

Question 1:

(a) A discrete-time (DT) linear time-invariant (LTI) system is described by the following input-output relationships:

If the input is $x_1[n] = 4\delta[n] + 2\delta[n-1]$ the output is $y_1[n] = 4\delta[n] + 6\delta[n-1] + 2\delta[n-2]$.

If input is $x_2[n] = -2\delta[n] - \delta[n-1] + \frac{1}{2}\delta[n-2]$ the output is $y_2[n] = -2\delta[n] - 3\delta[n-1] - \frac{1}{2}\delta[n-2] + \frac{1}{2}\delta[n-3]$

Find and sketch the impulse response $h[n]$ of this system.

(b) A continuous-time system is described by $y(t) = \exp[x(t+1)]$, where $x(t)$ is the input and $y(t)$ is the output. (i) Is this system stable? (ii) Is it causal? (iii) Is it time-invariant? Explain why or why not.

Question 2:

(a) Figure 1 describes a CT LTI system with input

$x(t)$ and output $y(t)$, where:

$$h_1(t) = u(t), \quad h_2(t) = \delta(t-1) + \delta(t+1),$$

$$h_3(t) = -\delta(t) - \delta(t-2), \quad \text{and} \quad h_4(t) = \delta(t-1).$$

Find the overall impulse response $h(t)$ of the system so that $y(t) = x(t) * h(t)$.

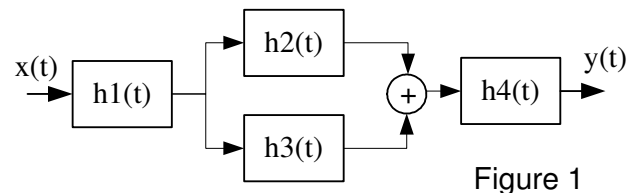


Figure 1

(b) A CT signal $z(t)$ is shown in Figure 2.

(i) Determine and sketch the autocorrelation of $z(t)$

(ii) Find the energy of $z(t)$ by using the autocorrelation.

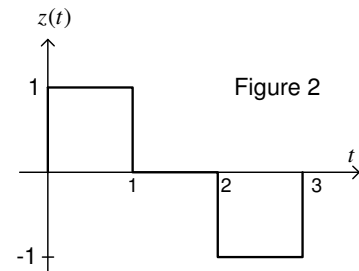


Figure 2

Question 3:

(a) $x(t)$ is a periodic signal. One period is given by $x(t) = \exp(-|t|)$ for $-1 \leq t \leq 1$ then the same pattern repeats every 2 seconds. (i) Find the Fourier series coefficients a_k of $x(t)$

(ii) Find the dc component of $x(t)$

(iii) Find the Fourier transform of $x(t)$.

(b) Consider the DT filter $h[n] = \begin{cases} 2-|n| & -2 \leq n \leq 2 \\ 0 & \text{elsewhere} \end{cases}$.

(i) Find the frequency response $H(e^{j\omega})$ of this system.

(ii) Determine and sketch the magnitude and phase of $H(e^{j\omega})$.

(iii) Is this system low-pass, band-pass, or high-pass?

Question 4:

Let $x(t)$ be a CT signal with Fourier transform $X(\omega) = \begin{cases} 1-\omega^2 & -1 \leq \omega \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

Using the above information, choose the correct answers from the following and **explain why**:

(a) To avoid aliasing when sampling the signal $x(t)$, the sampling angular frequency ω_s should be

- (i) $\omega_s > 1$ (ii) $\omega_s < 1$ (iii) $\omega_s > 2$ (iv) $\omega_s < 2$ (v) $\omega_s > 4$ (vi) $\omega_s < 4$

(b) The energy of $x(t)$ is

- (i) $\frac{4}{3}$ (ii) $\frac{2}{3\pi}$ (iii) $\frac{6}{13}$ (iv) $\frac{6}{13\pi}$ (v) $\frac{16}{15\pi}$ (vi) $\frac{8}{15\pi}$

(c) To avoid aliasing when sampling the signal $y(t) = x(2t)$, the sampling angular frequency ω_s should be

- (i) $\omega_s > 1$ (ii) $\omega_s < 1$ (iii) $\omega_s > 2$ (iv) $\omega_s < 2$ (v) $\omega_s > 4$ (vi) $\omega_s < 4$

Question 5:

A linear time invariant discrete system is shown in Figure 3:

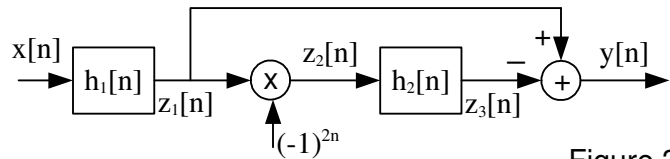


Figure 3

where

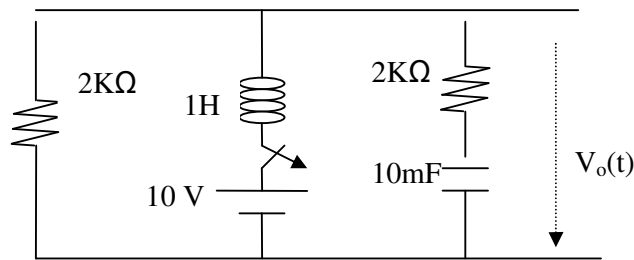
$$H_1(e^{j\omega}) = \begin{cases} 1 & |\omega| < \pi/2 \\ 0 & \text{otherwise} \end{cases} ; H_2(e^{j\omega}) = \begin{cases} 1 & |\omega| < \pi/4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the overall frequency response $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$ and draw it against ω .

(b) Compute $y(n)$ if $x(n) = \cos(3\pi n/8) + 2 \sin(\pi n/8) + \cos(3\pi n/4)$

Question 6:

Given the RLC circuit as shown in figure. The switch is closed for long time at $t < 0$. The switch is opened at $t = 0$.



(a) Compute $V_o(t)$ for all $t > 0$

(b) Compute the steady state value for $V_o(t)$

(c) Compute the transient response for $V_o(t)$