

Question 1:

(a) Determine whether or not each of the following signals is periodic (explain why). If yes, determine its fundamental period.

i) $x[n] = 2 \cos\left(\frac{2\pi}{4}n\right) - \sin\left(\frac{2\pi}{3}n\right)$

ii) $y(t) = \sin(2\pi t + 3) + 5 \cos(3t)$

(b) Let $x(t) = \begin{cases} 4 & \text{if } 1 < t < 2 \\ 8 - 2t & \text{if } 2 < t < 4 \\ 0 & \text{elsewhere} \end{cases}$

i) Sketch the odd part of $x(t)$ and carefully label the axes.

ii) Sketch $y(t) = x(2 - 2t)$ and carefully label the axes.

Question 2:

(a) Consider a discrete-time system with the input $x[n]$ and the output $y[n]$ related as:

$y[n] = (x[n - 1])^3 n$. Is this system linear? Is it time-invariant? Explain why.

(b) The impulse response of an LTI system is given as $h[n] = \cos(\pi n)u[n]$. Is this system causal? Is it stable? Explain why.

(c) Consider the difference equation $2y[n - 1] - y[n] = 3x[n]$ that describes the relationship between the input $x[n]$ and the output $y[n]$ of an LTI system.

i) Draw a block diagram of the system.

ii) Find the impulse response of the system. Assume that the system is initially at rest, i.e.,

$y[-1] = 0$. (Hint: Find $y[0]$ then $y[1]$ then, ..., $y[n]$)

Question 3:

(a) The impulse response for a LTI system is given by: $h(t) = \begin{cases} \frac{1}{2}(t-1) & 3 \geq t \geq 1 \\ 0 & \text{otherwise} \end{cases}$

i) Compute and draw the unit step response $y(t)$

ii) Compute the autocorrelation function for the signal $x(t) = u(t-1) - u(t-3)$

(b) Compute the cross-correlation function $R_{xy}(n)$ where $x(n)$ and $y(n)$ are given by:

$x(n) = 2\delta(n) - \delta(n-1)$; $y(n) = \delta(n-1) + \delta(n)$

hint:- $y(t) = \int_{-\infty}^{\infty} h(\tau) u(t - \tau) d\tau$; $R_{xy}(n) = \sum_{k=-\infty}^{\infty} x(k - n)y(k)$