# Final Exam

**Instructors:** A. Sheta & S. Aldosari  
**Date:** 21/5/1428  
**Time:** 8:00-11:00 pm

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**Total Mark**
**Problem I**  (20 points)

(a) Determine whether or not each of the following signals is periodic. If the signal is periodic find the fundamental period.

i. \( x_1(t) = 3 \cos 2\pi t^2 \)

ii. \( x_2[n] = e^{j\frac{3\pi}{4} n} + e^{-j\frac{5\pi}{2} n} \)

(b) the signal \( x_3(t) \) is defined by

\[
x_3(t) = \begin{cases} 
-\frac{1}{4} t + 1 & 0 < t < 2 \\
0 & \text{otherwise}
\end{cases}
\]

i. sketch the signal \( x_3(2t-1) \)

ii. find the energy and power in the signal \( x_3(t-5) \)
(c) Discuss the invertible and linear time invariant properties of the signals:

i. \( y(t) = x^2(t) \)

ii. \( y[n] = x[2n] + 1 \)
Problem II  (20 points)

(a) The impulse response of an LTI system is given as  \( h(t) = 3e^{-|t|}u(t) \)

Find the output \( y(t) \) if the input is  \( x(t) = e^{-|t|} \)
(b)

i. The unit step response can be used to characterize an LTI system. Derive an expression that relates the impulse response to unit step response for DT system.

ii. find the unit step response $s[n]$ of an LTI system where its impulse response is given by

$$h[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1]$$
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(a) For the above continuous-time periodic signal:

(i) Find the Fourier series coefficients $a_k$ of $x(t)$

(ii) Find the dc component of $x(t)$

Hint: $\int a^u du = \frac{1}{\ln(a)} a^u$
(b) For the above discrete-time periodic signal:
   (i) Find the 4th harmonic component of $x[n]$
   (ii) Find the 8th harmonic component of $x[n]$. What is the relationship between the 4th and the 8th harmonic components? Explain why?
(a) The Fourier transform of \( x(t) = e^{-t^2/2} \) is given by \( X(\omega) = \sqrt{2\pi} e^{-\omega^2/2} \).

Using this information find \( y(t) \) the inverse Fourier transform of \( Y(\omega) = \omega e^{-\omega^2/2} \).
(b) Find the frequency response $H(e^{j\omega})$ of the following discrete-time system:
Problem V  (20 points)

Let $X(\omega), H_1(\omega), H_2(\omega)$ be the Fourier transforms of $x(t), h_1(t), h_2(t)$ respectively. In the following questions choose the correct answers and explain why?

(a) Suppose that the signal $x(t)$ is sampled at a sampling angular frequency $\omega_s = 9$, what should be the cutoff-frequency $\omega_c$ of the reconstruction filter to reconstruct the original signal?

(i) $\omega_c > 8$  (ii) $\omega_c < 8$  (iii) $\omega_c > 4$  (iv) $\omega_c < 4$  (v) $\omega_c > 3$  (vi) $\omega_c < 3$

(vii) $\omega_c > 0$  (viii) $3 < \omega_c < 4$  (ix) $4 < \omega_c < 5$  (x) $5 < \omega_c < 6$  (xi) $2 < \omega_c < 4$  (xii) $4 < \omega_c < 8$

(b) In the following system we would like to sample the signal $y_1(t)$. What is the minimum sampling angular frequency $\omega_s$?

(i) 0  (ii) 1  (iii) 2  (iv) 3  (v) 4  (vi) 5

(vii) 6  (viii) 7  (ix) 8  (x) 9  (xi) 10  (xii) 2/3
(c) In the following system we would like to sample the signal $y_2(t)$. What is the minimum sampling angular frequency $\omega_s$?

(i) 0  (ii) 1  (iii) 2  (iv) 3  (v) 4  (vi) 5
(vii) 6  (viii) 7  (ix) 8  (x) 9  (xi) 10  (xii) 2/3

(c) In the following system the input is $z(t) = x\left(\frac{3}{4}t\right)$ and we would like to sample the signal $y_3(t)$. What is the minimum sampling angular frequency $\omega_s$?

(i) 3/4  (ii) 1  (iii) 2  (iv) 3  (v) 4  (vi) 5
(vii) 6  (viii) 7  (ix) 8  (x) 16/3  (xi) 10  (xii) 4/3
Problem VI  (20 points)

(a) The transfer function of a continuous-time system is given by

\[ H(s) = \frac{7(s - 3)}{(s + 1)(s^2 - 6s + 8)} \]

(i) Sketch the zero-pole plot and find all possibilities for the region of convergence (ROC).
(ii) For each ROC discuss the causality and stability of the system.
(iii) For each ROC find the impulse response \( h(t) \) of this system.
(b) In the following circuit, assume that \( v_{in}(t) = 0 \) for \( t < 0 \).

i) Find the transfer function of this system
\[ H(s) = \frac{V_{out}(s)}{V_{in}(s)} \]

ii) sketch the zeros-pole plot of this system showing the ROC (this is a causal system)