 0.745 A and 167 W respectively.

**S.C. Test**

Suppose S.C. test is performed by short-circuiting the LV, winding i.e. the secondary so that all instruments are in primary.

\[ R_{eq1} = R_1 + R'_2 = R_1 + a^2 R_2 = 0.2 + 4 \times 0.5 = 2.2 \ \Omega \]

\[ X_{eq1} = X_1 + X'_2 = X_1 + a^2 X_2 = 0.4 + 4 \times 0.1 = 0.8 \ \Omega \]

Then, \[ Z_{eq1} = \sqrt{R_{eq1}^2 + X_{eq1}^2} = \sqrt{2.2^2 + 0.8^2} = 2.341 \ \Omega \]

Full-load primary current

\[ I_1 = \frac{\text{Rated } kVA}{\text{Rated Primary voltage}} = \frac{10000}{500} = 20 \ A \]

Then \[ V_{sc} = I_1 \times Z_{eq1} = 20 \times 2.431 = 46.8V \]

Power absorbed \[ = I_1^2 \times R_{eq1} = 20^2 \times 2.2 = 880W \]

Primary instruments will read: 468 V, 20 A, 880 W.
3.10 Efficiency

Equipment is desired to operate at a high efficiency. Fortunately, losses in transformers are small. Because the transformer is a static device, there are no rotational losses such as windage and friction losses in a rotating machine. In a well-designed transformer the efficiency can be as high as 99%. The efficiency is defined as follows:

\[
\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{P_{out} + \text{Losses}} \times 100 \quad (3.14)
\]

The losses in the transformer are the core loss \( P_c \) and copper loss \( P_{cu} \). Therefore,

\[
\eta = \frac{P_{out}}{P_{out} + \text{Losses}} = \frac{P_{out}}{P_{out} + P_c + P_{cu}} \quad (3.15)
\]

The copper loss can be determined if the winding currents and their resistances are known:

\[
P_{cu} = I_1^2 R_1 + I_2^2 R_2
\]

\[
= I_1^2 R_{eq1} = I_2^2 R_{eq2} \quad (3.16)
\]

The copper loss is a function of the load current.

The core loss depends on the peak flux density in the core, which in turn depends on the voltage applied to the transformer. Since a transformer remains connected to an essentially constant voltage, the core loss is almost constant and can be obtained from the
no-load test of a transformer. Therefore, if the parameters of the equivalent circuit of a transformer are known, the efficiency of the transformer under any operating condition may be determined. Now,

\[ P_{out} = V_2 I_2 \cos \phi_2 \]

Therefore,

\[
\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_c + I_2^2 R_{eq2}} \times 100 \quad (3.17)
\]

\[
\eta = \frac{V'_2 I'_2 \cos \phi_2}{V'_2 I'_2 \cos \phi_2 + P_c + I'_2^2 R_{eq1}} \times 100 \quad (3.18)
\]

### 3.11 Maximum Efficiency

For constant values of the terminal voltage \( V_2 \) and load power factor angle \( \phi_2 \), the maximum efficiency occurs when:

\[
\frac{d \eta}{d I_2} = 0 \quad (3.19)
\]

If this condition is applied to Eqn. (3.17) the condition for maximum efficiency is:

\[
P_c = I_2^2 R_{eq2} \quad (3.20)
\]

That is, core loss = copper loss. For full load condition,

\[
P_{cu,FL} = I_{2,FL}^2 R_{eq2} \quad (3.21)
\]
Let $x = \frac{I_2}{I_{2,FL}} = \text{per unit loading}$ \hspace{1cm} (3.22)

From Eqns. (3.20), (3.21) and (3.22).

$$P_c = x^2 P_{cu,FL}$$ \hspace{1cm} (3.23)

Then, $x = \sqrt{\left( \frac{P_c}{P_{cu,FL}} \right)}$ \hspace{1cm} (3.24)

For constant values of the terminal voltage $V_2$ and load current $I_2$, the maximum efficiency occurs when:

$$\frac{d\eta}{d\varphi_2} = 0$$ \hspace{1cm} (3.25)
If this condition is applied to Eq.(3.17), the condition for maximum efficiency is

$$\varphi_2 = 0 \text{ Then, } \cos \varphi_2 = 1$$

that is, load power factor = 1

Therefore, maximum efficiency in a transformer occurs when the load power factor is unity (i.e., resistive load) and load current is such that copper loss equals core loss. The variation of efficiency with load current and load power factor is shown in Fig.3.17.
Example 3.8 For the transformer in Example 3.4, determine
(a) Efficiency at 75% rated output and 0.6 PF.
(b) Power output at maximum efficiency and the value of
maximum efficiency. At what percent of full load does this
maximum efficiency occur?

Solution:

\[ P_{\text{out}} = V_2 I_2 \cos \varphi_2. \]

(a) \[ = 0.75 \times 10000 \times 0.6 = 4500 \, W \]

\[ P_c = 100 \, W, \]

\[ P_{\text{cu}} = I_1^2 R_{\text{eq}} \]

\[ = (0.75 \times 4.55)^2 \times 10.4 = 121 \, W \]

\[ \eta = \frac{4500}{4500 + 100 + 121} \times 100 = 95.32\% \]

(b) At maximum efficiency

\[ P_{\text{core}} = P_{\text{cu}} \quad \text{and} \quad PF = \cos \varphi_2 = 1 \]

Now, \[ P_{\text{core}} = 100 \, W = I_2^2 R_{\text{eq2}} = P_{\text{cu}} \]

Then, \[ I_2 = \left( \frac{100}{0.104} \right)^{1/2} = 31 \, A \]

\[ P_{\text{out}} |_{\eta_{\text{max}}} = V_2 I_2 \cos \varphi_2 = 220 \times 31 \times 1 = 6820 \, W \]
\[ \eta_{\text{max}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_c + P_{\text{cu}}} = \frac{6820}{6820 + 100 + 100} \times 100 \]
\[ = 97.15\% \]

output kVA=6.82 and Rated kVA=10

Then, \( \eta_{\text{max}} \) occurs at 68.2% full load.

**Another Method**

From Example 3.4, \( P_{\text{cu,FL}} = 215 W \)

Then \( X = \sqrt{\frac{P_c}{P_{\text{cu,FL}}}} = \sqrt{\frac{100}{215}} = 0.68 \)

**Example 3.9** Obtain the equivalent circuit of a 8kVA 200/400 V, 50 Hz, 1 phase transformer from the following test a :-

O.C. test : 200 V, 0.8 A, 80W,
S.C. test : 20 V, 20 A, 100 W

Calculate the secondary voltage when delivering 6 kW at 0.7 power factor lagging, the primary voltage being 200 V.

**From O.C. Test**

\[ P_o = V_o I_o \cdot \cos \varphi_o \]

\[ \therefore \cos \varphi_o = \frac{P_o}{V_o I_o} = \frac{80}{200 \times 0.8} = 0.5 \]

Then \( \varphi_o = \cos^{-1} 0.5 = 60^\circ \)
Then $I_{cl} = I_o \cos \varphi_o = 0.8 \times 0.5 = 0.4 A$

$I_{m1} = I_o \sin \varphi_o = 0.8 \times 0.866 = 0.69282 A$

Then, $R_{cl} = \frac{V_{o1}}{I_{cl}} = \frac{200}{0.4} = 500 \Omega$

And $X_{m1} = \frac{V_{o1}}{I_{cl}} = \frac{200}{0.69282} = 288.675 \Omega$

**From Short Circuit test:**

It may be noted that in this test instruments have been placed in the secondary i.e. high voltage winding and the low voltage winding i.e. primary has been short-circuited.

Now,

$$Z_{eq2} = \frac{V_{2sc}}{I_{2sc}} = \frac{20}{20} = 1 \Omega$$

Also, $P_{sc} = I_{2sc}^2 \times R_{eq2}$

Then, $R_{eq2} = \frac{100}{20^2} = 0.25 \Omega$

Then, $X_{eq2} = \sqrt{Z_{eq2}^2 - R_{eq2}^2} = \sqrt{1^2 - 0.25^2} = 0.968246 \Omega$

Output current $I_2 = \frac{6000}{0.7 \times 400} = 21.4286 A$
Now, from the approximate equivalent circuit referred to secondery:

\[ V_2 \angle 0^\circ = V_1' \angle \delta^\circ - I_2 \angle \phi^\circ * Z_{eq} \]

Then,

\[ V_2 \angle 0^\circ = 400 \angle \delta^\circ - 21.4286 \angle -45.573^\circ \cdot (0.25 + j0.968246) \]

\[ V_2 \angle 0^\circ = 400 \angle \delta^\circ - 21.43 \angle 29.9495^\circ \]

From the above equation we have two unknown variables \( V_2 \) and \( \delta^\circ \) it need two equations to get both of them. The above equation is a complex one so we can get two equations out of it. If we equate the real parts together and the equate the imaginary parts:

**So from the Imaginary parts:**

\[ |V_2| \sin(0) = 400 \sin(\delta^\circ) - 21.43 \sin(29.9495^\circ) \]

\[ 0 = 400 \sin(\delta^\circ) - 10.6986 \]

Then, \( \delta^\circ = 1.533^\circ \)

**So from the Real parts:**

\[ |V_2| \cos(0) = 400 \cos(1.533^\circ) - 21.43 \cos(29.9495^\circ) \]

Then, \( |V_2| = 381.288 \) \( V \)
Example: 3.10 A 6kVA, 250/500 V, transformer gave the following test results
short-circuit: 20 V; 12 A, 100 W and Open-circuit test: 250 V, 1 A, 80 W

I. Determine the transformer equivalent circuit.
II. Calculate applied voltage, voltage regulation and efficiency when the output is 10 A at 500 volt and 0.8 power factor lagging.
III. Maximum efficiency, at what percent of full load does this maximum efficiency occur? (At 0.8 power factor lagging).
IV. At what percent of full load does the efficiency is 95% at 0.8 power factor lagging.

Solution:
(I) From O.C. Test

\[ P_o = V_o I_o \cos \varphi_o \]

\[ \therefore \cos \varphi_o = \frac{P_o}{V_o I_o} = \frac{80}{250*1.0} = 0.32 \]

Then \( \varphi_o = \cos^{-1} 0.32 = 71.3371^\circ \)

Then \( I_{cl} = I_o \cos \varphi_o = 1.0 * 0.32 = 0.32 A \)

\( I_{m1} = I_o \sin \varphi_o = 1.0 * 0.7953 = 0.7953 A \)
Then \( R_{c1} = \frac{V_{o1}}{I_{c1}} = \frac{250}{0.32} = 781.25 \Omega \)

And \( X_{m1} = \frac{V_{o1}}{I_{m1}} = \frac{250}{0.7953} = 314.35 \Omega \)

As shown in Fig.3.16, these values refer to primary i.e. low-voltage side.

**From Short Circuit test:**

The rated current of the secondary side is:

\[ I_2 = \frac{6000}{500} = 12 \text{ A} \]

It is clear that in this test instruments have been placed in the secondary i.e. high-voltage winding and the low-voltage winding i.e. primary has been short-circuited.

Now,

\[ Z_{eq2} = \frac{V_{2sc}}{I_{2sc}} = \frac{20}{12} = 1.667 \Omega \]

\[ Z_{eq1} = a^2 \times Z_{eq2} = \left(\frac{1}{2}\right)^2 \times 1.667 = 0.4167 \Omega \]

Also, \( P_{sc} = I_{2sc}^2 \times R_{eq2} \)

Then, \( R_{eq2} = \frac{100}{12^2} = 0.694 \Omega \)
Then, \( R_{eq1} = a^2 \times R_{eq2} = \left( \frac{1}{2} \right)^2 \times 0.694 = 0.174 \ \Omega \)

Then, \( X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = \sqrt{0.4167^2 - 0.174^2} = 0.3786 \ \Omega \)

As shown in the following figure, these values refer to primary i.e. low-voltage side

The parameters of series branch can be obtained directly by modifying the short circuit test data to be referred to the primary side as following:

SC test 20 V ; 12 A, 100 W (referred to secondary)
SC test 20/\(a=10\) V ; 12/\(a=24\) A, 100 W (referred to Primary)

So, \( Z_{eq1} = \frac{V_{1sc}}{I_{1sc}} = \frac{10}{24} = 0.4167 \Omega \)

Also, \( P_{sc} = I_{1sc}^2 \times R_{eq1} \)

Then, \( R_{eq1} = \frac{100}{24^2} = 0.174 \Omega \)
Then, \[ X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = \sqrt{0.4167^2 - 0.174^2} = 0.3786 \ \Omega \]

It is clear the second method gives the same results easily.

(II) Output KVA = 10 * 500 * 0.8 = 4 kVA

Now, from the approximate equivalent circuit referred to secondary:

\[ V_1 \angle \delta^o = V_2' \angle 0^o + I_2' \angle \varphi^o * Z_{eq1} \]

Then,

\[ V_1 \angle \delta^o = 250 \angle 0^o + 20 \angle -36.87^o * (0.174 + j0.3786) = 257.358 \angle 0.89^o \]

\[ V_R = \frac{V_1 - V_2'}{V_2'} = \frac{257.358 - 250}{250} * 100 = 2.943\% \]

\[ P_{out} = 10 * 500 * 0.8 = 4kW \]

\[ P_i = P_{oc} = 80 W, \ and, \]

\[ P_{cu} = 10^2 * R_{eq2} = 100 * 0.694 = 69.4 W \ or \]

\[ P_{cu} = P_{sc} * \left( \frac{I_2}{I_{2sc}} \right)^2 = 100 * \left( \frac{10}{12} \right)^2 = 69.4 W \]

\[ \eta = \frac{P_{out}}{P_{out} + P_i + P_{cu}} = \frac{4000}{4000 + 80 + 69.4} * 100 = 96.4\% \]
(III) maximum efficiency occurs when \( P_c = P_{cu} = 80W \)

The percent of the full load at which maximum efficiency occurs is:

\[
X = \sqrt{\frac{P_c}{P_{cu,FL}}} = \sqrt{\frac{80}{100}} = 0.8945\%
\]

Then, the maximum efficiency is:

\[
\eta = \frac{6000 \cdot 0.8945 \cdot 0.8}{6000 \cdot 0.8945 \cdot 0.8 + 80 + 80} \cdot 100 = 96.41\%
\]

(IV)

\[
\eta = \frac{P_{out}}{P_{out} + P_i + P_{cu}} = 0.95
\]

\[
= \frac{6000 \cdot 0.8 \cdot x}{6000 \cdot 0.8 \cdot x + 80 + 100 \cdot x^2} = 0.95
\]

Then,

\[95x^2 - 240x + 76 = 0\]

Then, \( x = 2.155 \) \( \text{ (Unacceptable)} \)

Or \( x = 0.3712 \)

Then to get 95% efficiency at 0.8 power factor the transformer must work at 37.12% of full load.
3.12 All-Day (Or Energy) Efficiency, $\eta_{ad}$

The transformer in a power plant usually operates near its full capacity and is taken out of circuit when it is not required. Such transformers are called power transformers, and they are usually designed for maximum efficiency occurring near the rated output. A transformer connected to the utility that supplies power to your house and the locality is called a distribution transformer. Such transformers are connected to the power system for 24 hours a day and operate well below the rated power output for most of the time. It is therefore desirable to design a distribution transformer for maximum efficiency occurring at the average output power.

A figure of merit that will be more appropriate to represent the efficiency performance of a distribution transformer is the "all-day" or "energy" efficiency of the transformer. This is defined as follows:

$$\eta_{ad} = \frac{\text{energy output over 24 hours}}{\text{energy input over 24 hours}} \times 100 \quad (3.26)$$

$$\eta_{ad} = \frac{\text{energy output over 24 hours}}{\text{energy output over 24 hours} + \text{Losses over 24 hours}}$$

If the load cycle of the transformer is known, the all day efficiency can be determined.
Example 3.11 A 50 kVA, 2400/240 V transformer has a core loss $P_c = 200$ W at rated voltage and a copper loss $P_{cu} = 500$ W at full load. It has the following load cycle.

<table>
<thead>
<tr>
<th>%Load</th>
<th>0.0%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
<th>110%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Factor</td>
<td>1</td>
<td>0.8Lag</td>
<td>0.9Lag</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Hours</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Determine the all-day efficiency of the transformer.

Solution

Energy output 24 hours is

$$0.5 \times 50 \times 6 + 0.75 \times 50 \times 0.8 \times 6 + 1 \times 50 \times 0.9 \times 3 + 1.1 \times 50 \times 1 \times 3 = 630 \text{ kWh}$$

Energy losses over 24 hours:

- Core loss = $0.2 \times 24 = 4.8 \text{ kWh}$
- Copper losses = $0.5^2 \times 0.5 \times 6 + 0.75^2 \times 0.5 \times 6 + 1^2 \times 0.5 \times 3 + 1.1^2 \times 0.5 \times 3 = 5.76 \text{ kWh}$

Total energy loss = 4.8 + 5.76 = 10.56 kWh

Then, $\eta_{AD} = \frac{630}{630 + 10.56} \times 100 = 98.35\%$

3.13 Regulation of a Transformer

(1) When a transformer is loaded with a constant primary voltage, then the secondary terminal voltage drops because of its internal resistance and leakage reactance.

Let $V_{2o} =$ Secondary terminal voltage at no-load
Because at no-load the impedance drop is negligible.

\[ V_2 = \text{Secondary terminal voltage on full-load.} \]

The change in secondary terminal voltage from no-load to full-load is \( V_{20} - V_2 \). This change divided by \( V_{20} \) is known as regulation down. If this change is divided by \( V_2 \), i.e. full-load secondary terminal voltage, then it is called regulation up.

\[
\% \text{reg} = \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100 \tag{3.27}
\]

\[
\% \text{reg} = \frac{(V_2)_{\text{no-load}} - (V_2)_{\text{load}}}{(V_2)_{\text{load}}} \times 100 \tag{3.28}
\]

\[
\% \text{reg} = \frac{V_1' - (V_2)_{\text{load}}}{(V_2)_{\text{load}}} \times 100 = \frac{V_1 - (V_2')_{\text{load}}}{(V_2')_{\text{load}}} \times 100 \tag{3.29}
\]

As the transformer is loaded, the secondary terminal voltage falls (for a lagging power factor). Hence, to keep the output voltage constant, the primary voltage must be increased. The rise in primary voltage required to maintain rated output voltage from no-load to full-load at a given power factor expressed as percentage of rated primary voltage gives the regulation of the transformer.

Vector diagram for the voltage drop in the transformer for different load power factor is shown in Fig.3.18. It is clear that the only way to get \( V_1 \) less than \( V_2' \) is when the power factor is leading.
which means the load has capacitive reactance (i.e. the drop on $Z_{eq1}$ will be negative, which means the regulation may be negative).

![Vector diagram for transformer for different power factor (a) lagging PF (b) Unity PF (c) Leading PF.](image)

Fig.3.18 Vector diagram for transformer for different power factor (a) lagging PF (b) Unity PF (c) Leading PF.
Example 3.12 A 250/500 V, transformer gave the following test results

Short-circuit test: with low-voltage winding shorted.
short-circuit 20 V; 12 A, 100 W

Open-circuit test: 250 V, 1 A, 80 W on low-voltage side.

Determine the circuit constants, insert these on the equivalent circuit diagram and calculate applied voltage, voltage regulation and efficiency when the output is 5 A at 500 volt and 0.8 power factor lagging.

Solution

Open circuit test

\[
\cos \varphi_o = \frac{P_{oc}}{V_{oc}I_{oc}} = \frac{80}{250 \times 1} = 0.32
\]

\[
I_{cl} = I_o \cos \varphi_o = 1 \times 0.32 = 0.32 A
\]

\[
I_{m1} = \sqrt{I_o^2 - I_c^2} = \sqrt{1^2 - 0.32^2} = 0.95 A
\]

\[
R_{cl} = \frac{V_{loc}}{I_c} = \frac{250}{0.32} = 781.3 \text{Ω}
\]

\[
X_{m1} = \frac{V_{loc}}{I_m} = \frac{250}{0.95} = 263.8 \text{Ω}
\]
**Short circuit test**

As the primary is short-circuited, all values refer to secondary winding. So we can obtain $R_{eq2}$ and $X_{eq2}$ and then refer them to primary to get $R_{eq1}$ and $X_{eq1}$ as explained before in **Example 3.5** or we can modify the short circuit data to the primary and then we can calculate $R_{eq1}$ and $X_{eq1}$ directly. Here will use the two method to compare the results.

**First method**

$$R_{eq2} = \frac{P_{sc}}{I_{2sc}^2} = \frac{100}{12^2} = 0.694 \, \Omega$$

$$Z_{eq2} = \frac{V_{sc}}{I_{2sc}} = \frac{20}{12} = 1.667 \, \Omega$$

Then, $X_{eq2} = \sqrt{Z_{eq2}^2 - R_{eq2}^2} = \sqrt{1.667^2 - 0.694^2} = 1.518 \, \Omega$

As $R_c$ and $X_m$ refer to primary, hence we will transfer these values ($R_{eq2}$, $X_{eq2}$, and $Z_{eq2}$) to primary with the help of transformation ratio.

Then

$$R_{eq1} = a^2 \times R_{eq2} = 0.5^2 \times 0.694 = 0.174 \Omega$$

$$X_{eq1} = a^2 \times X_{eq2} = 0.5^2 \times 1.518 = 0.38 \Omega$$

$$Z_{eq1} = a^2 \times Z_{eq2} = 0.5^2 \times 1.667 = 0.417 \Omega$$
Second method

Short-circuited results referred to secondary are 20 V, 12 A, 100 W

Then, Short-circuited results referred to primary are 10 V, 24 A, 100 W

Then, \[ R_{eq1} = \frac{P_{sc}}{I^2_{1sc}} = \frac{100}{24^2} = 0.174 \, \Omega \]

\[ Z_{eq1} = \frac{V_{1sc}}{I_{1sc}} = \frac{10}{24} = 0.417 \, \Omega \]

Then, \[ X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = \sqrt{0.417^2 - 0.174^2} = 0.38 \, \Omega \]

Applied voltage

\[ V_1 \, \delta^o = V_2' \, 0^o + I_2' \, \varphi^o \cdot Z_{eq1} \]

Then, \[ V_1 \, \delta^o = 250 \, 0^o + 10 \, - \cos^{-1} 0.8 \cdot (0.174 + j0.38) \]

\[ V_1 \, \delta^o = 250 \, 0^o + 10 \, -36.24^o \cdot 0.418 \, 65.4^o \]

\[ V_1 \, \delta^o = 250 \, 0^o + 4.18 \, 29.16^o \]

\[ V_1 \, \delta^o = 250 \, 0^o + 3.65 + j2.04 = 253.65 + j2.04 \]

\[ = 253.7 \, 0.47^o \, V \]
Voltage regulation

\[ \%_{\text{reg}} = \frac{(V_1) - (V'_2)_{\text{load}}}{(V'_2)_{\text{load}}} \times 100 \]

\[ (V'_2)_{\text{load}} = 250 \, 0^\circ \]

\[ \%_{\text{reg}} = \frac{253.7 - 250}{250} \times 100 = 1.48\% \]

Efficiency

\[ \eta = \frac{V'_2 \, I'_2 \, \cos \varphi}{V'_2 \, I'_2 \, \cos \varphi + P_{\text{cu}} + P_{\text{iron}}} \times 100 \]

\[ \eta = \frac{250 \times 10 \times 0.8}{250 \times 10 \times 0.8 + 10^2 \times 0.174 + 80} \times 100 = 95.356\% \]

Example 3.13 A 1φ, 10 kVA, 2400/240 V, 60 Hz distribution transformer has the following characteristics: Core loss at full voltage =100 W and Copper loss at half load =60 W (a) Determine the efficiency of the transformer when it delivers full load at 0.8 power factor lagging. (b) Determine the rating at which the transformer efficiency is a maximum. Determine the efficiency if the load power factor is 0.9. (c) The transformer has the following load cycle:

No load for 6 hours, 70% full load for 10 hours at 0.8 PF and 90% full load for 8 hours at 0.9 PF
**Solution:**

(a) \[ P_{out} = 10 \times 0.8 = 8 \text{ kW} \]

\[ P_{core} = 100 \text{ W}, \quad P_{cu,FL} = 60 \times 2^2 = 240\text{W} \]

\[ \eta = \frac{8000}{8000 + 100 + 240} \times 100 = 95.92\% \]

(b) \[ x = \sqrt{\frac{100}{240}} = 0.6455 \]

\[ \eta_{\text{max}} = \frac{10 \times 10^3 \times 0.6455 \times 0.9}{\left(10^4 \times 0.6455 \times 0.9\right) + 100 + 100} = 96.67\% \]

Output energy in 24 hours is:

\[ E_{24hrs} = 0 + 10 \times 0.7 \times 0.8 \times 10 + 10 \times 0.9 \times 0.9 \times 8 = 120.8\text{kWh} \]

Energy losses in the core in 24 hours is

\[ E_{\text{core}} = 100 \times 24 \times 10^{-3} = 2.4\text{kWh} \]

Energy losses in the cupper in 24 hours is

\[ E_{\text{cu}} = \left(240 \times 0.7^2 \times 10 + 240 \times 0.9^2 \times 8\right) \times 10^{-3} = 2.7312\text{kWh} \]

Then, \[ \eta_{\text{all day}} = \frac{120.8}{120.8 + 2.4 + 2.7312} \times 100 = 95.93\% \]
3.14 Percentage Resistance, Reactance and Impedance

These quantities are usually measured by the voltage drop at full-load current expressed as a percentage of the normal voltage of the winding on which calculations are made.

(i) Percentage resistance at full load

\[
\% R = \frac{I_{1}^{*} R_{eq1} \times 100}{V_{1}} = \frac{I_{1}^{2} R_{eq1} \times 100}{V_{1} I_{1}} \]

\[
= \frac{I_{2}^{2} R_{eq2} \times 100}{V_{2} I_{2}} \]

\begin{equation}
(3.30)
\end{equation}

\[\text{Loss at full load} \]

Percentage reactance at full load:

\[
\% X = \frac{I_{1}^{*} X_{eq1} \times 100}{V_{1}} = \frac{I_{2}^{2} X_{eq2} \times 100}{V_{2}} \]

\begin{equation}
(3.31)
\end{equation}

\[
\% Z = \frac{I_{1}^{*} Z_{eq1} \times 100}{V_{1}} = \frac{I_{2}^{2} Z_{eq2} \times 100}{V_{2}} \]

\begin{equation}
(3.32)
\end{equation}

\[
\% Z = \sqrt{\% R^2 + \% X} \]

\begin{equation}
(3.33)
\end{equation}
3.15 Autotransformer

This is a special connection of the transformer from which a variable AC voltage can be obtained at the secondary. A common winding as shown in Fig.3.19 is mounted on core and the secondary is taken from a tap on the winding. In contrast to the two-winding transformer discussed earlier, the primary and secondary of an autotransformer are physically connected. However, the basic principle of operation is the same as that of the two-winding transformer.

![Autotransformer Diagram](image)

Fig.3.19 Step down autotransformer.
Since all the turns link the same flux in the transformer core,

\[
\frac{V_1}{V_2} = \frac{N_1}{N_2} = a
\]

(3.34)

If the secondary tapping is replaced by a slider, the output voltage can be varied over the range \(0 < V_2 < V_1\).

The ampere-turns provided by the upper half (i.e., by turns between points a and b) are:

\[
(N_1 - N_2)I_1 = \left(1 - \frac{1}{a}\right)N_1I_1
\]

(3.35)

The ampere-turns provided by the lower half (i.e., by turns between points b and c) are:

\[
N_2(I_2 - I_1) = \frac{N_1}{a} (I_2 - I_1)
\]

(3.36)

from amper turn balance, from equations (3.35) and (3.36)

\[
\left(1 - \frac{1}{a}\right)N_1I = \frac{N_1}{a} (I_2 - I_1)
\]

(3.37)

Then,

\[
\frac{I_1}{I_2} = \frac{1}{a}
\]

(3.38)

Equations (3.34) and (3.37) indicate that, viewed from the terminals of the autotransformer, the voltages and currents are related by the same turns ratio as in a two-winding transformer.

The advantages of an autotransformer connection are lower leakage reactances, lower losses, lower exciting current, increased
kVA rating (see Example 3.11), and variable output voltage when a sliding contact is used for the secondary. The disadvantage is the direct connection between the primary and secondary sides.

**Example 3.14** A 1 φ, 100 kVA, 2000/200 V two-winding transformer is connected as an autotransformer as shown in Fig.E2.6 such that more than 2000 V is obtained at the secondary. The portion \(ab\) is the 200 V winding, and the portion \(bc\) is the 2000 V winding. Compute the kVA rating as an autotransformer.

![Diagram showing an autotransformer with labels: \(I_H = 500\ A\), \(I_L = 550\ A\), \(V_L = 2000\ V\), \(V_H = 2200\ V\)](image.png)
Solution:

The current ratings of the windings are

\[
I_{ab} = \frac{100,000}{200} \text{ A} = 500 \text{ A}
\]

\[
I_{bc} = \frac{100,000}{2000} = 50 \text{ A}
\]

Therefore, for full-load operation of the autotransformer, the terminal currents are:

\[
I_H = 500 \text{ A}
\]

\[
I_L = 500 + 50 = 550 \text{ A}
\]

Now, \( V_L = 2000 \text{ V} \) and

\[
V_H = 2000 + 200 = 2200 \text{ V}
\]

Therefore,

\[
kVA_L = \frac{2000 \times 550}{1000} = 1100
\]

\[
kVA_H = \frac{2200 \times 500}{1000} = 1100
\]

A single-phase, 100 kVA, two-winding transformer when connected as an autotransformer can deliver 1100 kVA. Note that this higher rating of an autotransformer results from the conductive connection. Not all of the 1100 kVA is transformed by electromagnetic induction. Also note that the 200 V winding must have sufficient insulation to withstand a voltage of 2200 V to ground.
Example 3.15 A single phase, 50 kVA, 2400/460 V, 50 Hz transformer has an efficiency of 0.95% when it delivers 45kW at 0.9 power factor. This transformer is connected as an auto-transformer to supply load to a 2400 V circuit from 2860 V source.

(a) Show the transformer connection.

(b) Determine the maximum kVA the autotransformer can supply to 2400 V circuit. (c) Determine the efficiency of the autotransformer for full load at 0.9 power factor.

Solution:

(a) 

(b) \[ I_{s,2w} = \frac{50 \times 10^3}{2460} = 108.7 \, A \]
\[ kVA_{Auto} = 108.782860 = 310.87 \text{ kW} \]

(c) \[ \eta_2 = \frac{50 \times 10^3 \times 0.9}{50 \times 10^3 \times 0.9 + P_i + P_{cu,FL}} = 0.95 \]

Then, \[ P_i + P_{cu,FL} = 2368.42 \text{ W} \]

\[ \eta_{Auto} = \frac{310870 \times 0.9}{310870 \times 0.9 + 2368.42} = 99.61\% \]

### 3.16 Three-Phase Transformers

#### 3.16.1 Introduction

Power is distributed throughout the world by means of 3-phase transmission lines. In order to transmit this power efficiently and economically, the voltages must be at appropriate levels. These levels (13.8 kV to 1000 kV) depend upon the amount of power that has to be transmitted and the distance it has to be earned. Another aspect is the appropriate voltage levels used in factories and homes. These are fairly uniform, ranging from 120/240 V single-phase systems to 480 V, 3-phase systems. Clearly, this requires the use of 3-phase transformers to transform the voltages from one level to another. The transformers may be inherently 3-phase, having three primary windings and three secondary windings mounted on a 3-legged core. However, the same result can be achieved by using three single-phase transformers connected together to form a 3-phase transformer bank.
3.16.2 Basic Properties Of 3-Phase Transformer Banks

When three single-phase transformers are used to transform a 3-phase voltage, the windings can be connected in several ways. Thus, the primaries may be connected in delta and the secondaries in wye, or vice versa. As a result, the ratio of the 3-phase input voltage to the 3-phase output voltage depends not only upon the turns ratio of the transformers, but also upon how they are connected.

A 3-phase transformer bank can also produce a phase shift between the 3-phase input voltage and the 3-phase output voltage. The amount of phase shift depends again upon the turns ratio of the transformers, and on how the primaries and secondaries are interconnected. Furthermore, the phaseshift feature enables us to change the number of phases. Thus, a 3-phase system can be converted into a 2-phase, a 6-phase, or a 12-phase system. Indeed, if there were a practical application for it, we could even convert a 3-phase system into a 5-phase system by an appropriate choice of single-phase transformers and interconnections.

In making the various connections, it is important to observe transformer polarities. An error in polarity may produce a short-circuit or unbalance the line voltages and currents.

The basic behavior of balanced 3-phase transformer banks can be understood by making the following simplifying assumptions:
The exciting currents are negligible.

2. The transformer impedances, due to the resistance and leakage reactance of the windings, are negligible.

3. The total apparent input power to the transformer bank is equal to the total apparent output power.

Furthermore, when single-phase transformers are connected into a 3-phase system, they retain all their basic single-phase properties, such as current ratio, voltage ratio, and flux in the core. Given the polarity marks $X_1, X_2$ and $H_1, H_2$, the phase shift between primary and secondary is zero.

### 3.16.3 Delta-Delta Connection

The three single-phase transformers P, Q, and R of Fig.3.21 transform the voltage of the incoming transmission line A, B, C to a level appropriate for the outgoing transmission line 1, 2, 3. The incoming line is connected to the source, and the outgoing line is connected to the load. The transformers are connected in delta-delta. Terminal $H_1$ of each transformer is connected to terminal $H_2$ of the next transformer. Similarly, terminals $X_1$ and $X_2$ of successive transformers are connected together. The actual physical layout of the transformers is shown in Fig.3.21. The corresponding schematic diagram is given in Fig.3.22. The schematic diagram is drawn in such a way to show not only the connections, but also the phasor
relationship between the primary and secondary voltages. Thus, each secondary winding is drawn parallel to the corresponding primary winding to which it is coupled. Furthermore, if source G produces voltages $E_{AB}, E_{BC}, E_{CA}$ according to the indicated phasor diagram, the primary windings are oriented the same way, phase by phase. For example, the primary of transformer P between lines A and B is oriented horizontally, in the same direction as phasor $E_{AB}$.

![Diagram of a delta-delta connection of three single-phase transformers.](image)

Fig. 3.21 Delta-delta connection of three single-phase transformers. The incoming lines (source) are A, B, C and the outgoing lines (load) are 1, 2, 3.
In such a delta-delta connection, the voltages between the respective incoming and outgoing transmission lines are in phase. If a balanced load is connected to lines 1-2-3, the resulting line currents are equal in magnitude. This produces balanced line currents in the incoming lines A-B-C. As in any delta connection, the line currents are 43 times greater than the respective currents \( I_p \) and \( I_s \) flowing in the primary and secondary windings (Fig.3.22). The power rating of the transformer bank is three times the rating of a single transformer.

Note that although the transformer bank constitutes a 3-phase arrangement, each transformer, considered alone, acts as if it were
placed in a singlephase circuit. Thus, a current $I_p$ flowing from $H_1$ $H_2$ in the primary winding is associated with a current $I_s$ flowing from $X_2$ to $X_1$ in the secondary.

**Example 3.16** Three single-phase transformers are connected in delta-delta to step down a line voltage of 138 kV to 4160 V to supply power to a manufacturing plant. The plant draws 21 MW at a lagging power factor of 86 percent.

Calculate a. The apparent power drawn by the plant b. The apparent power furnished by the HV line c. The current in the HV lines d. The current in the LV lines e. The currents in the primary and secondary windings of each transformer f. The load carried by each transformer

**Solution:**

a. The apparent power drawn by the plant is:

$$S = P / \cos \phi = 21/0.86 = 24.4 \text{ MVA}$$

b. The transformer bank itself absorbs a negligible amount of active and reactive power because the $I^2 R$ losses and the reactive power associated with the mutual flux and the leakage fluxes are small. It follows that the apparent power furnished by the HV line is also 24.4 MVA.
c. The current in each HV line is:

\[ I_1 = \frac{S}{\sqrt{3} \cdot V_1} = \frac{24.4 \times 10^6}{3 \cdot 13800} = 102 \ A \]

d. The current in the LV lines is:

\[ I_2 = \frac{S}{\sqrt{3} \cdot V_2} = \frac{24.4 \times 10^6}{3 \cdot 4160} = 3386 \ A \]

e. Referring to Fig.3.19, the current in each primary winding is:

\[ I_p = \frac{102}{\sqrt{3}} = 58.9 \ A \]

The current in each secondary winding is:

\[ I_s = \frac{3386}{\sqrt{3}} = 1955 \ A \]

f. Because the plant load is balanced, each transformer carries one-third of the total load, or \(24.4/3 = 8.13\) MVA.

The individual transformer load can also be obtained by multiplying the primary voltage times the primary current:

\[ S = E_p I_p = 138000 \times 58.9 = 8.13 \text{ MVA} \]

Note that we can calculate the line currents and the currents in the transformer windings even though we do not know how the 3-phase load is connected. In effect, the plant load (shown as a box in Fig.3.22) is composed of hundreds of individual loads, some of which are connected in delta, others in wye. Furthermore, some are single-phase loads operating at much lower voltages than 4160 V,
powered by smaller transformers located inside the plant. The sum total of these loads usually results in a reasonably well-balanced 3-phase load, represented by the box.

### 3.16.4 Delta-wye connection

When the transformers are connected in delta-wye, the three primary windings are connected the same way as in Fig.3.21. However, the secondary windings are connected so that all the $X_2$ terminals are joined together, creating a common neutral N (Fig.3.23). In such a delta-wye connection, the voltage across each primary winding is equal to the incoming line voltage. However, the outgoing line voltage is 3 times the secondary voltage across each transformer.

The relative values of the currents in the transformer windings and transmission lines are given in Fig.3.24. Thus, the line currents in phases A, B, and C are $\sqrt{3}$ times the currents in the primary windings. The line currents in phases 1, 2, 3 are the same as the currents in the secondary windings. A delta-wye connection produces a 30 phase shift between the line voltages of the incoming and outgoing transmission lines. Thus, outgoing line voltage $E_{12}$ is 30 degrees ahead of incoming line voltage $E_{AB}$, as can be seen from the phasor diagram. If the outgoing line feeds an isolated group of
loads, the phase shift creates no problem. But, if the outgoing line has to be connected in parallel with a line coming from another source, the 30 degrees shift may make such a parallel connection impossible, even if the line voltages are otherwise identical.

One of the important advantages of the wye connection is that it reduces the amount of insulation needed inside the transformer. The HV winding has to be insulated for only $\frac{1}{\sqrt{3}}$, or 58 percent of the line voltage.

![Delta-wye connection of three single-phase transformers.](image)

Fig. 3.23 Delta-wye connection of three single-phase transformers.
Example 3.17 Three single-phase step-up transformers rated at 90 MVA, 13.2 kV/80 kV are connected in delta-wye on a 13.2 kV transmission line (Fig.3.25). If they feed a 90 MVA load, calculate the following:

a. The secondary line voltage
b. The currents in the transformer windings
c. The incoming and outgoing transmission line currents
Solution
The easiest way to solve this problem is to consider the windings of only one transformer, say, transformer P.

a. The voltage across the primary winding is obviously 13.2 kV
The voltage across the secondary is, therefore, 80 kV.
The voltage between the outgoing lines 1, 2, and 3 is:
\[ V_2 = 80 \times \sqrt{3} = 139 \text{kV} \]
b. The load carried by each transformer is
\[ S = \frac{90}{3} = 30 \text{MVA} \]
\[ E_s = 80 \sqrt{3} = 139 \text{kV} \]

b. The load carried by each transformer is
\[ S = \frac{90}{3} = 30 \text{ MVA} \]
The current in the primary winding is
\[ I_p = \frac{30 \text{ MVA}}{13.2 \text{kV}} = 2273 \text{ A} \]
The current in the secondary winding is
\[ I_s = \frac{30 \text{ MVA}}{80 \text{kV}} = 375 \text{ A} \]
3.16.5 Wye-delta connection

The currents and voltages in a wye-delta connection are identical to those in the delta-wye connection. The primary and secondary connections are simply interchanged. In other words, the $H_2$ terminals are connected together to create a neutral, and the $X_1, X_2$ terminals are connected in delta. Again, there results a 30 degrees phase shift between the voltages of the incoming and outgoing lines.

3.16.6 Wye-wye connection

When transformers are connected in wye-wye, special precautions have to be taken to prevent severe distortion of the line-to-neutral voltages. One way to prevent the distortion is to connect the neutral of the primary to the neutral of the source, usually by way of the ground (Fig.3.26). Another way is to provide each transformer with a third winding, called tertiary winding. The tertiary windings of the three transformers are connected in delta (Fig.3.27). They often provide the substation service voltage where the transformers are installed.

c. The current in each incoming line A, B, C is

$$I = 2273 \sqrt{3} = 3937 \text{ A}$$

The current in each outgoing line 1, 2, 3 is

$$I = 375 \text{ A}$$
Note that there is no phase shift between the incoming and outgoing transmission line voltages of a wye-wye connected transformer.

![Fig.3.26 Wye-wye connection with neutral of the primary connected to the neutral of the source.](image-url)

![Fig.3.27 Wye-wye connection using a tertiary winding.](image-url)

**Example 3.18** Three single phase, 30 kVA, 2400/240 V, 50 Hz transformers are connected to form 3 ϕ, 2400/416 V transformer bank. The equivalent impedance of each transformer referred to the high voltage side is 1.5+j2 S2. The transformer delivers 60 kW at 0.75 power factor (leading).
(a) Draw schematic diagram showing the transformer connection.

(b) Determine the transformer winding current

(c) Determine the primary voltage.

(d) Determine the voltage regulation.

**Solution:**

(a)

(b) \( kVA = \frac{60}{0.75} = 80kVA \)

\( I_s = \frac{80 \times 10^3}{\sqrt{3} \times 416} = 111.029 \text{ A} \)

\( a = \frac{2400}{240} = 10 \)

\( I_{1ph} = \frac{111.029}{10} = 11.103 \text{ A} \)

\( I_{1L} = 11.103 \times \sqrt{3} = 19.231 \text{ A} \)
\[ V'_2 = 2400 \angle 0^\circ \text{ V} \ , \ I'_2 = 11.103 \angle 41.41^\circ \text{ A} \]

\[ V_1 = V'_2 + I'_2 \cdot (Z_{eq1}) \]

\[ = 2400 \angle 0 + 11.103 \angle 41.41^\circ \cdot (1.5 + j2) = 2397.96 \angle 0.66^\circ \text{ V} \]

\[ V_R = \frac{V_1 - V'_2}{V'_2} \cdot 100 \]

\[ = \frac{2397.96 - 2400}{2400} \cdot 100 = -0.0875\% \]
Problems:

1. A 10 kVA, 1000/100 V transformer gave the following test results: open-circuit test 100 V, 6.0 A, 400 W short-circuit test 50 V, 100 A, 1800 W
   
   (a) Determine the rated voltage and rated current for the HV and LV sides.
   
   (b) Derive an approximate equivalent circuit referred to the HV side.
   
   (c) Determine the voltage regulation at full load, 0.6 PF leading.
   
   (d) Draw the phasor diagram for condition (c).

2. A 1.25 kVA, 220/440 V, 60 Hz transformer gave the following test results.

   Open circuit test : 220 V, 9.5 A, 650 W  Short-circuit test : 37.5 V, 55 A, 950 W
   
   (a) Derive the approximate equivalent circuit in per-unit values.
   
   (b) Determine the voltage regulation at full load, 0.8 PF lagging.
   
   (c) Draw the phasor diagram for condition (b).

3. A 1 10 kVA, 2400/120 V, 60 Hz transformer has the following equivalent circuit parameters:  \( Z_{eq1} = 5 + j25 \) \( \Omega \),  \( R_{c1} \).
Standard no-load and short-circuit tests are performed on this transformer. Determine the following:

No-load test results: $V_{oc}, I_{oc}, P_{oc}$

Short-circuit test results: $V_{sc}, I_{sc}, P_{sc}$

A single-phase, 250 kVA, 11 kV/2.2 kV, 60 Hz transformer has the following parameters. $R_{HV} = 1.3 \Omega$, $X_{HV} = 4.5 \Omega$, $R_{LV} = 0.05 \Omega$, $X_{LV} = 0.16$, $R_{c1} = 2.4 \Omega$, $X_{m2} = 0.8 \Omega$

(a) Draw the approximate equivalent circuit (i.e., magnetizing branch, with $R_{c1}$ and $X_{m}$ connected to the supply terminals) referred to the HV side and show the parameter values.

(b) Determine the no load current in amperes (HV side) as well as in per unit.

(c) If the low-voltage winding terminals are shorted, determine
   (i) The supply voltage required to pass rated current through the shorted winding.
   (ii) The losses in the transformer.

(d) The HV winding of the transformer is connected to the 11 kV supply and a load, $Z_L = 15^\circ - 90^\circ \Omega$ is connected to the low voltage winding. Determine:
   (i) Load voltage. (ii) Voltage regulation.
5  A 1-ϕ, 10 kVA, 2400/240 V, 60 Hz distribution transformer has the following characteristics: Core loss at full voltage = 100 W  Copper loss at half load = 60 W
(a) Determine the efficiency of the transformer when it delivers full load at 0.8 power factor lagging. (b) Determine the per unit rating at which the transformer efficiency is a maximum. Determine this efficiency if the load power factor is 0.9. The transformer has the following load cycle: No load for 6 hours 70% full load for 10 hours at 0.8 PF 90% full load for 8 hours at 0.9 PF Determine the all-day efficiency of the transformer.

6  The transformer of Problem 5 is to be used as an autotransformer (a) Show the connection that will result in maximum kVA rating. (b) Determine the voltage ratings of the high-voltage and low-voltage sides. (c) Determine the kVA rating of the autotransformer. Calculate for both high-voltage and low-voltage sides.

7  A 1-ϕ, 10 kVA, 460/120 V, 60 Hz transformer has an efficiency of 96% when it delivers 9 kW at 0.9 power factor. This transformer is connected as an
autotransformer to supply load to a 460 V circuit from a 580 V source.

(a) Show the autotransformer connection.

(b) Determine the maximum kVA the autotransformer can supply to the 460 V circuit.

(c) Determine the efficiency of the autotransformer for full load at 0.9 power factor.

8 Reconnect the windings of a 1φ, 3 kVA, 240/120 V, 60 Hz transformer so that it can supply a load at 330 V from a 110 V supply. (a) Show the connection.

(b) Determine the maximum kVA the reconnected transformer can deliver.

9 Three 1¢, 10 kVA, 460/120 V, 60 Hz transformers are connected to form a 3φ 460/208 V transformer bank. The equivalent impedance of each transformer referred to the high-voltage side is 1.0 + j2.0 Ω. The transformer delivers 20 kW at 0.8 power factor (leading).

(a) Draw a schematic diagram showing the transformer connection. (b) Determine the transformer winding current. (c) Determine the primary voltage. (d) Determine the voltage regulation.
10 A 1Φ 200 kVA, 2100/210 V, 60 Hz transformer has the following characteristics. The impedance of the high-voltage winding is $0.25 + j 1.5 \ \Omega$ with the low-voltage winding short-circuited. The admittance (i.e., inverse of impedance) of the low-voltage winding is $0.025 - j 0.075$ mhos with the high-voltage winding open-circuited.

(a) Taking the transformer rating as base, determine the base values of power, voltage, current, and impedance for both the high-voltage and low-voltage sides of the transformer.

(b) Determine the per-unit value of the equivalent resistance and leakage reactance of the transformer.

(c) Determine the per-unit value of the excitation current at rated voltage.

(d) Determine the per-unit value of the total power loss in the transformer at full-load output condition.
Chapter Four

Three Phase Induction Machine

4.1 Introduction

Three-phase induction motors are the motors most frequently encountered in industry. They are simple, rugged, low-priced, and easy to maintain. They run at essentially constant speed from zero to full-load. The speed is frequency-dependent and, consequently, these motors are not easily adapted to speed control. However, variable frequency electronic drives are being used more and more to control the speed of commercial induction motors.

In this chapter we cover the basic principles of the 3-phase induction motor and develop the fundamental equations describing its behavior. We then discuss its general construction and the way the windings are made.

Squirrel-cage, wound-rotor ranging from a few horsepower to several thousand horsepower permit the reader to see that they all operate on the same basic principles.
4.2 Principal components

A 3-phase induction motor (Fig.4.1) has two main parts: a stationary stator and a revolving rotor. The rotor is separated from the stator by a small air gap that ranges from 0.4 mm to 4 mm, depending on the power of the motor.

The stator (Fig.4.2) consists of a steel frame that supports a hollow, cylindrical core made up of stacked laminations. A number of evenly spaced slots, punched out of the internal circumference of the laminations, provide the space for the stator winding.

The rotor is also composed of punched laminations. These are carefully stacked to create a series of rotor slots to provide space for the rotor winding. We use two types of rotor windings: (1) conventional 3-phase windings made of insulated wire and (2)
squirrel-cage windings. The type of winding gives rise to two main classes of motors: *squirrel cage* induction motors (also called *cage* motors) and *wound-rotor* induction motors.

A *squirrel-cage* rotor is composed of bare copper bars, slightly longer than the rotor, which are pushed into the slots. The opposite ends are welded to two copper end-rings, so that all the bars are short-circuited together. The entire construction (bars and end-rings) resembles a squirrel cage, from which the name is derived. In small and medium-size motors, the bars and end-rings are made of diecast aluminum, molded to form an integral block.
A wound rotor has a 3-phase winding, similar to the one on the stator. The winding is uniformly distributed in the slots and is usually connected in 3 wire wye. The terminals are connected to three slip rings, which turn with the rotor. The revolving slip-rings and associated stationary brushes enable us to connect external resistors in series with the rotor winding. The external resistors are mainly used during the start up period; under normal running conditions, the three brushes are short-circuited.

4.3 Principle of operation

The operation of a 3-phase induction motor is based upon the application of Faraday Law and the Lorentz force on a conductor. The behavior can readily be understood by means of the following example.

Consider a series of conductors of length $l$, whose extremities are short-circuited by two bars $A$ and $B$ (Fig.4.3 a). A permanent magnet placed above this conducting ladder, moves rapidly to the right at a speed $v$, so that its magnetic field $B$ sweeps across the conductors. The following sequence of events then takes place:

1. A voltage $E = Blv$ is induced in each conductor while it is being cut by the flux (Faraday law).
2. The induced voltage immediately produces a current $I$, which flows down the conductor underneath the pole face, through the end-bars, and back through the other conductors.

3. Because the current carrying conductor lies in the magnetic field of the permanent magnet, it experiences a mechanical force (Lorentz force).

4. The force always acts in a direction to drag the conductor along with the magnetic field.
   If the conducting ladder is free to move, it will accelerate toward the right. However, as it picks up speed, the conductors will be cut less rapidly by the moving magnet, with the result that the induced voltage $E$ and the current $I$ will diminish. Consequently, the force acting on the conductors will also decreases. If the ladder were to move at the same speed as the magnetic field, the induced voltage $E$, the current $I$, and the force dragging the ladder along would all become zero.

In an induction motor the ladder is closed upon itself to form a squirrel-cage (Fig.4.3b) and the moving magnet is replaced by a rotating field. The field is produced by the 3-phase currents that flow in the stator windings, as we will now explain.
4.4 The Rotating Field and Induced Voltages

Consider a simple stator having 6 salient poles, each of which carries a coil having 5 turns (Fig.4.4). Coils that are diametrically opposite are connected in series by means of three jumpers that respectively connect terminals a-a, b-b, and c-c. This creates three identical sets of windings AN, BN, CN, that are mechanically spaced at 120 degrees to each other. The two coils in each winding produce magnetomotive forces that act in the same direction.
Fig. 4.4 Elementary stator having terminals A, B, C connected to a 3-phase source (not shown). Currents flowing from line to neutral are considered to be positive.

The three sets of windings are connected in wye, thus forming a common neutral N. Owing to the perfectly symmetrical arrangement, the line to neutral impedances are identical. In other words, as regards terminals A, B, C, the windings constitute a balanced 3-phase system.

For a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the center of the magnetic poles. The result is illustrated in Fig. 4.5. The rotating field will induce voltages in the phase coils aa', bb', and cc'. Expressions for the induced voltages can be obtained by using Faraday laws of induction.
The flux density distribution in the air gap can be expressed as:

\[ B(\theta) = B_{\text{max}} \cos \theta \]  \hspace{1cm} (4.1)

The air gap flux per pole, \( \phi_p \), is:

\[ \phi_p = \int_{-\pi/2}^{\pi/2} B(\theta)lr d\theta = 2B_{\text{max}} lr \]  \hspace{1cm} (4.2)

Where,

- \( l \) is the axial length of the stator.
- \( r \) is the radius of the stator at the air gap.

Let us consider that the phase coils are full-pitch coils of \( N \) turns (the coil sides of each phase are 180 electrical degrees apart as shown in Fig.4.5). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil \( aa' \) will be maximum \((= N \phi_p \text{ at } \omega t = 0^\circ)\) (Fig.4.5a) and zero at \( \omega t = 90^\circ \). The flux linkage \( \lambda_a(\omega t) \) will vary as the cosine of the angle \( \omega t \). Hence;
\[ \lambda_a(\omega t) = N\phi_p \cos \omega t \]  

(4.3)

Therefore, the voltage induced in phase coil \( \text{aa}' \) is obtained from Faraday law as:

\[ e_a = -\frac{d\lambda_a(\omega t)}{dt} = \omega N\phi_p \sin \omega t = E_{\text{max}} \sin \omega t \]  

(4.4)

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

\[ e_b = E_{\text{max}} \sin(\omega t - 120) \]  

(4.5)

\[ e_c = E_{\text{max}} \sin(\omega t + 120). \]  

(4.6)

From Equation (4.4), the \textit{rms} value of the induced voltage is:

\[ E_{\text{rms}} = \frac{\omega N\phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\phi_p = 4.44 fN\phi_p \]  

(4.7)

Where \( f \) is the frequency in hertz. Equation (4.7) has the same form as that for the induced voltage in transformers. However, \( \phi_p \) in Equation (4.7) represents the flux per pole of the machine.

Equation (4.7) shows the \textit{rms} voltage per phase. The \( N \) is the total number of series turns per phase with the turns forming a concentrated full-pitch winding. In an actual AC machine each phase winding is distributed in a number of slots for better use of
the iron and copper and to improve the waveform. For such a distributed winding, the \( EMF \) induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the \( EMF \) is less than their numerical sum when they are connected in series for the phase winding. A reduction factor \( K_w \), called the winding factor, must therefore be applied. For most three-phase machine windings \( K_w \) is about 0.85 to 0.95. Therefore, for a distributed phase winding, the \( rms \) voltage per phase is

\[
E_{rms} = 4.44 f N_{ph} \phi_p K_w
\]

Where \( N_{ph} \) is the number of turns in series per phase.

### 4.5 Running Operation

If the stator windings are connected to a three-phase supply and the rotor circuit is closed, the induced voltages in the rotor windings produce rotor currents that interact with the air gap field to produce torque. The rotor, if free to do so, will then start rotating. According to \( Lens \ law \), the rotor rotates in the direction of the rotating field such that the relative speed between the rotating field and the rotor winding decreases. The rotor will eventually reach a steady-state speed \( n \) that is less than the synchronous speed \( n_s \) at which the stator rotating field rotates in the air gap. It is
obvious that at \( n = n_s \) there will be no induced voltage and current in the rotor circuit and hence no torque.

In a \( P \)-pole machine, one cycle of variation of the current will make the \( mmf \) wave rotate by \( 2/P \) revolutions. The revolutions per minute \( n \) (rpm) of the traveling wave in a \( P \)-pole machine for a frequency \( f \) cycles per second for the currents are:

\[
n = \frac{2}{P} f \times 60 = \frac{120 f}{p} \quad (4.9)
\]

The difference between the rotor speed \( n \) and the synchronous speed \( n_s \) of the rotating field is called the slip \( s \) and is defined as

\[
s = \frac{n_s - n}{n_s} \quad (4.10)
\]

If you were sitting on the rotor, you would find that the rotor was slipping behind the rotating field by the slip \( rpm = n_s - n = sn_s \). The frequency \( f_2 \) of the induced voltage and current in the rotor circuit will correspond to this slip \( rpm \), because this is the relative speed between the rotating field and the rotor winding. Thus, from Equation (4.9):

\[
f_2 = \frac{p}{120} (n_s - n) = \frac{p}{120} sn_s = sf_1 \quad (4.11)
\]
This rotor circuit frequency $f_2$ is also called *slip frequency*. The voltage induced in the rotor circuit at slip $s$ is:

$$E_{2s} = 4.44 f_2 N_2 \phi_p K \omega_2 = 4.44 s f_1 N_2 \phi_p K \omega_2 = s E_2$$ (4.12)

Where $E_2$ is the induced voltage in the rotor circuit at standstill, that is, at the stator frequency $f_1$.

The induced currents in the three-phase rotor windings also produce a rotating field. Its speed (rpm) $n_2$ with respect to rotor is:

$$n_2 = \frac{120 f_2}{p} = \frac{120 s f_1}{p} = sn_s$$ (4.13)

Because the rotor itself is rotating at $n$ rpm, the induced rotor field rotates in the air gap at speed $n + n_2 = (1 - s)n_s + sn_s = n_s$ rpm. Therefore, both the stator field and the induced rotor field rotate in the air gap at the same synchronous speed $n_s$. The stator magnetic field and the rotor magnetic field are therefore stationary with respect to each other. The interaction between these two fields can be considered to produce the torque. As the magnetic fields tend to align, the stator magnetic field can be visualized as dragging the rotor magnetic field.
Example 4.1 A 3-phase, 460 V, 100 hp, 60 Hz, four-pole induction machine delivers rated output power at a slip of 0.05. Determine the:

(a) Synchronous speed and motor speed.

(b) Speed of the rotating air gap field.

(c) Frequency of the rotor circuit.

(d) Slip rpm.

(e) Speed of the rotor field relative to the (i) rotor structure. (ii) Stator structure. (iii) Stator rotating field.

(f) Rotor induced voltage at the operating speed, if the stator-to-rotor turns ratio is 1 : 0.5.

Solution:

(a) $n_s = \frac{120f}{p} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$,

$n = (1 - s)n_s = (1 - 0.05) \times 1800 = 1710 \text{ rpm}$

(b) 1800 rpm (same as synchronous speed)

(c) $f_2 = sf_1 = 0.05 \times 60 = 3 \text{ Hz}$.

(d) slip rpm = $s n_s = 0.05 \times 1800 = 90 \text{ rpm}$

(e) (i) 90 rpm (ii) 1800 rpm (iii) 0 rpm

(f) Assume that the induced voltage in the stator winding is the same as the applied voltage. Now,

$E_{2s} = sE_2 = s \frac{N_2}{N_1} E_1 = 0.05 \times 0.5 \times \frac{460}{\sqrt{3}} = 6.64V / \text{Phase}$
4.6 Equivalent Circuit of the Induction Motor

In this section we develop the equivalent circuit from basic principles. We then analyze the characteristics of a low-power and high-power motor and observe their basic differences.

Finally, we develop the equivalent circuit of an asynchronous generator and determine its properties under load.

A 3-phase wound-rotor induction motor is very similar in construction to a 3-phase transformer. Thus, the motor has 3 identical primary windings and 3 identical secondary windings one set for each phase. On account of the perfect symmetry, we can consider a single primary winding and a single secondary winding in analyzing the behavior of the motor.

When the motor is at standstill, it acts exactly like a conventional transformer, and so its equivalent circuit (Fig.4.6) is the same as that of a transformer, previously developed.

Fig.4.6 Equivalent circuit of a wound-rotor induction motor at standstill.
In the case of a conventional 3-phase transformer, we would be justified in removing the magnetizing branch composed of $jX_m$ and $R_m$ because the exciting current $I_o$ is negligible compared to the load current $I_p$. However, in a motor this is no longer true: $I_o$ may be as high as 40 percent of $I_p$ because of the air gap. Consequently, we cannot eliminate the magnetizing branch.

The stator and rotor winding can be represented as shown in Fig.4.7 (a) and (b), where,

$V_1$ = per-phase terminal voltage.

$R_1$ = per-phase stator winding resistance.

$L_1$ = per-phase stator leakage inductance.

$E_1$ = *per-phase* induced voltage in the stator winding

$L_m$ = *per-phase* stator magnetizing inductance

$R_c$ = *per-phase* stator core loss resistance.

$E_2$ = Per-phase induced voltage in rotor at standstill (i.e., at stator frequency $f_s$)

$R_2$ = Per-phase rotor circuit resistance

$L_2$ = Per-phase rotor leakage inductance

Note that there is no difference in form between this equivalent circuit and that of the transformer primary winding. The difference
lies only in the magnitude of the parameters. For example, the excitation current $I_o$ is considerably larger in the induction machine because of the air gap. In induction machines it is as high as 30 to 50 percent of the rated current, depending on the motor size, whereas it is only 1 to 5 percent in transformers. Moreover, the leakage reactance $X_1$ is larger because of the air gap and also because the stator and rotor windings are distributed along the periphery of the air gap rather than concentrated on a core, as in the transformer.
Fig. 4.7 Development of the induction machine equivalent circuit

Note that the rotor circuit frequency and current are: \( f_2 \) and \( I_2 \).

\[
I_2 = \frac{sE_2}{R_2 + jsX_2}
\]  

(4.14)

The power involved in the rotor circuit is:

\[
P_2 = I_2^2 R_2
\]  

(4.15)
Which represents the rotor copper loss per phase. Equation (4.14) can be rewritten as:

\[
I_2 = \frac{E_2}{(R_2/s) + jX_2} \tag{4.16}
\]

Equation (4.16) suggests the rotor equivalent circuit of Fig.4.7b. Although the magnitude and phase angle of \(I_2\) are the same in Equations (4.14) and (4.16), there is a significant difference between these two equations and the circuits (Figs.4.7b and 4.14c) they represent. The current \(I_2\) in Equation (4.14) is at slip frequency \(f_2\), whereas \(I_2\) in Equation (4.16) is at line frequency \(f_1\). In Equation (4.14) the rotor leakage reactance \(sX_2\) varies with speed but resistance \(R_2\) remains fixed, whereas in Equation (4.16) the resistance \(R_2/s\) varies with speed but the leakage reactance \(X_2\) remains unaltered. The per phase power associated with the equivalent circuit of Fig.4.7c is:

\[
P = I_2^2 \frac{R_2}{s} = \frac{P_2}{s} \tag{4.17}
\]

Because induction machines are operated at low slips (typical values of slip \(s\) are 0.01 to 0.05) the power associated with Fig.4.7c is considerably larger. Note that the equivalent circuit of Fig.4.7c is at the stator frequency, and therefore this is the rotor
equivalent circuit as seen from the stator. The power in Equation (4.17) therefore represents the power that crosses the air gap and thus includes the rotor copper loss as well as the mechanical power developed. Equation (4.17) can be rewritten as:

\[ P = P_{ag} = I_s^2 \left( R_2 + \frac{R_2}{s} (1 - s) \right) = I_s^2 \frac{R_2}{s} \]  

(4.18)

The corresponding equivalent circuit is shown in Fig. 4.7d. The speed dependent resistance \( R_2 (1 - s) / s \) represents the mechanical power developed by the induction machine.

\[ P_{mech} = I_s^2 \frac{R_2}{s} (1 - s) \]  

(4.19)

\[ P_{mech} = (1 - s) * P_{ag} \]  

(4.20)

\[ P_{mech} = \frac{(1 - s)}{s} P_2 \]  

(4.21)

and, \( P_2 = I_2^2 R_2 = sP_{ag} \)  

(4.22)

Thus, \( P_{ag} : P_2 : P_{mech} = 1 : s : (1 - s) \)  

(4.23)

Equation (4.23) indicates that, of the total power input to the rotor (i.e., power crossing the air gap, \( P_{ag} \)), a fraction is dissipated in the resistance of the rotor circuit (known as rotor copper loss) and the fraction 1 - s is converted into mechanical power.
Therefore, for efficient operation of the induction machine, it should operate at a low slip so that more of the air gap power is converted into mechanical power. Part of the mechanical power will be lost to overcome the windage and friction. The remainder of the mechanical power will be available as output shaft power.

4.7 IEEE-Recommended Equivalent Circuit

In the induction machine, because of its air gap, the exciting current $I_0$ is high—of the order of 30 to 50 percent of the full-load current. The leakage reactance $X_l$ is also high. The IEEE recommends that, in such a situation, the magnetizing reactance $X_m$ not be moved to the machine terminals but be retained at its appropriate place, as shown in Fig. 4.8. The resistance $R_c$ is, however, omitted, and the core loss is lumped with the windage and friction losses. This equivalent circuit (Fig. 4.8) is to be preferred for situations in which the induced voltage $E_1$ differs appreciably from the terminal voltage $V_1$. 
4.8 Thevenin Equivalent Circuit

In order to simplify computations, $V_1$, $R_1$, $X_1$, and $X_m$ can be replaced by the Thevenin equivalent circuit values $V_{th}$, $R_{th}$, and $X_{th}$, as shown in Fig.4.9, where:

$$V_{th} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} V_1 \quad (4.24)$$

If $R_1^2 \ll (X_1 + X_m)^2$ as is usually case,

$$V_{th} \approx \frac{X_m}{X_1 + X_m} V_1 = K_{th} V_1 \quad (4.25)$$

The Thevenin impedance is:

$$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} = R_{th} + jX_{th} \quad (4.26)$$
If \( R_1^2 \ll (X_1 + X_m)^2 \), then,

\[
R_{th} \approx \left( \frac{X_m}{X_1 + X_m} \right)^2 R_1 = K_{th}^2 R_1
\]  \hspace{1cm} (4.27)

and since \( X_1 \ll X_m \), then,

\[
X_{th} \approx X_1
\]  \hspace{1cm} (4.28)

**Fig. 4.9 Thevenin equivalent circuit.**

### 4.9 Tests To Determine The Equivalent Circuit

The approximate values of \( R_1, R_2, X_m, R_m \) and \( X \) in the equivalent circuit can be found by means of the following tests:

**No-load test** When an induction motor runs at no load, the slip is exceedingly small. Referring to Fig. 4.7, this means that the value of \( R_2/s \) is very high and so current \( I_1 \) is negligible compared to \( I_o \). Thus, at no-load the circuit consists essentially
of the magnetizing branch $X_m, R_m$. Their values can be determined by measuring the voltage, current, and power at no-load, as follows:

a. Measure the stator resistance $R_{LL}$ between any two terminals. Assuming a wye connection, the value of $R_1$ is:

$$R_1 = \frac{R_{LL}}{2}$$  \hspace{1cm} (4.29)

b. Run the motor at no-load using rated line-to-line voltage, $V_{NL}$ (Fig.4.10). Measure the no load current $I_{NL}$ and the total 3-phase active power $P_{NL}$.

Fig.4.10 A no-load test permits the calculation of $R_m$ and $R_m$ of the magnetizing branch.

*IEEE-recommended equivalent circuit.* For the no-load condition, $R'_s / s$ is very high. Therefore, in the equivalent circuit of Fig.4.9,
the magnetizing reactance $X_m$ is shunted by a very high resistive branch representing the rotor circuit. The reactance of this parallel combination is almost the same as $X_m$. Therefore the total reactance $X_{NL}$, measured at no load at the stator terminals, is essentially $X_1 + X_m$. The equivalent circuit at no load is shown in Fig.4.11a.

(a) No-load equivalent circuit  (b) Blocked-rotor equivalent circuit.

(c) Blocked-rotor equivalent circuit for improved value for $R_2$.

The primary phase voltage can be obtained from the following equation:
\[ V_1 = \frac{V_{LL}}{\sqrt{3}} V / \text{Phase} \]  
\[ (4.30) \]

Then the no-load impedance can be obtained as following:

\[ Z_{NL} = \frac{V_1}{I_1} \]  
\[ (4.31) \]

The no-load resistance is:

\[ R_{NL} = \frac{P_{NL}}{3I_1^2} \]  
\[ (4.32) \]

The no-load reactance is:

\[ X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2} \]  
\[ (4.33) \]

In the IEEE recommended equivalent circuit we assume that

\[ X_1 + X_m = X_{NL} \]  
\[ (4.35) \]

Then from no load test we only get the value of \( X_1 + X_m \)

**Locked-rotor test** Under rated line voltage, when the rotor of an induction motor is locked, the stator current \( I_p \) is almost six times its rated value. Furthermore, the slip \( s \) is equal to one. This means that \( R_2 / s \) is equal to \( R_2 \), where \( R_2 \) is the resistance of the rotor reflected into the stator. Because \( I_p \) is much greater than the exciting current \( I_o \), we can neglect the magnetizing branch. This leaves us with the circuit of Fig.4.8 (without magnetizing branch),
Three-Phase Induction Machine

composed of the leakage reactance $X$, the stator resistance $R_1$, and the reflected rotor resistance $R_2 / s$. Their values can be determined by measuring the voltage, current, and power under locked-rotor conditions, as follows:

a. Apply reduced 3-phase voltage to the stator and gradually increase it from zero until the stator current is about equal to its rated value. Sometimes it is recommended to use lower frequency than the rated to avoid the errors due to skin effect in the rotor circuit.

b. Take readings of $V_{LL} \big|_{BL}$ (line-to-line), $I_1 \big|_{BL}$, and the total 3-phase power $P_{BL}$ (Fig.4.12).

So, for the blocked-rotor test the slip is 1. In the equivalent circuit of Fig.4.9, the magnetizing reactance $X_m$ is shunted by the low-impedance branch $R'_2 + jX'_2$. Because $|X_m| >> |R'_2 + jX'_2|$, the impedance $X_m$ can be neglected and the equivalent circuit for the blocked-rotor test reduces to the form shown in Fig.4.11b.

From the blocked-rotor test, the blocked-rotor resistance is:

$$R_{BL} = \frac{P_{BL}}{3I_1^2 \big|_{BL}}$$  \hspace{1cm} (4.36)

The blocked-rotor impedance at frequency of blocked rotor test is:
The blocked-rotor reactance at frequency of blocked rotor test is:

\[ X_{BL} \big|_{fBL} = \sqrt{\left( \frac{Z_{BL}^2 \big|_{fBL}}{I_1 \big|_{BL}} - R_{BL}^2 \right)} \]  \hspace{1cm} (4.38)

Its value at rated frequency is:

\[ X_{BL} = X_{BL} \big|_{fBL} \times \frac{\text{Rated Frequency}}{\text{Frequency at blocked rotor test}} \]  \hspace{1cm} (4.39)

\[ X_{BL} \cong X_1 + X_2' \]  \hspace{1cm} (4.40)

assume, \( X_1 = X_2' \) (at rated frequency)

then \( X_1 \) and \( X_2' \) can be obtained.

From no load test we know that \( X_1 + X_m = X_{NL} \) and \( X_1 \) are known then the magnetizing reactance is:

\[ X_m = X_{NL} - X_1 \]  \hspace{1cm} (4.41)

Comments: The rotor equivalent resistance \( R'_2 \) plays an important role in the performance of the induction machine. So, an accurate determination of \( R'_2 \) is recommended by the IEEE as follows:

The blocked resistance \( R_{BL} \) is the sum of \( R_1 \) and an equivalent resistance, say \( R \), which is the resistance of \( R'_2 + jX'_2 \) in parallel with \( X_m \) as shown in Fig.4.11c; therefore,
Three-Phase Induction Machine

\[ R = \frac{X_m^2}{R_2^2 + (X'_2 + X_m)^2} R'_2 \]  \hspace{1cm} (4.42)

If \( X'_2 + X_m \gg R'_2 \), as is usually the case,

\[ R'_2 = \left( \frac{X'_2 + X_m}{X_m} \right)^2 R \]  \hspace{1cm} (4.43)

or \( R \approx \left( \frac{X_m}{X'_2 + X_m} \right)^2 R'_2 \)  \hspace{1cm} (4.44)

Now \( R = R_{BL} - R_1 \). So, we can use this value of \( R \) to determine \( R'_2 \) from equation (4.43).

More elaborate tests are conducted on large machines, but the above-mentioned procedure gives results that are adequate in most cases.

Fig.4.12 A locked-rotor test permits the calculation of the total leakage reactance \( x \) and the total resistance \( (R_1 + R_2) \). From these results we can determine the equivalent circuit of the induction motor.
Example 4.2 A no-load test conducted on a 30 hp, 835 r/min, 440 V, 3-phase, 60 Hz squirrel-cage induction motor yielded the following results:

No-load voltage (line-to-line): 440 V
No-load current: 14 A
No-load power: 1470 W
Resistance measured between two terminals: 0.5 Ω

The locked-rotor test, conducted at reduced voltage, gave the following results:

Locked-rotor voltage (line-to-line): 163 V
Locked-rotor power: 7200 W
Locked-rotor current: 60 A

Determine the equivalent circuit of the motor.

Solution:

Assuming the stator windings are connected in wye, the resistance per phase is:

\[ R_1 = \frac{0.5}{2} = 0.25 \, \Omega \]

From the no-load test:

The primary phase voltage can be obtained from the following equation:

\[ V_1 = \frac{V_{LL}}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254 \, V / \text{Phase} \]
Then, the no-load impedance can be obtained as following:

\[ Z_{NL} = \frac{V_1}{I_1} = \frac{254}{14} = 18.143 \, \Omega \]

The no-load resistance is:

\[ R_{NL} = \frac{P_{NL}}{3I_1^2} = \frac{1470}{3 \times 14^2} = 2.5 \, \Omega \]

The no-load reactance is:

\[ X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2} = \sqrt{18.143^2 - 2.5^2} = 17.97 \, \Omega \]

In the IEEE recommended equivalent circuit we assume that

\[ X_1 + X_m = X_{NL} = 17.97 \, \Omega \]

From the blocked-rotor test, the blocked-rotor resistance is:

\[ R_{BL} = \frac{P_{BL}}{3I_1^2_{BL}} = \frac{7200}{3 \times 60^2} = 0.6667 \, \Omega \]

Note that in this example it does not mentioned the frequency of the blocked rotor test so we can use the rated frequency as the frequency of the blocked rotor test.

The blocked-rotor impedance at frequency of blocked rotor test is:

\[ Z_{BL} = \frac{V_{BL}}{I_{BL}} = \frac{163 / \sqrt{3}}{60} = 1.5685 \, \Omega \]

The blocked-rotor reactance is:
\[ X_{BL} = \sqrt{(Z_{BL}^2 - R_{BL}^2)} = \sqrt{1.5685^2 - 0.6667^2} = 1.42 \ \Omega \]

\[ X_{BL} \approx X_1 + X_2' = 1.42 \ \Omega \]

Assume, \( X_1 = X_2' \) (at rated frequency)

then \( X_1 = X_2' = 0.71 \ \Omega \)

From no load test we know that \( X_1 + X_m = X_{NL} \) and \( X_1 = 0.71 \ \Omega \), then the magnetizing reactance is:

\[ X_m = X_{NL} - X_1 = 17.97 - 0.71 = 17.26 \ \Omega \]

Now \( R = R_{BL} - R_1 = 0.6667 - 0.25 = 0.4167 \ \Omega \).

Form equation (4.43):

\[
R_2' = \left( \frac{X_2' + X_m}{X_m} \right)^2 \quad R = \left( \frac{0.71 + 17.26}{17.26} \right)^2 \times 0.4167 = 0.4517 \ \Omega
\]

**Example 4.3** The following test results are obtained from a three-phase 60 hp, 2200 V, six-pole, 60 Hz squirrel-cage induction motor.

**1) No-load test:**

Supply frequency = 60 Hz, Line voltage = 2200 V

Line current = 4.5 A, Input power = 1600 W

**2) Blocked-rotor test:**

Frequency = 15 Hz, Line voltage = 270 V

Line current = 25 A, Input power = 9000 W
(3) Average DC resistance per stator phase: \( R_1 = 2.8 \, \Omega \)

(a) Determine the no-load rotational loss.

(b) Determine the parameters of the IEEE-recommended equivalent circuit of Fig. 4.9.

(c) Determine the parameters \( (V_{th}, R_{th}, X_{th}) \) for the Thevenin equivalent circuit of Fig. 4.10.

**Solution:**

(a) From the no-load test, the no-load power is:

\[
P_{NL} = 1600 \, W
\]

The no-load rotational loss is:

\[
P_{rot} = P_{NL} - 3I_1^2R_1
\]

\[
P_{rot} = 1600 - 3 \times 4.5^2 \times 2.8 = 1429.9W
\]

(b) **IEEE-recommended equivalent circuit.** For the no-load condition, \( R'_{2s} \) is very high. Therefore, in the equivalent circuit of Fig. 4.9, the magnetizing reactance \( X_m \) is shunted by a very high resistive branch representing the rotor circuit. The reactance of this parallel combination is almost the same as \( X_m \). Therefore the total reactance \( X_{NL} \), measured at no load at the stator terminals, is essentially \( X_1 + X_m \). The equivalent circuit at no load is shown in Fig. 4.11a.
The no-load impedance is:

\[ Z_{NL} = \frac{V_1}{I_1} = \frac{1270.2}{4.5} = 282.27 \ \Omega \]

The no-load resistance is:

\[ R_{NL} = \frac{P_{NL}}{3I_1^2} = \frac{1600}{3 \times 4.5^2} = 26.34 \ \Omega \]

The no-load reactance is:
Thus, $X_1 + X_m = X_{NL} = 281 \Omega = 281.0 \Omega$.

For the blocked-rotor test the slip is 1, the magnetizing reactance $X_m$ is shunted by the low-impedance branch $R'_2 + jX'_2$. Because $|X_m| >> |R'_2 + jX'_2|$, the impedance $X_m$ can be neglected and the equivalent circuit for the blocked-rotor test reduces to the form shown in Fig.4.11b. From the blocked-rotor test, the blocked-rotor resistance is:

$$R_{BL} = \frac{P_{BL}}{3I_1^2} = \frac{9000}{3 \times 25^2} = 4.8 \Omega$$

Therefore, $R'_2 = R_{BL} - R_1 = 4.8 - 2.8 = 2 \Omega$, The blocked-rotor impedance at 15 Hz is: $Z_{BL} = \frac{V_1}{I_1} = \frac{270}{\sqrt{3} \times 25} = 6.24 \Omega$

The blocked-rotor reactance at 15 Hz is:

$$X_{BL} = \sqrt{(6.24^2 - 4.8^2)} = 3.98 \Omega$$

Its value at 60 Hz is $X_{BL} = 3.98 \times \frac{60}{15} = 15.92 \Omega$

$$X_{BL} \approx X_1 + X'_2$$

Hence, $X_1 = X'_2 = \frac{15.92}{2} = 7.96 \Omega$ (at 60 Hz)

The magnetizing reactance is therefore:
$X_m = 281 - 7.96 = 273.04 \, \Omega$

Now $R = R_{BL} - R_1 = 4.8 - 2.8 = 2 \, \Omega$. So,

$R'_2 = \left(\frac{7.96 + 273.04}{273.04}\right)^2 2 = 2.12 \, \Omega$

(c) From Equation (4.31)

$V_{th} \cong \frac{273.04}{7.96 + 273.04} V_1 = 0.97 \, V_1$

From Equation (4.34)

$R_{th} \cong 0.97^2 R_1 = 0.97^2 \times 2.8 = 2.63 \, \Omega$

From Equation (4.35)

$X_{th} \cong X_1 = 7.96 \, \Omega$.

### 4.10 Performance Characteristics

The equivalent circuits derived in the preceding section can be used to predict the performance characteristics of the induction machine. The important performance characteristics in the steady state are the efficiency, power factor, current, starting torque, maximum (or pull-out) torque, and so forth.

The mechanical torque developed $T_{mech}$ per phase is given by

$$P_{mech} = T_{mech} \omega_{mech} = I_2^2 \frac{R_2}{s} (1 - s)$$

(4.45)
Where \( \omega_{mech} = \frac{2\pi n}{60} \) \hspace{2cm} (4.46)

The mechanical speed \( \omega_{mech} \) is related to the synchronous speed by:

\[ \omega_{mech} = (1 - s) \omega_{syn} \] \hspace{2cm} (4.47)

\( \omega_{mech} = \frac{n_{syn}}{60} \cdot 2\pi (1 - s) \) \hspace{2cm} (4.48)

and

\[ \omega_{syn} = \frac{120 f}{60 P} \cdot 2\pi = \frac{4\pi f_1}{P} \] \hspace{2cm} (4.49)

From Equations (4.45), (4.47), and (4.18)

\[ T_{mech} \omega_{syn} = I_2^2 \frac{R_2}{s} = P_{ag} \] \hspace{2cm} (4.50)

Then,

\[ T_{mech} = \frac{1}{\omega_{syn}} P_{ag} \] \hspace{2cm} (4.51)

\[ T_{mech} = \frac{1}{\omega_{syn}} I_2^2 \frac{R_2}{s} \] \hspace{2cm} (4.52)

\[ T_{mech} = \frac{1}{\omega_{syn}} I'^2_2 \frac{R'_2}{s} \] \hspace{2cm} (4.53)

From the equivalent circuit of Fig.4.11 and Equation (4.53)

\[ T_{mech} = \frac{1}{\omega_{syn}} \frac{V_{th}^2}{(R_{th} + R'_2 / s)^2 + (X_{th} + X'_2)^2} \cdot \frac{R'_2}{s} \] \hspace{2cm} (4.54)
Note that if the approximate equivalent circuits (Fig. 4.13) are used to determine $I_2$, in Equation (4.54), $V_{th}$, $R_{th}$, and $X_{th}$ should be replaced by $V_1$, $R_1$, and $X_1$, respectively. The prediction of performance based on the approximate equivalent circuit (Fig. 4.13) may differ by 5 percent from those based on the equivalent circuit of Fig. 4.8 or 4.16.

For a three-phase machine, Equation (4.54) should be multiplied by three to obtain the total torque developed by the machine. The torque-speed characteristic is shown in Fig. 4.14. At low values of slip,

$$R_{th} + \frac{R'_s}{s} \gg X_{th} + X'_2$$

and

$$\frac{R'_s}{s} \gg R_{th}$$

(4.55)

Thus $T_{mech} \cong \frac{1}{\omega_{syn}} \frac{V_{th}^2}{R'_2} s$.

(4.56)

The linear torque-speed relationship is evident in Fig. 4.14 near the synchronous speed. Note that if the approximate equivalent circuits (Fig. 4.13) are used, in Equation (4.56) $V_{th}$ should be replaced by $V_1$. At larger values of slip,

$$R_{th} + \frac{R'_s}{s} \ll X_{th} + X'_2$$

(4.57)

And $T_{mech} = \frac{1}{\omega_{syn}} \frac{V_{th}^2}{(X_{th} + X'_2)^2} \frac{R'_2}{s}$.

(4.58)
The torque varies almost inversely with slip near $s = 1$, as seen from Fig. 4.14.

Equation (4.49) also indicates that at a particular speed (i.e., a fixed value of $s$) the torque varies as the square of the supply voltage $V_{th}$ (hence $V_1$ Fig. 4.14) shows the $T-n$ profile at various supply voltages.

An expression for maximum torque can be obtained by setting $dT / ds = 0$. Differentiating Equation (4.54) with respect to slip $s$ and equating the results to zero gives the following condition for maximum torque:

$$
\frac{R_2'}{S_{T_{\text{max}}}} = \sqrt{R_{th}^2 + \left(X_{th} + X_2'\right)^2}
$$  \hspace{1cm} (4.59)
This expression can also be derived from the fact that the condition for maximum torque corresponds to the condition for maximum air gap power (Equation (4.51)). This occurs, by the familiar impedance-matching principle in circuit theory, when the impedance of $\frac{R'_2}{s}$ equals in magnitude the impedance between it and the supply voltage $V_1$ (Fig. 4.19) as shown in Equation (4.59).

The slip $S_{T_{\text{max}}}$ at maximum torque $T_{\text{max}}$ is:

$$S_{T_{\text{max}}} = \frac{R'_2}{\sqrt{R^2_{th} + (X_{th} + X'_2)^2}}$$  \hspace{1cm} (4.60)

The maximum torque per phase from Equations (4.49) and (4.60) is:

$$T_{\text{max}} = \frac{1}{2\omega_{\text{syn}}} \frac{V^2_{th}}{R_{th} + \sqrt{R^2_{th} + (X_{th} + X'_2)^2}}$$ \hspace{1cm} (4.61)

Equation (4.61) shows that the maximum torque developed by the induction machine is independent of the rotor circuit resistance. However, from Equation (4.60) it is evident that the value of the rotor circuit resistance $R_2$ determines the speed at which this maximum torque will occur. The torque speed characteristics for various values of $R_2$ are shown in Fig. 4.15. In a wound rotor induction motor, external resistance is added to the rotor circuit to make the maximum torque occur at standstill so that high starting torque can be obtained. As the motor speeds up,
the external resistance is gradually decreased and finally taken out completely. Some induction motors are, in fact, designed so that maximum torque is available at start, that is, at zero speed.

![Figure 4.15 Torque speed characteristics for varying $R_2$.](image)

If the stator resistance $R_1$ is small (hence $R_{th}$ is negligibly small), from Equations (4.60) and (4.61),

$$s_{T_{\text{max}}} \approx \frac{R'_2}{X_{th} + X'_2}$$  \hspace{1cm} (4.62)

$$T_{\text{max}} = \frac{1}{2\omega_{\text{syn}}} \frac{V_{th}^2}{X_{th} + X'_2}$$ \hspace{1cm} (4.63)
Equation (4.63) indicates that the maximum torque developed by an induction machine is inversely proportional to the sum of the leakage reactances.

From Equation (4.54), the ratio of the maximum developed torque to the torque developed at any speed is:

\[
\frac{T_{\text{max}}}{T} = \frac{\left( R_{th} + R_2' / s \right)^2 + \left( X_{th} + X_2' \right)^2}{\left( R_{th} + R_2' / s_{T_{\text{max}}} \right)^2 + \left( X_{th} + X_2' \right)^2} \frac{s}{s_{T_{\text{max}}}}
\]

(4.64)

If \( R_1 \) (hence \( R_{th} \)) is negligibly small,

\[
\frac{T_{\text{max}}}{T} = \frac{\left( R_2' / s \right)^2 + \left( X_{th} + X_2' \right)^2}{\left( R_2' / s_{T_{\text{max}}} \right)^2 + \left( X_{th} + X_2' \right)^2} \frac{s}{s_{T_{\text{max}}}}
\]

(4.65)

From Equations (4.62) and (4.65)

\[
\frac{T_{\text{max}}}{T} = \frac{\left( R_2' / s \right)^2 + \left( R_2' / s_{T_{\text{max}}} \right)^2}{2\left( R_2' / s_{T_{\text{max}}} \right)^2} \frac{s}{s_{T_{\text{max}}}}
\]

(4.66)

\[
\frac{T_{\text{max}}}{T} = \frac{s_{T_{\text{max}}}^2 + s^2}{2 * s_{T_{\text{max}}} * s}
\]

(4.67)

Equation (4.67) shows the relationship between torque at any speed and the maximum torque in terms of their slip values.
4.11 Efficiency

In order to determine the efficiency of the induction machine as a power converter, the various losses in the machine are first identified. These losses are illustrated in the power flow diagram of Fig. 4.16. For a 3 \( \phi \) machine the power input to the stator is:

\[
P_{in} = 3V_1I_1 \cos \theta_1
\]  
(4.68)

The power loss in the stator winding is:

\[
P_1 = 3I_1^2 R_1
\]  
(4.69)

Where \( R_1 \) is the AC resistance (including skin effect) of each phase winding at the operating temperature and frequency.

Power is also lost as hysteresis and eddy current loss in the magnetic material of the stator core.

The remaining power, \( P_{ag} \), crosses the air gap. Part of it is lost in the resistance of the rotor circuit.

\[
P_2 = 3I_2^2 R_2
\]  
(4.70)

Where \( R_2 \) is the ac resistance of the rotor winding. If it is a wound-rotor machine, \( R_2 \) also includes any external resistance connected to the rotor circuit through slip rings.

Power is also lost in the rotor core. Because the core losses are dependent on the frequency \( f_2 \) of the rotor, these may be negligible at normal operating speeds, where \( f_2 \) is very low.
Fig. 4.16 Power flow in an induction motor.

The remaining power is converted into mechanical form. Part of this is lost as windage and friction losses, which are dependent on speed. The rest is the mechanical output power $P_{out}$, which is the useful power output from the machine.

The efficiency of the induction motor is: \[ \eta = \frac{P_{out}}{P_{in}} \] (4.71)

The efficiency is highly dependent on slip. If all losses are neglected except those in the resistance of the rotor circuit,

\begin{align*}
P_{ag} &= P_{in} \quad \text{(4.72)} \\
P_2 &= sP_{ag} \quad \text{(4.73)} \\
P_{out} &= P_{mecj} = P_{ag} (1 - s) \quad \text{(4.74)}
\end{align*}

And the ideal efficiency is: \[ \eta_{ideal} = \frac{P_{out}}{P_{in}} = (1 - s) \] (4.75)
Sometimes $\eta_{\text{ideal}}$ is also called the \textit{internal efficiency} as it represents the ratio of the power output to the air gap power. It indicates that an induction machine must operate near its synchronous speed if high efficiency is desired. This is why the slip is very low for normal operation of the induction machine.

If other losses are included, the actual efficiency is lower than the ideal efficiency of Equation (4.75). The full-load efficiency of a large induction motor may be as high as 95 percent.

\section*{4.12 Power Flow In Three Modes Of Operation}

The induction machine can be operated in three modes: \textit{motoring}, \textit{generating}, and \textit{plugging}. The power flow in the machine will depend on the mode of operation. However, the equations derived in Section 4.11 for various power relationships hold good for all modes of operation. If the appropriate sign of the slip $s$ is used in these expressions, the sign of the power will indicate the actual power flow. For example, in the generating mode, the slip is negative. Therefore, from Equation (4.185) the air gap power $P_{ag}$ is negative (note that the copper loss $P_2$ in the rotor circuit is always positive). This implies that the actual power flow across the air gap in the generating mode is from rotor to stator.
The power flow diagram in the three modes of operation is shown in Fig. 4.17. The core losses and the friction and windage losses are all lumped together as a constant rotational loss.

In the motoring mode, slip $s$ is positive. The air gap power $P_{ag}$, equation (4.18) and the developed mechanical power $P_{mech}$ Equation (4.27) are positive, as shown in Fig. 4.17a.

In the generating mode $s$ is negative and therefore both $P_{ag}$, and $P_{mech}$ are negative, as shown in Fig. 4.17b. In terms of the equivalent circuit of Fig. 4.7e the resistance $\left(\frac{(1-s)}{s}\right)R_2$ is negative, which indicates that this resistance represents a source of energy.

In the plugging mode, $s$ is greater than one and therefore $P_{ag}$ is positive but $P_{mech}$ is negative as shown in Fig. 4.17c. In this mode the rotor rotates opposite to the rotating field and therefore mechanical energy must be put into the system. Power therefore flows from both sides, and as a result the loss in the rotor circuit, $P_2$, is enormously increased. In terms of the equivalent circuit of Fig. 4.7e, the resistance $\left(\frac{(1-s)}{s}\right)R_2$ is negative and represents a source of energy.
Fig. 4.17 Power flow for various modes of operation of an induction machine. (a) Motoring mode, $0 < s < 1$. (b) Generating mode, $s < 0$. (c) Plugging mode, $s > 1$. 
Example 4.4 A three-phase, 460 V, 1740 rpm, 60 Hz, four-pole wound-rotor induction motor has the following parameters per phase:

\[ R_1 = 0.25 \, \Omega, \quad R'_2 = 0.2 \, \Omega, \quad X_1 = X'_2 = 0.5 \, \Omega, \quad X_m = 30 \, \Omega \]

The rotational losses are 1700 watts. With the rotor terminals short-circuited, find

(a) (i) Starting current when started direct on full voltage.
(ii) Starting torque.

(b) (i) Full-load slip.
(ii) Full-load current.
(iii) Ratio of starting current to full-load current.
(iv) Full-load power factor.
(v) Full-load torque.
(iv) Internal efficiency and motor efficiency at full load.

(c) (i) Slip at which maximum torque is developed.
(ii) Maximum torque developed.

(d) How much external resistance per phase should be connected in the rotor circuit so that maximum torque occurs at start?

Solution:

(a) \[ V_1 = \frac{460}{\sqrt{3}} = 265.6 \text{ volts/phase} \]

(i) At start \( s = 1 \). The input impedance is
\[ Z_1 = 0.25 + j0.5 + \frac{j30(0.2 + j0.5)}{0.2 + j30.5} = 1.08/66^\circ \Omega \]

\[ I_{st} = \frac{265.6}{1.08/66^\circ} = 245.9/-66^\circ \text{ A} \]

(ii) \[ \omega_{syn} = \frac{1800}{60} \times 2\pi = 188.5 \text{ rad/sec} \]

\[ V_{th} = \frac{265.6(j30.0)}{(0.25 + j30.5)} \approx 261.3 \text{ V} \]

\[ Z_{th} = \frac{j30(0.25 + j0.5)}{0.25 + j30.5} = 0.55/63.9^\circ \]

\[ = 0.24 + j0.49 \]

\[ R_{th} = 0.24 \Omega \]

\[ X_{th} = 0.49 \approx X_1 \]

\[ T_{st} = \frac{P_{ag}}{\omega_{syn}} = \frac{I_{st}^2R_{th}'}{\omega_{syn}} \]

\[ = \frac{3 \times 261.3^2}{188.5 (0.24 + 0.2)^2 + (0.49 + 0.5)^2} \times \frac{0.2}{1} \]

\[ = \frac{3}{188.5} \times (241.2)^2 \times \frac{0.2}{1} \]

\[ = 185.2 \text{ N} \cdot \text{m} \]

(i) \[ s = \frac{1800 - 1740}{1800} = 0.0333 \]
(ii) \[
\frac{R'_2}{s} = \frac{0.2}{0.0333} = 6.01 \, \Omega
\]

\[
Z_1 = (0.25 + j0.5) + \frac{(j30)(6.01 + j0.5)}{6.01 + j30.5}
\]

\[
= 0.25 + j0.5 + 5.598 + j1.596
\]

\[
= 6.2123/19.7^\circ \, \Omega
\]

\[
I_{FL} = \frac{265.6}{6.2123/19.7}
\]

\[
= 42.754/-19.7^\circ
\]

(iii) \[
\frac{I_{st}}{I_{fl}} = \frac{245.9}{42.754} = 5.75
\]

(iv) PF = \cos(19.7^\circ) = 0.94 (lagging)

(v) \[
T = \frac{3}{188.5} \left( \frac{(261.3)^2}{(0.24 + 6.01)^2 + (0.49 + 0.5)^2} \times 6.01 \right)
\]

\[
= \frac{3}{188.5} \times 41.29^2 \times 6.01
\]

=163.11 N.m

(vi) Air gap power:

\[
P_{ag} = T\omega_{syn} = 163.11 \times 188.5 = 30,746.2 \, \text{W}
\]

Rotor copper loss:

\[
P_z = sP_{ag} = 0.0333 \times 30,746.2 = 1023.9 \, \text{W}
\]

\[
P_{mech} = (1 - 0.0333)30,746.2 = 29,722.3 \, \text{W}
\]

\[
P_{out} = P_{mech} - P_{rot} = 29,722.3 - 1700 = 28,022.3 \, \text{W}
\]

\[
P_{input} = 3V_1I_1 \cos \theta_1
\]

\[
= 3 \times 265.6 \times 42.754 \times 0.94 = 32,022.4 \, \text{W}
\]
Three-Phase Induction Machine

\[
\eta_{\text{motor}} = \frac{28022.3}{32022.4} \times 100 = 87.5\%
\]

\[
\eta_{\text{internal}} = (1 - s) \times 100 = (1 - 0.0333) \times 100 = 96.7\%
\]

(c) (i) From Equation (4.60)

\[
s_{r_{\text{max}}} = \frac{0.2}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{0.2}{1.0187} = 0.1963
\]

(i) From Equation (4.61)

\[
T_{\text{max}} = \frac{3}{2 \times 188.5} \left[ \frac{261.3^2}{0.24 + [0.24^2 + (0.49 + 0.5)^2]^{1/2}} \right]
\]

\[
= 431.68 \text{ N} \cdot \text{m}
\]

\[
\frac{T_{\text{max}}}{T_{\text{FL}}} = \frac{431.68}{163.11} = 2.65
\]

(d) \[
s_{r_{\text{max}}} = 1 = \frac{R_2' + R'_{\text{ext}}}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{R_2' + R'_{\text{ext}}}{1.0186}
\]

\[
R'_{\text{ext}} = 1.0186 - 0.2 = 0.8186 \Omega/\text{phase}
\]

Note that for parts (a) and (b) it is not necessary to use Thevenin equivalent circuit. Calculation can be based on the equivalent circuit of Fig. 4.13 as follows:
Example 4.5 A three-phase, 460 V, 60 Hz, six-pole wound-rotor induction motor drives a constant load of 100 N·m at a speed of 1140 rpm when the rotor terminals are short-circuited. It is required to reduce the speed of the motor to 1000 rpm by inserting resistances in the rotor circuit. Determine the value of the resistance if the rotor winding resistance per phase is 0.2 ohms. Neglect rotational losses. The stator-to-rotor turns ratio is unity.

Solution:

The synchronous speed is

\[ n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm} \]

Slip at 1140 rpm:

\[ s_1 = \frac{1200 - 1140}{1200} = 0.05 \]

Slip at 1000 rpm:

\[ s_2 = \frac{1200 - 1000}{1200} = 0.167 \]
From the equivalent circuits, it is obvious that if the value of $R_2'/s$ remains the same, the rotor current $I_2$ and the stator current $I_1$ will remain the same, and the machine will develop the same torque (Equation (4.54)). Also, if the rotational losses are neglected, the developed torque is the same as the load torque. Therefore, for unity turns ratio,

$$\frac{R_2}{s_1} = \frac{R_2 + R_{\text{ext}}}{s_2}$$

$$\frac{0.2}{0.05} = \frac{0.2 + R_{\text{ext}}}{0.167}$$

$$R_{\text{ext}} = 0.468 \, \Omega/\text{phase}$$

**Example 4.6** The following test results are obtained from three phase 100hp, 460 V, eight pole star connected induction machine.

**No-load test**: 460 V, 60 Hz, 40 A, 4.2 kW. **Blocked rotor test** is 100V, 60Hz, 140A 8kW. Average DC resistor between two stator terminals is 0.152 $\Omega$

(a) Determine the parameters of the equivalent circuit.

(b) The motor is connected to $3\varphi$, 460 V, 60 Hz supply and runs at 873 rpm. Determine the input current, input power, air gap power, rotor copper loss, mechanical power developed, output power and efficiency of the motor.
Determine the speed of the rotor field relative to stator structure and stator rotating field

**Solution:**

**From no load test:**

(a) \( Z_{NL} = \frac{460 / \sqrt{3}}{40} = 6.64 \ \Omega \)

\( R_{NL} = \frac{P_{NL}}{3*I_1^2} = \frac{4200}{3*40^2} = 0.875 \ \Omega \)

\( X_{NL} = \sqrt{6.64^2 - 0.875^2} = 6.58 \ \Omega \)

Then, \( X_1 + X_m = 6.58 \ \Omega \)

**From blocked rotor test:**

\( R_{BL} = \frac{8000}{3*140^2} = 0.136 \ \Omega \)

\( R_1 = \frac{0.152}{2} = 0.076 \ \Omega \) (from resistance between two stator terminals).

\( Z_{BL} = \frac{100 / \sqrt{3}}{140} = 0.412 \ \Omega \)

\( X_{BL} = \sqrt{0.412^2 - 0.136^2} = 0.389 \ \Omega \)

Then, \( X_1 + X'_2 = 0.389 \ \Omega \)
Then, \( X_1 = X'_2 = \frac{0.389}{2} = 0.1945 \Omega \)

\( X_m = 6.58 - 0.1945 = 6.3855 \Omega \)

\( R = R_{BL} - R_1 = 0.136 - 0.076 = 0.06 \Omega \)

Then, \( R'_2 = \left( \frac{0.1945 + 6.3855}{6.3855} \right)^2 \times 0.06 = 0.0637 \Omega \)

\[
\begin{align*}
0.076 \Omega & \quad j0.195 \Omega & \quad j0.195 \Omega \\
& \quad j6.386 \Omega & 0.0637 \frac{\Omega}{s}
\end{align*}
\]

(b) \( n_s = \frac{120 f}{P} = \frac{120 \times 60}{8} = 900 \text{rpm} \)

\( s = \frac{n_s - n}{n_s} = \frac{900 - 873}{900} = 0.03 \)

\( \frac{R'_2}{s} = \frac{0.0637}{0.03} = 2.123 \Omega \)

Input impedance \( Z_1 \)

\[ Z_1 = 0.076 + j0.195 + \frac{(j6.386)(2.123 + j0.195)}{2.123 + j(6.386 + 0.195)} = 2.121 \angle 27.16^\circ \]

\[ I_1 = \frac{V_1}{Z_1} = \frac{460}{\sqrt{3}} \times \frac{2.12 \angle 27.16}{2.12 \angle 27.16} = 125.22 \angle -27.16^\circ \]
Input power:
\[ P_{in} = 3 * \frac{460}{\sqrt{3}} * 125.22 \cos(27.16^\circ) = 88.767 \text{ kW} \]

Stator CU losses:
\[ P_{sl} = 3 * 125.22^2 * 0.076 = 3.575 \text{ kW} \]

Air gap power
\[ P_{ag} = 88.767 - 3.575 = 85.192 \text{ kW} \]

Rotor CU losses
\[ P_2 = sP_{ag} = 0.03 * 85.192 = 2.556 \text{ kW} \]

Mechanical power developed:
\[ P_{mech} = (1 - s)P_{ag} = (1 - 0.03) * 85.192 = 82.636 \text{ kW} \]

\[ P_{out} = P_{mech} - P_{rot} \]

From no load test:
\[ P_{rot} = P_{NL} - 3I_1^2 * R_1 = 4200 - 3 * 40^2 * 0.076 = 3835.2 \text{ W} \]
\[ P_{out} = 82.636 * 10^3 - 3835.2 = 78.8 \text{ kW} \]

Then the efficiency of the motor is:
\[ \eta = \frac{P_{out}}{P_{in}} * 100 = \frac{78.8}{88.767} * 100 = 88.77 \% \]
Example 4.7 A three phase, 460 V 1450 rpm, 50 Hz, four pole wound rotor induction motor has the following parameters per phase \( R_1 = 0.2 \Omega \), \( R'_2 = 0.18 \Omega \), \( X_1 = X'_2 = 0.2 \Omega \), \( X_m = 40 \Omega \). The rotational losses are 1500 W. Find,

(a) Starting current when started direct on full load voltage. Also find starting torque.

(b) Slip, current, power factor, load torque and efficiency at full load conditions.

(c) Maximum torque and slip at which maximum torque will be developed.

(d) How much external resistance per phase should be connected in the rotor circuit so that maximum torque occurs at start?

Solution:

(a) \( V_1 = \frac{460}{\sqrt{3}} = 265.6 \text{ V} / \text{phase} \)

\[ Z_1 = 0.2 + j0.2 + \frac{j40*(0.18 + 0.2)}{0.18 + j40.2} = 0.55 \angle 46.59^\circ \Omega \]

Then, \( I_{st} = \frac{V_1}{I_1} = \frac{265.6}{0.55 \angle 46.59^\circ} = 482.91 \angle -46.3^\circ \Omega \)

(b) \( s = \frac{1500 - 1450}{1500} = 0.0333 \)
\[
\frac{R_2'}{s} = \frac{0.18}{0.0333} = 5.4 \Omega
\]

\[
Z_1 = 0.2 + j0.2 + \frac{j40 \times (5.4 + j0.2)}{5.4 + j45.4} = 4.959 \angle 10.83^\circ \Omega
\]

Then \( I_{1|FL} = \frac{265.6}{4.959 \angle 10.83^\circ} = 53.56 \angle -10.83^\circ A \)

Then the power factor is: \( \cos 10.83^\circ = 0.9822 \) lag.

\[
\omega_{sys} = \frac{1500}{60} \times 2\pi = 157.08 \text{ rad/sec.}
\]

\[
V_{th} = \frac{265.6 \times (j40)}{0.2 + j40.2} = 264.275 \angle 0.285^\circ V
\]

Then,

\[
Z_{th} = \frac{j40 \times (0.2 + j0.2)}{0.2 + j40.2} = 0.281432 \angle 45.285^\circ = 0.198 + j0.2 \Omega
\]

Then,

\[
T = \frac{3 \times (264.275)^2 \times 5.4}{157.08 \times (0.198 + 5.4)^2 + (0.2 + 0.2)^2} = 228.68 \text{ Nm}
\]

Then, \( P_{ag} = T \times \omega_{sys} = 228.68 \times 157.08 = 35921.1 W \)

Then, \( P_2 = sP_{ag} = 0.0333 \times 35921.1 = 1197 W \)

And, \( P_m = (1 - s)P_{ag} = 34723.7 W \)

Then, \( P_{out} = P_m - P_{rot} = 34723.7 - 1500 = 33223.7 W \)
Three-Phase Induction Machine

\[ P_{in} = 3 \times 265.6 \times 53.56 \times 0.9822 = 41917 \text{ W} \]

Then, \( \eta = \frac{P_{out}}{P_{in}} = \frac{33223.7}{41914} = 79.26\% \)

Then,

\[ T_m = \frac{3 \times (264.275)^2}{2 \times 188.5 \left[ 0.198 + \left( 0.198^2 + (0.2 + 0.2)^2 \right) \right]^{1/2}} = 862.56 \text{ Nm} \]

\[ s_{T_{\text{max}}} = \frac{0.18}{\left[ 0.198^2 + (0.2 + 0.2)^2 \right]^{1/2}} = 0.4033 \]

(d) \( s_{T_{\text{max}}} = 1 = \frac{R'_2 + R'_{\text{ext}}}{\left[ 0.198^2 + (0.2 + 0.2)^2 \right]^{1/2}} \)

Then, \( R'_2 + R'_{\text{ext}} = 0.446323 \)

Then, \( R'_{\text{ext}} = 0.446323 - 0.18 = 0.26632 \Omega \)

**Example 4.8** The rotor current at start of a three-phase, 460 volt, 1710 rpm, 60 Hz, four pole, squirrel-cage induction motor is six times the rotor current at full load.

(a) Determine the starting torque as percent of full load torque.

(b) Determine the slip and speed at which the motor develops maximum torque.

(c) Determine the maximum torque developed by the motor as percent of full load torque.
Solution:

Note that the equivalent circuit parameters are not given. Therefore equivalent circuit parameters cannot be used directly for computation.

(a) The synchronous speed is

$$\omega_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

The full-load slip is

$$s_{FL} = \frac{1800 - 1710}{1800} = 0.05$$

From Equation (4.57)

$$T = \frac{I_2^2 R_2}{s \omega_{syn}} \alpha \frac{I_2^2 R_2}{s}$$, Thus,

$$\frac{T_{st}}{T_{FL}} = \left| \frac{I_{2(st)}}{I_{2(FL)}} \right|^2 s_{FL}$$

$$T_{st} = 6^2 \times 0.05 \times T_{FL} = 1.8T_{FL}$$

$$= 180\% T_{FL}$$

From Equation (4.72)

$$\frac{T_{st}}{T_{max}} = \frac{2s_{T_{max}}}{1 + s_{T_{max}}^2}$$

$$\frac{T_{FL}}{T_{max}} = \frac{2s_{T_{max}} s_{FL}}{s_{T_{max}}^2 + s_{FL}^2}$$
Example 4.9 A 4 pole 50 Hz 20 hp motor has, at rated voltage and frequency a starting torque of 150% and a maximum torque of 200% of full load torque. Determine (i) full load speed (ii) speed at maximum torque.
Solution:

\[ \frac{T_{st}}{T_{FL}} = 1.5 \quad \text{and} \quad \frac{T_{max}}{T_{FL}} = 2 \quad \text{then,} \quad \frac{T_{st}}{T_{max}} = 1.5 \quad \text{and} \quad \frac{1}{2} = 0.75 \]

From above and equation (4.72)

\[ \frac{T_{st}}{T_{max}} = \frac{2s_{T_{max}}}{1 + s_{T_{max}}^{2}} = 0.75 \]

Then, \[ 0.75 s_{T_{max}}^{2} - 2 s_{T_{max}} + 0.75 = 0 \]

Then \[ s_{T_{max}} = 2.21525 \text{ (unacceptable)} \]

Or \[ s_{T_{max}} = 0.451416 \]

Also from Equation 4.72

\[ \frac{T_{max}}{T_{FL}} = \frac{s_{T_{max}}^{2} + s_{FL}^{2}}{2s_{T_{max}} s_{FL}} = 2 \]

But \[ s_{T_{max}} = 0.451416 \]

Then

\[ \frac{T_{max}}{T_{FL}} = \frac{0.451416^{2} + s_{FL}^{2}}{2 * 0.451416 s_{FL}} = 2 \]

\[ s_{FL}^{2} - 4 * 0.451416 \quad s_{FL} + 0.451416^{2} = 0 \]

\[ s_{FL}^{2} - 1.80566 \quad s_{FL} + 0.203777 = 0 \]

\[ s_{FL} = 1.6847 \text{ (unacceptable)} \]

or \[ s_{FL} = 0.120957 \]

\[ n_{s} = \frac{120 * 50}{4} = 1500 \text{ rpm} \]
then (a) \( n_{FL} = (1 - s_{FL}) \times n_s \)

\[
n_{FL} = (1 - 0.120957) \times 1500 = 1319 \text{ rpm}
\]

(b) \( n_{T_{\text{max}}} = (1 - s_{T_{\text{max}}}) \times n_s = (1 - 0.451416) \times 1500 = 823 \text{ rpm} \)

**Example 4.10** A 3φ, 280 V, 60 Hz, 20 hp, four-pole induction motor has the following equivalent circuit parameters.

\( R_1 = 0.12 \, \Omega, \, R'_2 = 0.1 \, \Omega, \, X_1 = X'_2 = 0.25 \, \Omega, \, \text{and} \, X_m = 10 \, \Omega \)

The rotational loss is 400 W. For 5% slip, determine (a) The motor speed in rpm and radians per sec. (b) The motor current. (c) The stator cu-loss. (d) The air gap power. (e) The rotor cu-loss. (f) The shaft power. (g) The developed torque and the shaft torque. (h) The efficiency.

**Solution:**

\[
n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}
\]

\[
\omega_s = \frac{1800}{60} \times 2\pi = 188.5 \text{ rad/sec}
\]
\[ Z_1 = 0.12 + j0.25 + R_e + X_e \]
\[ Z_1 = 0.12 + j0.25 + \frac{j10 \cdot (2 + j0.25)}{2 + j10.25} = 2.1314 \angle 23.55^\circ \Omega \]
\[ V_1 = \frac{208}{\sqrt{3}} = 120.1 \ V \]
\[ I_1 = \frac{120.1}{2.1314 \angle 23.55^\circ} = 2.1314 \angle -23.55^\circ \ A \]
(c) \[ P_1 = 3 \times 56.3479^2 \times 0.12 = 1143.031 \ W \]
(d) \[ P_s = 3 \times 120.1 \times 56.3479 \times \cos(-23.55^\circ) = 18610.9794 \ W \]
\[ P_{ag} = P_s - P_1 = 17467.9485 \ W \]
(e) \[ P_2 = sP_{ag} = 0.05 \times 17467.9785 = 873.3974 \ W \]
(f) \[ P_m = (1 - s)P_{ag} = 16594.5511 \ W \]
(g) \[ T = \frac{P_{ag}}{188.5} = \frac{17467.9485}{188.5} = 92.6682 \ N.m \]
\[ T_{\text{shaft}} = \frac{P_{\text{shaft}}}{188.5} = \frac{16194.5511}{188.5} = 85.9127 \, \text{Nm} \]

(h) \( \eta = \frac{P_{\text{shaft}}}{P_s} \times 100 = 87.02\% \)

For operation at low slip, the motor torque can be considered proportional to slip.

**Example 4.11** A 30, 100 WA, 460 V, 60 Hz, eight-pole induction machine has the following equivalent circuit parameters:

\[ R_1 = 0.07 \, \Omega, \quad R'_2 = 0.05 \, \Omega, \quad X_1 = X'_2 = 0.2 \, \Omega, \text{ and } X_m = 6.5 \, \Omega \]

(a) Derive the Thevenin equivalent circuit for the induction machine.

(b) If the machine is connected to a 30, 460 V, 60 Hz supply, determine the starting torque, the maximum torque the machine can develop, and the speed at which the maximum torque is developed.

(c) If the maximum torque is to occur at start, determine the external resistance required in each rotor phase. Assume a turns ratio (stator to rotor) of 1.2.
Solution:

\[ V_{th} = \frac{X_m}{X_1 + X_m} * V_1 = \frac{6.5}{0.2 + 6.5} * 265.6 = 257.7 \, V \]

\[ R_{th} + jX_{th} = \frac{(j6.5)(j0.2 + 0.07)}{0.07 + j0.2 + j6.5} = 0.06589 + j0.1947 \, \Omega \]

(b)

\[ T_{st} = \frac{3 * 257.7^2 * 0.05}{94.25 \left[ (0.06589 + 0.05)^2 + (0.1947 + 0.2)^2 \right]} = 624.7 \, Nm \]

\[ T_{\text{max}} = \frac{3 * 257.7^2}{2 * 94.25 \left[ 0.06589 + \sqrt{0.06589^2 + (0.1947 + 0.2)^2} \right]} = 2267.8 \, Nm \]

\[ s_{T_{\text{max}}} = \frac{0.05}{\sqrt{0.06589^2 + (0.1947 + 0.2)^2}} = 0.1249 \]

Speed in rpm for which max torque occurs

\[ = (1 - s_{T_{\text{max}}}) * n_s = (1 - 0.1249) * 900 = 787.5 \, rpm \]

(c) \[ s_{T_{\text{max}}} = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X'_2)^2}} \alpha R_2' \]
or \( R_2' \left|_{\text{start}} \right. = \frac{s_{\text{start}}}{s_{T_{\text{max}}}} \cdot R_2' = \frac{1}{0.1249} \cdot 0.05 = 0.4 \Omega \)

Then \( R_{\text{ext}} = (0.4 - 0.05)/1.2^2 = 0.243 \Omega \)

**1-speed characteristic of an induction machine**

We have seen that a 3-phase squirrel-cage induction motor can also function as a generator or as a brake. These three modes of operation—motor, generator, and brake—merge into each other, as can be seen from the torque-speed curve of Fig. 4.18. This curve, together with the adjoining power flow diagrams, illustrates the overall properties of a 3-phase squirrel-cage induction machine.

We see, for example, that when the shaft turns in the same direction as the revolving field, the induction machine operates in
either the motor or the generator mode. But to operate in the generator mode, the shaft must turn faster than synchronous speed. Similarly, to operate as a motor, the shaft must turn at less than synchronous speed.

Finally, in order to operate as a brake, the shaft must turn in the opposite direction to the revolving field.

Fig. 4.18 Complete torque speed curve of a 3-phase induction machine
Chapter Five

Synchronous Generators

5.1 Introduction

Three phase synchronous generators are the primary source of all the electrical energy we consume. These machines are the largest energy converters in the world. They convert mechanical energy into electrical energy, in powers ranging up to 1500 MW. In this chapter we will study the construction and characteristics of these large, modern generators. They are based upon the elementary principles.

5.2 Commercial Synchronous Generators

Commercial synchronous generators are built with either a stationary or a rotating DC magnetic field.

A stationary field synchronous generator has the same outward appearance as a DC generator. The salient poles create the DC field, which is cut by a revolving armature. The armature possesses a 3-phase winding whose terminals are connected to three slip-rings mounted on the shaft. A set of brushes, sliding on the slip-rings, enables the armature to be connected to an external 3-phase load. The armature is driven by a gasoline engine, or some other source of motive power. As it rotates, a 3-phase voltage is
induced, whose value depends upon the speed of rotation and upon the DC exciting current in the stationary poles. The frequency of the voltage depends upon the speed and the number of poles on the field. Stationary-field generators are used when the power output is less than 5 kVA. However, for greater outputs, it is cheaper, safer, and more practical to employ a revolving DC field.

A revolving field synchronous generator has a stationary armature called a stator. The 3-phase stator winding is directly connected to the load, without going through large, unreliable slip-rings and brushes. A stationary stator also makes it easier to insulate the windings because they are not subjected to centrifugal forces. Fig.5.1 is a schematic diagram of such a generator, sometimes called an alternator. The field is excited by a DC generator, usually mounted on the same shaft. Note that the brushes on the commutator have to be connected to another set of brushes riding on slip-rings to feed the DC current $I_X$ into the revolving field.

Fig.5.1 Schematic diagram and cross-section view of a typical 500 MW synchronous generator.
5.3 Number of Poles

The number of poles on a synchronous generator depends upon the speed of rotation and the frequency we wish to produce. Consider, for example, a stator conductor that is successively swept by the $N$ and $S$ poles of the rotor. If a positive voltage is induced when an $N$ pole sweeps across the conductor, a similar negative voltage is induced when the $S$ pole speeds by. Thus, every time a complete pair of poles crosses the conductor, the induced voltage goes through a complete cycle. The same is true for every other conductor on the stator; we can therefore deduce that the alternator frequency is given by

$$f = \frac{Pn}{120} \quad (5.1)$$

$f =$ frequency of the induced voltage [Hz].
$P =$ number of poles on the rotor.
$n =$ speed of the rotor [r/min].

Example 5.1 A hydraulic turbine turning at 200 r/min is connected to a synchronous generator. If the induced voltage has a frequency of 60 Hz, how many poles does the rotor have?

Solution:

From Eqn.(5.1), we have:

$$P = 120 * \frac{60}{200} = 36$$

= 36 poles, or 18 pairs of $N$ and $S$ poles.
5.4 Main Features Of The Stator

From an electrical standpoint, the stator of a synchronous generator is identical to that of a 3-phase induction motor. It is composed of a cylindrical laminated core containing a set of slots that carry a 3-phase lap winding. The winding is always connected in wye and the neutral is connected to ground. A wye connection is preferred to a delta connection because:

1. The voltage per phase is only $1/\sqrt{3}$ or 58% of the voltage between the lines. This means that the highest voltage between a stator conductor and the grounded stator core is only 58% of the line voltage. We can therefore reduce the amount of insulation in the slots that in turn, enables us to increase the cross section of the conductors. A larger conductor permits us to increase the current and, hence, the power output of the machine.

2. When a synchronous generator is under load, the voltage induced in each phase becomes distorted, and the waveform is no longer sinusoidal. The distortion is mainly due to an undesired third harmonic voltage whose frequency is three times that of the fundamental frequency. With a wye connection, the distorting line-to-neutral harmonics do not appear between the lines because they effectively cancel each other. Consequently, the line voltages remain sinusoidal under all load conditions. Unfortunately, when a delta connection is used, the harmonic voltages do not cancel, but
add up. Because the delta is closed on itself, they produce a third-harmonic circulating current, which increases the $I^2R$ losses.

The nominal line voltage of a synchronous generator depends upon its kVA rating. In general, the greater the power rating, the higher the voltage. However, the nominal line-to-line voltage seldom exceeds 25 kV because the increased slot insulation takes up valuable space at the expense of the copper conductors.

5.5 Main Features Of The Rotor

Synchronous generators are built with two types of rotors: salient pole rotors and smooth, cylindrical rotors. Salient pole rotors are usually driven by low-speed hydraulic turbines, and cylindrical rotors are driven by high-speed steam turbines.

1. Salient pole rotors. Most hydraulic turbines have to turn at low speeds (between 50 and 300 r/min) in order to extract the maximum power from a waterfall. Because the rotor is directly coupled to the waterwheel, and because a frequency of 50 Hz or 60 Hz is required, a large number of poles are required on the rotor. Low-speed rotors always possess a large diameter to provide the necessary space for the poles. The salient poles are mounted on a large circular steel frame which is fixed to a revolving vertical shaft. To ensure good cooling, the field coils are made of bare copper bars, with the turns insulated from each other by strips of
mica. The coils are connected in series, with adjacent poles having opposite polarities.

In addition to the DC field winding, we often add a squirrel-cage winding, embedded in the pole faces. Under normal conditions, this winding does not carry any current because the rotor turns at synchronous speed. However, when the load on the generator changes suddenly, the rotor speed begins to fluctuate, producing momentary speed variations above and below synchronous speed. This induces a voltage in the squirrel-cage winding, causing a large current to flow therein. The current reacts with the magnetic field of the stator, producing forces which dampen the oscillation of the rotor. For this reason, the squirrel-cage winding is sometimes called a damper winding.

The damper winding also tends to maintain balanced 3-phase voltages between the lines, even when the line currents are unequal due to unbalanced load conditions.

2. **Cylindrical rotors**. It is well known that high-speed steam turbines are smaller and more efficient than low-speed turbines. The same is true of high-speed synchronous generators. However, to generate the required frequency we cannot use less than 2 poles, and this fixes the highest possible speed. On a 60 Hz system it is 3600 r/min. The next lower speed is 1800 r/min, corresponding to a 4-pole machine. Consequently, these steam-turbine generators possess either 2 or 4 poles.
The rotor of a turbine-generator is a long, solid steel cylinder which contains a series of longitudinal slots milled out of the cylindrical mass. Concentric field coils, firmly wedged into the slots and retained by high-strength end-rings serve to create the N and S poles.

The high speed of rotation produces strong centrifugal forces, which impose an upper limit on the diameter of the rotor. In the case of a rotor turning at 3600 r/min, the elastic limit of the steel requires the manufacturer to limit the diameter to a maximum of 1.2 m. On the other hand, to build the powerful 1000 MVA to 1500 MVA generators the volume of the rotors has to be large. It follows that high-power, high-speed rotors have to be very long.

5.6 Field Excitation And Exciters

The DC field excitation of a large synchronous generator is an important part of its overall design. The reason is that the field must ensure not only a stable ac terminal voltage, but must also respond to sudden load changes in order to maintain system stability. Quickness of response is one of the important features of the field excitation. In order to attain it, two DC generators are used: a main exciter and a pilot exciter. Static exciters that involve no rotating parts at all are also employed.

The main exciter feeds the exciting current to the field of the synchronous generator by way of brushes and slip-rings. Under
normal conditions the exciter voltage lies between 125 V and 600 V. It is regulated manually or automatically by control signals that vary the current $I_c$, produced by the pilot exciter (Fig.5.1).

The power rating of the main exciter depends upon the capacity of the synchronous generator. Typically, a 25 kW exciter is needed to excite a 1000 kVA alternator (2.5% of its rating) whereas a 2500 kW exciter suffices for an alternator of 500 MW (only 0.5% of its rating).

Under normal conditions the excitation is varied automatically. It responds to the load changes so as to maintain a constant ac line voltage or to control the reactive power delivered to the electric utility system. A serious disturbance on the system may produce a sudden voltage drop across the terminals of the alternator. The exciter must then react very quickly to keep the ac voltage constant. For example, the exciter voltage may have to rise to twice its normal value in as little as 300 to 400 milliseconds. This represents a very quick response, considering that the power of the exciter may be several thousand kilowatts.

5.7 Brushless Excitation

Due to brush wear and carbon dust, we constantly have to clean, repair, and replace brushes, slip-rings, and commutators on conventional DC excitation systems. To eliminate the problem, brushless excitation systems have been developed. Such a system
Synchronous Generators consists of a 3-phase stationary field generator whose ac output is rectified by a group of rectifiers. The DC output from the rectifiers is fed directly into the field of the synchronous generator (Fig.5.2).

The armature of the ac exciter and the rectifiers are mounted on the main shaft and turn together with the synchronous generator. In comparing the excitation system of Fig.5.2 with that of Fig.5.1, we can see they are identical, except that the 3-phase rectifier replaces the commutator, slip rings, and brushes. In other words, the commutator (which is really a mechanical rectifier) is replaced by an electronic rectifier. The result is that the brushes and slip-rings are no longer needed.

The DC control current $I_c$ from the pilot exciter regulates the main exciter output $I_x$, as in the case of a conventional DC exciter. The frequency of the main exciter is generally two to three times...
the synchronous generator frequency (50 Hz). The increase in frequency is obtained by using more poles on the exciter than on the synchronous generator. Static exciters that involve no rotating parts at all are also employed.

5.8 Factors Affecting The Size Of Synchronous Generators

The prodigious amount of energy generated by electrical utility companies has made them very conscious about the efficiency of their generators. For example, if the efficiency of a 1000 MW generating station improves by only 1%, it represents extra revenues of several thousand dollars per day. In this regard, the size of the generator is particularly important because its efficiency automatically improves as the power increases. For example, if a small 1 kilowatt synchronous generator has an efficiency of 50%, a larger, but similar model having a capacity of 10 MW inevitably has an efficiency of about 90%. This improvement in efficiency with size is the reason why synchronous generators of 1000 MW and up possess efficiencies of the order of 99%.

Another advantage of large machines is that the power output per kilogram increases as the power increases. For example, if a 1 kW generator weighs 20 kg (yielding 1000 W/20 kg = 50 W/kg), a 10 MW generator of similar construction will weigh only 20 000
kg, thus yielding 500 W/kg. From a power standpoint, large machines weigh relatively less than small machines; consequently, they are cheaper. Section 16.24 at the end of this chapter explains why the efficiency and output per kilogram increase with size.

Everything, therefore, favors the large machines. However, as they increase in size, we run into serious cooling problems. In effect, large machines inherently produce high power losses per unit surface area (W/m$^2$); consequently, they tend to overheat. To prevent an unacceptable temperature rise, we must design efficient cooling systems that become ever more elaborate as the power increases. For example, a circulating cold air system is adequate to cool synchronous generators whose rating is below 50 MW but between 50 MW and 300 MW we have to resort to hydrogen cooling. Very big generators in the 1000 MW range have to be equipped with hollow, water-cooled conductors. Ultimately, a point is reached where the increased cost of cooling exceeds the savings made elsewhere, and this fixes the upper limit to size.

To sum up, the evolution of big alternators has mainly been determined by the evolution of sophisticated cooling techniques. Other technological breakthroughs, such as better materials, and novel windings have also played a major part in modifying the design of early machines.

As regards speed, low-speed generators are always bigger than high-speed machines of equal power. Slow-speed bigness
simplifies the cooling problem; a good air-cooling system, completed with a heat exchanger, usually suffices. For example, the large, slow-speed 500 MVA, 200 r/min synchronous generators installed in a typical hydropower plant are air-cooled whereas the much smaller high-speed 500 MVA, 1800 r/min units installed in a steam plant have to be hydrogen-cooled.

5.9 No-Load Saturation Curve

Fig.5.3 shows a 2-pole synchronous generator operating at no-load. It is driven at constant speed by a turbine. The leads from the 3-phase, wye-connected stator are brought out to terminals A, B, C, N, and a variable exciting current $I_x$ produces the flux in the air gap.

Fig.5.3 Generator operation at no-load.
Let us gradually increase the exciting current while observing the ac voltage $E_o$ between terminal A, say, and the neutral N. For small values of $I_x$, the voltage increases in direct proportion to the exciting current. However, as the iron begins to saturate, the voltage rises much less for the same increase in $I_x$. If we plot the curve of $E_o$ versus $I_x$, we obtain the no-load saturation curve of the synchronous generator.

Fig. 5.4 shows the actual no-load saturation curve of a 36 MW, 3-phase generator having a nominal voltage of 12 kV (line to neutral). Up to about 9 kV, the voltage increases in proportion to the current, but then the iron begins to saturate. Thus, an exciting current of 100 A produces an output of 12 kV, but if the current is doubled, the voltage rises only to 15 kV.
Fig. 5.5 is a schematic diagram of the generator showing the revolving rotor and the three phases on the stator.

Fig. 5.5 Electric circuit representing the generator of Fig. 5.3
5.10 Synchronous Reactance Equivalent Circuit Of An Ac Generator

Consider a 3-phase synchronous generator having terminals A, B, C feeding a balanced 3-phase load. The generator is driven by a turbine, and is excited by a DC current $I_x$. The machine and its load are both connected in wye, yielding the circuit of Fig.5.6. Although neutrals $N_1$ and $N_2$ are not connected, they are at the same potential because the load is balanced. Consequently, we could connect them together (as indicated by the short dash line) without affecting the behavior of the voltages or currents in the circuit.

The field carries an exciting current which produces a flux $\phi$. As the field revolves, the flux induces in the stator three equal voltages $E_o$ that are 120 degrees out of phase (Fig.5.7). Each phase of the stator winding possesses a resistance R and a certain inductance L. Because this

![Electric circuit representing the alternator connected with 3-phase load.](image)
Fig. 5.7 Voltage and impedances in a 3-phase generator and its connected load

The synchronous reactance of a generator is an internal impedance, just like its internal resistance $R$. The impedance is there, but it can neither be seen nor touched. The value of $X_s$ is typically 10 to 100 times greater than $R$; consequently, we can always neglect the resistance, unless we are interested in efficiency or heating effects.

We can simplify the schematic diagram of Fig. 5.7 by showing only one phase of the stator. In effect, the two other phases are identical, except that their respective voltages (and currents) are out of phase by 120 degrees. Furthermore, if we neglect the resistance of the windings, we obtain the very simple circuit of Fig. 5.8. A synchronous generator can therefore be represented by an equivalent circuit composed of an induced voltage $E_o$ in series with a reactance $X_s$. 
Fig. 5.8 Equivalent circuit of a 3-phase generator, showing only one phase.

In this circuit the exciting current $I_x$ produces the flux $\phi$ which induces the internal voltage $E_o$. For a given synchronous reactance, the voltage $E$ at the terminals of the generator depends upon $E_o$ and the load $Z$. Note that $E_o$ and $E$ are line-to-neutral voltages and $I$ is the line current.

**Example 5.2** A 3-phase synchronous generator produces an open-circuit line voltage of 6928 V when the do exciting current is 50 A. The ac terminals are then short-circuited, and the three line currents are found to be 800 A.

a. Calculate the synchronous reactance per phase.

b. Calculate the terminal voltage if three 12 Ω resistors are connected in wye across the terminals.

**Solution:**

a. The line-to-neutral induced voltage is

$$E_o = E_L / \sqrt{3} = 6928 / \sqrt{3} = 4000V$$
When the terminals are short-circuited, the only impedance limiting the current flow is that due to the synchronous reactance. Consequently,

$$X_s = E_o / I = 4000 / 80 = 5 \Omega$$

The synchronous reactance per phase is therefore 5 fl.

b. The equivalent circuit per phase is shown in Fig.5.9a.

The impedance of the circuit is:

$$Z = \sqrt{R^2 + X_s^2} = \sqrt{12^2 + 5^2} = 13 \Omega$$

The current is:

$$I = E_o / Z = 4000 / 13 = 308 A$$

The voltage across the load resistor is

$$E = IR = 308 \times 12 = 3696 V$$

The line voltage under load is:

$$E_L = \sqrt{3} E = \sqrt{3} \times 3696 = 6402 V$$

The schematic diagram of Fig.5.9b helps us visualize what is happening in the actual circuit.


5.11 Synchronous Generator Under Load

The behavior of a synchronous generator depends upon the type of load it has to supply. There are many types of loads, but they can all be reduced to two basic categories:

1. Isolated loads, supplied by a single generator
2. Consider a 3-phase generator that supplies power to a load having a lagging power factor. Fig. 5.10 represents the equivalent circuit for one phase. In order to construct the phasor diagram for this circuit, we list the following facts:

Fig. 5.10 Equivalent circuit of a generator under load.
1. Current $I$ lags behind terminal voltage $E$ by an angle $\theta$.

2. Cosine $\theta = \text{power factor of the load}$.

3. Voltage $E_X$ across the synchronous reactance leads current $I$ by 90 degrees. It is given by the expression $E_x = jIX_S$.

4. Voltage $E_o$ generated by the flux $\phi$ is equal to the phasor sum of $E$ plus $E_X$.

5. Both $E_o$ and $E_X$ are voltages that exist inside the synchronous generator windings and cannot be measured.

6. Flux $\phi$ is that produced by the DC exciting current $I_x$.

The resulting phasor diagram is given in Fig.5.11. Note that $E_o$ leads $E$ by $\delta$ degrees. Furthermore, the internally-generated voltage $E_o$ is greater than the terminal voltage, as we would expect.

In some cases the load is somewhat capacitive, so that current $I$ leads the terminal voltage by an angle $\theta$. What effect does this have on the phasor diagram? The answer is found in Fig.5.11. The voltage $E_X$ across the synchronous reactance is still 90 degrees ahead of the current. Furthermore, $E_O$ is again equal to the phasor sum of $E$ and $E_X$. However, the terminal voltage is now greater than the induced voltage $E_O$, which is a very surprising result. In
Synchronous Generators

Effect, the inductive reactance $X_S$ enters into partial resonance with the capacitive reactance of the load. Although it may appear we are getting something for nothing, the higher terminal voltage does not yield any more power.

If the load is entirely capacitive, a very high terminal voltage can be produced with a small exciting current. However, in later chapters, we will see that such under-excitation is undesirable.

**Example 5.3** A 36 MVA, 20.8 kV, 3-phase alternator has a synchronous reactance of 9Ω and a nominal current of 1 kA. The no-load saturation curve giving the relationship between $E_o$ and $I$, is given in Fig.5.5. If the excitation is adjusted so that the terminal
voltage remains fixed at 21 kV, calculate the exciting current required and draw the phasor diagram for the following conditions:

a. No-load
b. Resistive load of 36 MW
c. Capacitive load of 12 Mvar

**Solution:**

We shall immediately simplify the circuit to show only one phase.

The line-to-neutral terminal voltage for all cases is fixed at

\[ E = \frac{20.8}{\sqrt{3}} = 12 \text{ kV} \]

a. At no-load there is no voltage drop in the synchronous reactance; consequently,

\[ E_o = E = 12 \text{ kV} \]

The exciting current is:

\[ I_x = 100 \text{ A} \]

(See Fig. 5.5)

In no load \( E \) and \( E_o \) will be in phase

**With a resistive load of 36 MW:**

b. The power per phase is

\[ P = 36 / 3 = 12 \text{ MW} \]

The full load line current is

\[ I = P / E = 12 \times 10^6 / 12 \times 000 = 1000 \text{ A} \]
Synchronous Generators

The current is in phase with the terminal voltage.

The voltage across $X_S$ is:

$$E_x = jX_S = j1000 \times 9 = 9 \text{ kV} \angle 90^\circ$$

The voltage $E_o$ generated by $I_x$ is equal to the phasor sum of $E$ and $E_x$. Referring to the phasor diagram, its value is given by:

$$E_o = \sqrt{E^2 + E_x^2} = \sqrt{12^2 + 9^2} = 15 \text{ kV}$$

The required exciting current is

$$I_x = 200 \text{ A} \quad \text{(See Fig. 5.5)}$$

The phasor diagram is given in Fig. 5.13.

![Phasor Diagram](image)

Fig. 5.13 Phasor diagram with a unity power factor load.

With a capacitive load of 12 Mvar:

c. The reactive power per phase is

$$Q = 12/3 = 4 \text{ Mvar}.$$ 

The line current is

$$I = Q/E = 4 \times 10^6 / 12000 = 333 A$$
Chapter Five

The voltage across $X_S$ is

$$E_x = jX_S = j333 \times 9 = 3 \angle 90^\circ \text{kV}$$

As before $E_x$ leads $I$ by 90 degrees. (Fig.5.14)

![Phasor diagram with a capacitive load.](image)

The voltage $E_o$ generated by $I_x$ is equal to the phasor sum of $E_x$ and $E$.

$$E_o = E + E_x = 12 + (-3) = 9\text{kV}$$

The corresponding exciting current is

$$I_x = 70 \text{ A} \quad \text{(See Fig.5.5)}$$

Note that $E_o$ is again less than the terminal voltage $E$.

The phasor diagram for this capacitive load is given in Fig.5.14.