A New MSE Approach for Combined Linear-Viterbi Equalizers

Saeed A. Aldosari, Saleh A. Alshebeili and Abdulhameed M. Al-Sanie

Department of Electrical Engineering, King Saud University
P. O. Box 800, Riyadh 11421, Saudi Arabia

Abstract - Combined linear-Viterbi equalization (CLVE) is a technique that employs a linear pre-filter in conjunction with the Viterbi algorithm (VA) to mitigate the effects of intersymbol interference. The aim of the linear pre-filter is to shape the original channel impulse response to some shorter desired impulse response (DIR) in order to reduce the complexity of the VA. In this paper, we present a new MSE-based approach for optimizing CLVEs. This approach takes advantage of the recent modifications to the VA which are suitable for channels having coarsely located coefficients. Specifically, the new approach has the flexibility in choosing the positions and optimizing the values of nonzero coefficients of DIR. As a result, it includes the conventional MSE-based approaches as a special case. Simulation results have been presented to illustrate the performance of proposed method.

I. Introduction

Over the past decade, we have witnessed an increased demand for high-speed digital transmission. This has resulted from the need for fully integrated voice and data services, as well as from an increased demand of high-speed radio communications, such as indoor and mobile communications. On the other hand, the need to increase the efficiency of digital communication channels has led to the development of spectrally efficient high-level modulation schemes. High signaling rate digital communication systems utilizing high-level modulation schemes are very sensitive to intersymbol interference (ISI). The ISI degrades the performance of these systems and impose limitations on the data transmission rate [1, 2].

It is well known that maximum likelihood sequence estimation (MLSE) is the optimum method for the detection of data in the presence of ISI and additive white Gaussian noise (AWGN) provided that the channel impulse response (CIR) is known or can be precisely estimated [3]. Viterbi algorithm (VA) provides an efficient way of performing MLSE recursively when the length of CIR is finite. The symbol error rates of VA are often much lower than error rates of the symbol by symbol detectors. However, the use of MLSE is limited to channels having short delay spread. This is caused by the high computational demand, which is growing exponentially with the length of CIR. For real life channels, the use of MLSE may become impractical.

A considerable amount of research has been undertaken to reduce the complexity of MLSE using various techniques. Combined linear-Viterbi equalizer (CLVE) is a class of receivers employing a linear pre-filter before the VA in order to shape the original CIR to some shorter desired impulse response (DIR) which allows a lower complexity VA. When designing CLVEs it is often desirable to minimize the bit error rate (BER) of the receiver. However, such a criterion is difficult to optimize because of the lack of an exact mathematical model for the BER and the complexity and non-linearity of the existing approximate BER bounds. For this reason, several simplified optimization criteria were used to optimize the linear pre-filter and DIR. Minimization of the mean-square-error (MMSE) is among the simplest and most practical CLVE design approaches [4, 5]. The objective of the MMSE design approach is to minimize the MSE between the output of the pre-filter and the output of the DIR.

To the best of our knowledge, in all of the previous work related to the MMSE approach, the DIR was restricted to consist of a reduced number of *consecutive* coefficients. This was a direct result of the fact that the complexity of the classical VA is exponentially proportional to the length of the assumed DIR. In this paper, we introduce a new formulation for the MMSE criterion that allows us to choose freely the positions of nonzero coefficients of DIR. This new formulation is inspired by two main points:

- In general, we may not get the optimum settings of the linear filter and the DIR with respect to MSE if we have restricted the nonzero coefficients to be consecutive as considered in the conventional MMSE techniques [4, 5, 6].
- Recently, a number of simplifications to the VA have been proposed to take advantage of the sparse shape of some channels. These simplifications are suitable when the CIR consists only of few coarsely located coefficients. The complexity of such simplified schemes does not depend on the CIR length but only on the number of nonzero coefficients. The multitrellis Viterbi algorithm (MVA) proposed in [7, 8, 9] and the parallel trellis Viterbi algorithm (PTVA) [10, 11] are among these techniques.

Therefore, the main difference between the proposed approach (which will be denoted by MMSE-A) and the conventional approach (which will be denoted by

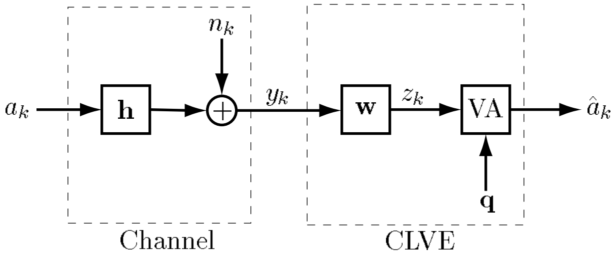

Figure 1: Combined linear-Viterbi equalizer (CLVE).

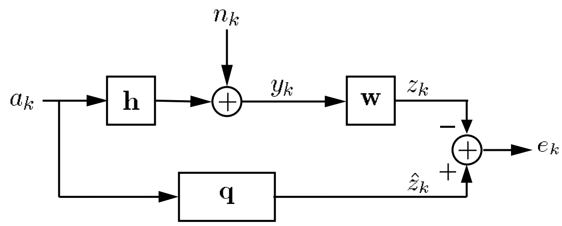

Figure 2: Optimization of linear-Viterbi equalizer using MSE criterion.

MMSE-B) is that the MMSE-A approach has no assumptions regarding the locations of the nonzero coefficients of the DIR while the conventional MMSE-B approach restricts the nonzero coefficients of the DIR to be consecutive. The conventional MMSE-B approach can therefore be viewed as a special case of the more general MMSE-A approach. As a result, the MSE achieved by the MMSE-A approach is always less than or equal to that of other MMSE approaches.

The organization of the paper is as follows. We first present the block diagram of CLVE systems and describe its basic concepts. Second, we derive the optimum settings of the linear pre-filter and the DIR. These settings include the results obtained in [4, 5] as special cases. Third, we evaluate the BER performance of the proposed approach as compared to the conventional MMSE approaches.

II. CLVE Model

The block diagram of a CLVE system is shown in Figure 1. The transmitted sequence a_k is assumed to be a zero mean M-ary sequence selected from the set $\{\pm 1, \pm 3, \ldots, \pm(M-1)\}$. The communication channel is modeled by a transversal filter with impulse response $\mathbf{h}$ of length $L+1$ and corrupted with additive white Gaussian noise (AWGN) sequence n_k with variance σ^2. The received samples y_k are processed by a linear pre-filter $\mathbf{w}$ and the output sequence z_k is, then, passed to the VA to perform MLSE. The function of the linear pre-filter $\mathbf{w}$ is to shape the original CIR $\mathbf{h}$ to some shorter DIR $\mathbf{q}$. With this arrangement, the complexity of the VA can be made lower by considering the shorter DIR $\mathbf{q}$ instead of the original longer CIR $\mathbf{h}$.

III. Optimizing CLVE Parameters

Figure 2 illustrates the principles of the MMSE design approach. The upper side of Figure 2 represents the model of the CLVE system while the lower side represents its equivalent DIR. The objective of the MMSE design approach is to minimize the MSE between the output of the pre-filter z_k and the output of the DIR $\hat{z}_k$ which is denoted by e_k in Figure 2.

In what follows, we present a general framework for obtaining the optimum setting of the linear pre-filter and the DIR with respect to the MSE criterion.

Over a block of N output symbols, the received signal y_k can be expressed in vector form by

$$\mathbf{y} = \mathbf{H}\mathbf{a} + \mathbf{n} \tag{1}$$

where

$$\mathbf{y} = \begin{bmatrix} y_k & y_{k-1} & \cdots & y_{k-N+1} \end{bmatrix}^T$$

$\cdots$ N column vector of channel outputs

$$\mathbf{a} = \begin{bmatrix} a_k & a_{k-1} & \cdots & a_{k-N-L+1} \end{bmatrix}^T$$

$\cdots$ $N+L-1$ column vector of channel inputs

$$\mathbf{n} = \begin{bmatrix} n_k & n_{k-1} & \cdots & n_{k-N+1} \end{bmatrix}^T$$

$\cdots$ N column vector of noise samples

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \cdots & h_L & 0 & \ldots & 0 \\ 0 & h_0 & h_1 & \cdots & h_L & 0 & \cdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_L \end{bmatrix}$$

$\cdots$ $N \times (N+L-1)$ channel toeplitz matrix

and $(\cdot)^T$ denotes the transposition operator. Note that the multiplication between the channel toeplitz matrix $\mathbf{H}$ and the vector of inputs $\mathbf{a}$ simply describes a convolution process. The channel output $\mathbf{y}$ is, then, processed using the linear filter $\mathbf{w}$ to produce the sequence z_k given by

$$z_k = \mathbf{w}^T\mathbf{y} \tag{2}$$

which will be passed to the VA to perform MLSE.

Instead of considering the original CIR, the VA implementation will be based on the knowledge of the equivalent DIR $\mathbf{q}$ given by

$$\mathbf{q} = \begin{bmatrix} q_0 & q_1 & \cdots & q_{N+L-1} \end{bmatrix}^T. \tag{3}$$

Basically, this means that the cascade of the CIR and the linear pre-filter has been modeled by an equivalent linear transversal filter $\mathbf{q}$ as shown in the lower part of Figure 2. It should be noted, however, that some constraints should be added regarding the choice of the DIR in order to insure that the complexity of the VA is reduced. In the MMSE-A approach that we propose, the DIR is selected such that the number of nonzero coefficients of $\mathbf{q}$ is limited to $V+1$, i.e.,

$$q_i = 0 \text{ for } i \notin \Theta = \{\theta_0, \theta_1, \ldots, \theta_V\} \tag{4}$$

where θ_j, $j = 0, 1, \ldots, V$ correspond to the locations of the $V + 1$ nonzero coefficients of the DIR.

The basic idea of the MMSE-A approach is to design the linear filter $\mathbf{w}$ and the DIR $\mathbf{q}$ in such a way to minimize the MSE between the output of the linear filter z_k and the output of the DIR $\hat{z}_k$, i.e., to minimize

$$\begin{aligned} \text{MSE} &\triangleq E\left[|e_k|^2\right] \\ &= E\left[|\hat{z}_k - z_k|^2\right] \end{aligned} \quad (5)$$

where $E[\cdot]$ denotes the expectation operation. Substituting Equation (2) into (5) and making use of the fact that $\hat{z}_k = \mathbf{q}^T \mathbf{a}$, it is straight forward to show that

$$\begin{aligned} \text{MSE} &= E\left[\left|\mathbf{q}^T\mathbf{a} - \mathbf{w}^T\mathbf{y}\right|^2\right] \\ &= \mathbf{q}^T\mathbf{R}_{aa}\mathbf{q} + \mathbf{w}^T\mathbf{R}_{yy}\mathbf{w} \\ &\quad -\mathbf{q}^T\mathbf{R}_{ay}\mathbf{w} - \mathbf{w}^T\mathbf{R}_{ya}\mathbf{q} \end{aligned} \quad (6)$$

where $\mathbf{R}_{yy} = E\left[\mathbf{y}\mathbf{y}^T\right]$ and $\mathbf{R}_{nn} = E\left[\mathbf{n}\mathbf{n}^T\right]$ are the $N \times N$ output and $N \times N$ noise correlation matrices. The other two terms $\mathbf{R}_{aa} = E\left[\mathbf{a}\mathbf{a}^T\right]$ and $\mathbf{R}_{ay} = E\left[\mathbf{a}\mathbf{y}^T\right]$ are the $(N + L - 1) \times (N + L - 1)$ input correlation and $(N + L - 1) \times N$ input-output cross-correlation matrices, respectively. Substituting Equation (1) into (6), it follows that

$$\begin{aligned} \text{MSE} &= \mathbf{q}^T\mathbf{R}_{aa}\mathbf{q} + \mathbf{w}^T\left(\mathbf{H}\mathbf{R}_{aa}\mathbf{H}^T + \mathbf{R}_{nn}\right)\mathbf{w} \\ &\quad -2\mathbf{q}^T\mathbf{R}_{aa}\mathbf{H}^T\mathbf{w}. \end{aligned} \quad (7)$$

Therefore, the MSE can be minimized by first taking its gradient with respect to the linear filter $\mathbf{w}$ and setting it equal to zero. By so doing, we obtain

$$\mathbf{w} = \left(\mathbf{H}\mathbf{R}_{aa}\mathbf{H}^T + \mathbf{R}_{nn}\right)^{-1}\mathbf{H}\mathbf{R}_{aa}\mathbf{q} \quad (8)$$

which represents the optimum setting of the linear filter $\mathbf{w}$, with respect to MSE, assuming a predefined DIR $\mathbf{q}$.

The MSE could be reduced further by proper selection of the DIR. In what follows, we consider the choice of the DIR $\mathbf{q}$ which minimizes the MSE. Substituting Equation (8) into (7) and making use of the matrix inversion lemma [6], we get

$$\text{MSE} = \mathbf{q}^T\left(\mathbf{R}_{aa}^{-1} + \mathbf{H}^T\mathbf{R}_{nn}^{-1}\mathbf{H}\right)^{-1}\mathbf{q}. \quad (9)$$

In Equation (9), we have implicitly assumed that $\mathbf{R}_{aa}$ and $\mathbf{R}_{nn}$ are invertible. Our objective here is to choose $\mathbf{q}$ such that the MSE, represented by Equation (9), is minimized. In the optimization process, we impose the following two constraints:

1. All the coefficients of the DIR $\mathbf{q}$ must be set equal to zero except $V + 1$ coefficients in order to limit the complexity of the VA. This can be done by multiplying $\mathbf{q}$ with a $(N + L - 1) \times (N + L - 1)$ diagonal matrix $\mathbf{C}$ whose elements are given by

$$\mathbf{C}_{ij} = \begin{cases} 1 & \text{for } i = j \in \Theta \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

Therefore, Equation (9) can be rewritten as

$$\text{MSE} = \mathbf{q}^T\mathbf{C}\left(\mathbf{R}_{aa}^{-1} + \mathbf{H}^T\mathbf{R}_{nn}^{-1}\mathbf{H}\right)^{-1}\mathbf{C}\,\mathbf{q}. \quad (11)$$

2. The DIR must be of unit energy, that is,

$$\mathbf{q}^T\mathbf{q} = 1 \quad (12)$$

in order to avoid the trivial solution of zero MSE which corresponds, of course, to no transmission through the channel.

To solve the above optimization problem, we use the method of Lagrange multipliers. It is not difficult to show that the optimum $\mathbf{q}$ is the eigenvector corresponding to the minimum eigenvalue of the Hermitian positive-definite matrix $\mathbf{B}$ defined as

$$\mathbf{B} = \mathbf{C}\left(\mathbf{R}_{aa}^{-1} + \mathbf{H}^T\mathbf{R}_{nn}^{-1}\mathbf{H}\right)^{-1}\mathbf{C}. \quad (13)$$

Assuming that the input data and noise component are independent and identically distributed (i.i.d.) sequences with zero-mean and variances 1 and σ^2, respectively, Equations (8) and (13) can be simplified to

$$\mathbf{w} = \left(\mathbf{H}\mathbf{H}^T + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{H}\,\mathbf{q} \quad (14)$$

and

$$\mathbf{B} = \mathbf{C}\left[\mathbf{I}_{N+L-1} + \mathbf{H}^T\mathbf{H}\right]^{-1}\mathbf{C} \quad (15)$$

where $\mathbf{I}_m$ denotes the $(m \times m)$ identity matrix.

The approaches developed by Qureshi and Newhall [4] and Falconer and Magee [5] (see also [6]) may be considered as a special case of the more general MMSE-A approach presented in this section. For this reason, the MSE achieved by the proposed MMSE-A approach is always less than or equal to that of other MMSE approaches. The results obtained in [4, 5, 6] can be easily verified by making the assumption of consecutive nonzero coefficients of the DIR and substituting into the general results obtained for the MMSE-A approach presented in this section.

It should be noted that the optimum solutions for the DIR derived in this section did not address the point of finding the optimum selection for the positions of the nonzero coefficients of the DIR. Of course, we can perform a search over all possible combinations in order to reach the optimum locations, which will result in the least MSE; however, this could be a very lengthy process. On the other hand, simulation results have shown that a good choice for the locations of nonzero coefficients of DIR is in the vicinity of the locations of the highest energy coefficients of the CIR.

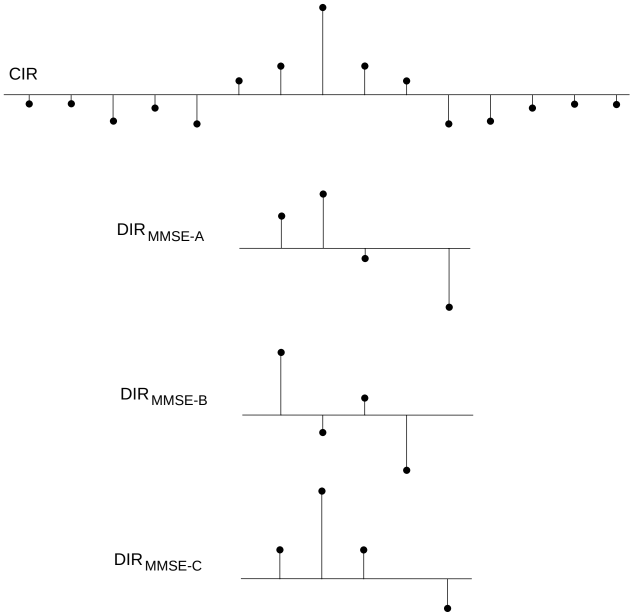

Figure 3: Impulse response of the original channel and the DIRs designed by MMSE-A, MMSE-B, and MMSE-C approaches.

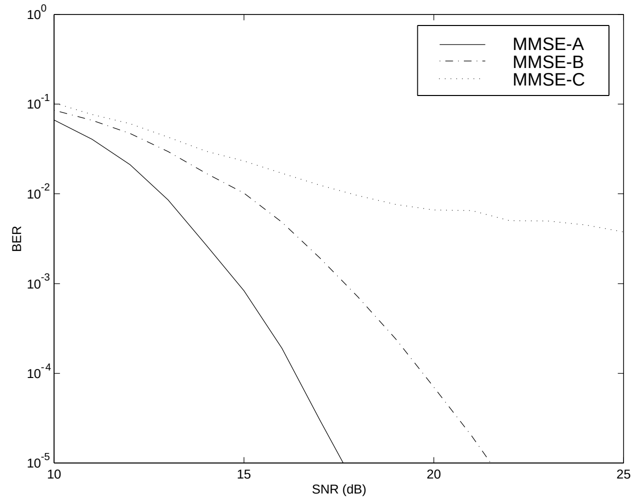

Figure 4: BER performance of MMSE-A, MMSE-B, and MMSE-C approaches.

IV. Performance Evaluation

Performance comparisons among several CLVE design approaches in terms of the BER at different signal-to-noise ratio (SNR) conditions are presented. In particular, three approaches were simulated. These approaches are similar in the sense that they optimize the linear pre-filter to minimize the MSE between the output of the CIR and that of a certain DIR. However, they differ in the choice of the DIR as outlined below:

MMSE-A: The positions of the DIR coefficients are chosen freely without any restrictions. The coefficient values are designed to minimize the MSE.

MMSE-B: This approach restricts the positions of the DIR coefficients to be consecutive. The DIR is chosen to minimize the MSE [5].

MMSE-C: The positions of the DIR coefficients are chosen freely without any restrictions. The coefficient values are assumed equal to the corresponding CIR coefficients [8, 9].

In the simulation, we have considered a 2-PAM data transmitted through channel-B used by Falconer and Magee in [5]. The impulse response of this channel is shown in Figure 3. This channel is characterized by its long impulse response ($L+1=15$) which prevents the direct implementation of the full complexity VA due to the high number of states required (no. of states $= 2^L = 2^{14}$). In order to limit the complexity of the VA, a 31-tap linear pre-filter was used to equalize the received sequence prior to passing it to the VA to shape the long CIR to a shorter DIR consisting of only 4 nonzero coefficients. The VA implementation is based on the knowledge of the equivalent short DIR $\mathbf{q}$ with 4 nonzero taps which means that only 2^3 states are required for VA computations. The tap gains of the linear pre-equalizer were designed such as to minimize the MSE between the output of the linear pre-filter and the output of a predefined DIR. The DIR itself was selected according to the three approaches outlined above. The resulting DIRs for the three approaches at SNR=15dB are shown in Figure 3. It can be seen that the DIR of the MMSE-B approach consists of 4 consecutive coefficients. For the other two approaches, the DIR consists of 4 non-consecutive nonzero coefficients. Note that the MMSE-C approach restricts the values of nonzero coefficients to be exactly equal to the corresponding coefficients of the CIR.

Performance curves for the three approaches are shown in Figure 4. As can be seen from the figure, the MMSE-A approach outperforms the conventional MMSE-B by about 4 dB at BER of 10^{-5} while the MMSE-C approach fails to produce an acceptable performance for the channel considered. The reason for the performance gain of the proposed algorithm over the other approaches may due to the fact that the MMSE-B and MMSE-C approaches fix either the positions or values of DIR while the MMSE-A approach allows these two design variables to take their optimal values.

V. Conclusion

Recently, reduced complexity Viterbi algorithms for use as sequence estimators for linear ISI channels with coarsely located coefficients have been developed [7, 10]. In contrast to classical VA whose computational complexity increases exponentially with the channel length, these algorithms have a computational complexity primarily governed by the number of nonzero coefficients of CIR. In this paper, a new approach for designing the DIR of CLVEs has been presented. This approach is basically based on optimizing the linear pre-filter so that the DIR consists of only few nonzero coefficients that makes it well suited for use with the previously mentioned computationally efficient Viterbi algorithms. The main feature of proposed scheme is that the positions and values of DIR can be optimized; hence, better performance can be achieved. In addition, the methods of [4] and [5] which are based on the use of consecutive DIR coefficients are special cases of the formulation presented.

VI. References

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