

COMBINED LINEAR-DECISION FEEDBACK SEQUENCE ESTIMATION: AN IMPROVED SYSTEM DESIGN

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ABSTRACT

Decision feedback sequence estimation (DFSE) is a reduced state alternative to maximum likelihood sequence estimation (MLSE). In this paper, we examine the performance of a system composed of a linear pre-filter in conjunction with a DFSE. In addition, we present a new optimization approach for the linear pre-filter. This approach has an advantage over MSE-based approaches in that it takes into account the effects of noise correlation and error distance variations. The performance of the proposed optimization approach is evaluated and compared with several existing optimization techniques.

1. INTRODUCTION

It is well known that maximum likelihood sequence estimation (MLSE) is the optimum method for the detection of data in the presence of intersymbol interference (ISI) and additive white Gaussian noise (AWGN) [1]. Viterbi algorithm (VA) provides an efficient way of performing MLSE recursively when the length of the channel impulse response (CIR) is finite. However, the practical implementation of VA for MLSE is limited by the high computational complexity, which grows exponentially with the length of CIR.

Decision feedback sequence estimation (DFSE) is a popular technique for reducing the complexity of the VA, proposed by Duel-Hallen et al. [2] and Eyuboglu et al. [3]. The DFSE technique is based on truncating the CIR in order to reduce the trellis searched by the VA. However, DFSE feeds back previous decisions in order to reduce the effects of residual ISI resulting from the truncation process. With this arrangement, the DFSE algorithm combines structures of the VA and the DFE.

The performance of DFSE highly depends on the structure of the channel under consideration. Specifically, DFSE is well suited for channels having most of their energy concentrated at the beginning of the CIR. For channels that do not satisfy this condition, such as nonminimum-phase channels, the performance of DFSE may become poor [2]. Several approaches have been attempted to improve the performance of DF-based equalization techniques under nonminimum-phase channel conditions [4, 5]. One of the most popular approaches is to process the received signal by a linear pre-filter prior to DFSE as shown in Figure 1. The aim of the pre-filter is to shape the original CIR into a one that has more energy concentration at the beginning of the impulse response to make it more suitable for DFSE. This

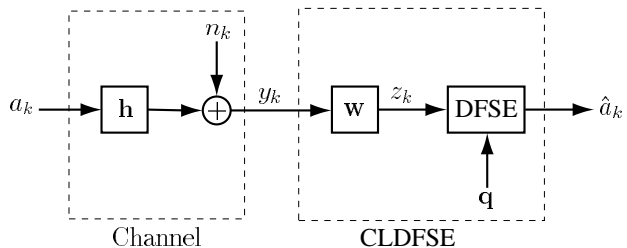


Figure 1: Combined linear-DFSE (CLDFSE).

technique will be termed combined linear-DFSE (CLDFSE). In designing CLDFSEs, it is essential to optimize the linear pre-filter in a way that minimizes the BER of the overall system. For example, in [4], the linear pre-filter was optimized according to the MSE criterion. Although this criterion is simple to optimize, it does not take into account the effects of noise correlations nor does it take error distance variations caused by the pre-filtering process which may cause further performance degradation.

In this paper, we propose a new optimization method that is much linked to system BER performance. This optimization method has an advantage over MSE-based optimization methods in that it takes into account the effects of noise correlation and error distance variations. In the following section, the model of CLDFSE is described. Second, an upper bound on the error performance of CLDFSE is derived and then used in the optimization of the linear pre-filter. Simulation results and conclusions are presented in Section 5.

2. MODEL OF CLDFSE

The CLDFSE is modeled as shown in Figure 1. The transmitted sequence $\{a_k\}$ is an i.i.d. zero-mean M -ary sequence selected from the set $\mathcal{A} = \{\pm\delta/2, \pm3\delta/2, \pm5\delta/2, \dots, \pm(M-1)\delta/2\}$, where δ is a real positive constant. The transmitted sequence passes through a channel modeled by a linear transversal filter \mathbf{h} with length $L+1$. It is also assumed that the channel corrupts the received signal with an AWGN n_k of variance σ and, hence, the received signal can be represented in vector form by

$$\mathbf{y} = \mathbf{a} * \mathbf{h} + \mathbf{n} \quad (1)$$

where \mathbf{y} , \mathbf{a} , and \mathbf{n} are the vector forms of y_k , a_k , and n_k , respectively. The received signal y_k is, then, processed using a linear FIR filter \mathbf{w} with length $N+1$ to produce the output of the pre-filter \mathbf{z} .

After pre-processing the received signal, the filtered signal is passed to DFSE to produce the estimates \hat{a}_k of the transmitted sequence a_k . The DFSE is performed under the assumption that the noise is white and that \mathbf{q} given by

$$\mathbf{q} = [q_0, q_1, \dots, q_\vartheta]. \quad (2)$$

is the overall impulse response of the system. The parameter $\vartheta + 1$ is the length of the desired impulse response (DIR). In order to make the model more general, we also introduce a delay parameter Δ between the output of the DIR and the output of the pre-filter. This delay has been shown to improve the performance of CLDFSEs [4].

3. UPPER BOUND ON THE PERFORMANCE OF CLDFSE

CLDFSE can be analyzed in the same way as the normal DFSE, i.e. by defining three error subevents (E_1 , E_2 , and E_3) and analyzing their probability of occurrence. The reader is referred to [2] for the definition of these subevents. In fact, most of the analysis that has been presented in the literature for DFSE is applicable to the case of CLDFSE. The only difference between the two schemes is in the evaluation of the probability of subevent E_3 . E_3 is the event at which the sum of the branch metrics of the estimated path exceeds the sum of the branch metrics of the correct path. In DFSE, the noise component corrupting the input signal was assumed to be white and Gaussian, which simplifies the analysis of DFSE. On the other hand, when analyzing CLDFSE, one must consider the effect of two new factors, which are: noise correlations due to pre-filtering; and residual ISI resulting from truncation of the overall impulse response \mathbf{g} to one with a shorter extent \mathbf{q} . Such factors complicate the analysis of CLDFSE as compared to DFSE. In what follows we derive a bound on the probability of subevent E_3 .

The output of the pre-filter can be written as

$$\mathbf{z} = \mathbf{a} * \mathbf{q} + \zeta \quad (3)$$

where ζ represents the undesired term while $\mathbf{a} * \mathbf{q}$ represents the desired one. The undesired term is composed of two components; the colored Gaussian noise χ given by $\chi = \mathbf{w} * \mathbf{n}$, and the untreated residual ISI ψ given by $\psi = \mathbf{b} * \mathbf{a}$ where the coefficients of \mathbf{b} are given by $b_k = g_k$ for $k \notin \{\Delta, \Delta + 1, \dots, \Delta + \vartheta\}$ and 0 otherwise, where the term g_k denotes the k th element of \mathbf{g} . It can be shown that the probability of subevent E_3 is given by [6]

$$\Pr(E_3) = \Pr(\zeta_e \geq \frac{1}{2}d_e) \quad (4)$$

where ζ_e is the projection of undesired term on the direction of the error signal \mathbf{u} , whose elements are u_0 , u_1, \dots , and $u_{\lambda-1}$, where λ is the length of the error event. The elements of the vector \mathbf{u} can be determined using

$$u_k = \sum_{i=0}^{\lambda-1} \epsilon_i q_{k-i} \quad \text{for } 0 \leq k \leq \lambda - 1 \quad (5)$$

and zero otherwise. Note that the length of the error signal \mathbf{u} of an error event e is nothing but the Euclidean distance of that error event d_e , i.e., we can define d_e as $d_e = \|\mathbf{u}\|$. The unit vector in the direction of the error signal is denoted by \mathbf{v} and is given by $\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|}$. The projections of the correlated noise χ and the residual ISI ψ in the direction of an error event e are denoted by χ_e and ψ_e , respectively, and are given by

$$\chi_e = \mathbf{v} * \chi \quad (6)$$

$$\psi_e = \mathbf{v} * \psi \quad (7)$$

Since the noise component \mathbf{n} introduced by the channel is assumed to be white and Gaussian, the corresponding noise χ at the output of the linear pre-filter is also Gaussian but it is no longer white which means that its variance is not the same in all directions. It can be shown that the correlated noise projection χ_e has zero mean while its variance is given by

$$\sigma_{\chi_e}^2 = \sigma^2 \mathbf{v} \mathbf{C}_{\mathbf{w}\mathbf{w}} \mathbf{v}^T \quad (8)$$

It can also be shown that the residual ISI projection ψ_e has zero mean (since the transmitted sequence is assumed to have zero mean) while its variance is given by

$$\sigma_{\psi_e}^2 = \sigma_a^2 \mathbf{v} \mathbf{C}_{\mathbf{b}\mathbf{b}} \mathbf{v}^T \quad (9)$$

where σ_a^2 is the variance of the transmitted sequence. However, the residual ISI projection ψ_e is non-Gaussian and its distribution is unknown (but we know that its probability density function (pdf) has even symmetry since the input sequence is drawn from an even symmetric set). In the last two equations, the matrices $\mathbf{C}_{\mathbf{w}\mathbf{w}}$ and $\mathbf{C}_{\mathbf{b}\mathbf{b}}$ are λ square matrices and their elements are given by

$$C_{\mathbf{w}\mathbf{w}}(i, j) = \sum_i \sum_j w_i w_{i-j} \quad (10)$$

$$C_{\mathbf{b}\mathbf{b}}(i, j) = \sum_i \sum_j b_i b_{i-j} \quad (11)$$

respectively.

Having characterized the distortion components, we can now proceed with the derivation of the bound. To find the probability of subevent E_3 , which is our main task, we need to evaluate the probability in Equation 4, i.e. we need to evaluate

$$\Pr(E_3) = \Pr(\chi_e + \psi_e \geq \frac{d_e}{2}). \quad (12)$$

It is clear that the pdf of χ_e and ψ_e is even and, hence, the one-sided tail probability of Equation 12 can be replaced by the two-sided tail probability given by

$$\Pr(E_3) = \frac{1}{2} \Pr\left(|\chi_e + \psi_e| \geq \frac{d_e}{2}\right) \quad (13)$$

Using the Chebyshev inequality, Chebyshev bound can be derived from 13 and is given by

$$\Pr(E_3) = \frac{1}{2} \Pr(|\zeta_e| \geq \frac{d_e}{2}) \leq \frac{2\sigma_{\zeta_e}^2}{d_e^2} \quad (14)$$

which clearly shows how the probability of the system is critically affected by two factors; the squared error distance d_e^2 and the variance of the noise projection $\sigma_{\zeta_e}^2$ where $\sigma_{\zeta_e}^2$ is given by

$$\sigma_{\zeta_e}^2 = \sigma_{\chi_e}^2 + \sigma_{\psi_e}^2. \quad (15)$$

4. OPTIMIZATION OF CLDFSE

Most of the design approaches related to the design of pre-filters for CLDFSE are based on the classical MSE criterion. Such a criterion is simple to implement and analytically tractable. However, it does not consider all the factors affecting the performance of CLDFSE. It neglects the fact that the noise is no longer white and, thus, its variance can not be considered the same in all directions (as can be seen from Equations 8 and 9). Moreover, optimization methods based on MSE completely neglect the effect of pre-filtering on the distances of error events d_e which has a critical influence on the overall performance of CLDFSE. Neglecting these two important factors may result in designs that have unsatisfactory performance while being optimal only in the MSE sense. Therefore, it is necessary to develop other optimization criteria that are more related to the performance of CLDFSE. This can be done by incorporating most factors affecting the performance of CLDFSE into the structure of the developed criterion.

By virtue of the Chebyshev bound presented in Equation 14, we propose a new optimization method that is based on the following cost function

$$\Phi \triangleq \sup_{e \in \mathcal{E}'} \frac{\sigma_{\zeta_e}^2}{d_e^2} \quad (16)$$

The term \mathcal{E}' denotes the subset containing the most critical error events.

The cost function Φ as defined above is nonlinear, hence nonlinear technique is needed for its optimization. In the optimization process, we may search directly for the optimum pre-filter \mathbf{w} that minimizes Φ . However, it can be shown that, for a given DIR \mathbf{q} , MSE-optimized pre-filters are also optimum with respect to Φ . Therefore, we can design the pre-filter \mathbf{w} using classical MSE-approaches while letting nonlinear optimization techniques search for the optimum setting of the DIR \mathbf{q} which minimizes the cost function Φ .

5. SIMULATION RESULTS

In the simulation, we have considered a system transmitting a BPSK i.i.d. sequence through channel with impulse response shown in Figure 2. At the receiver, the equalizer is based on a 4-state DFSE, where each state considers only the first 3-coefficients of the channel impulse response while

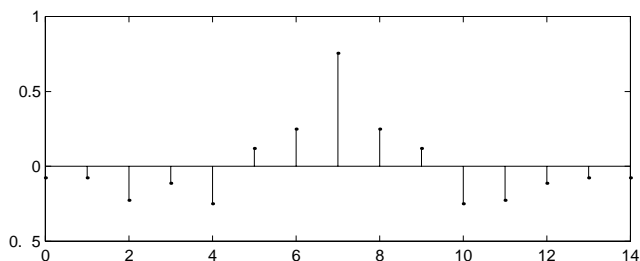


Figure 2: Channel impulse response.

the rest of coefficients are fed back to eliminate any residual ISI. In addition to the original DFSE we have simulated four techniques which are used for improving the performance of DFSE. These techniques are outlined below:

CLDFSE-MSE A 50-tap FIR pre-filter is used before DFSE.

The filter is designed according to the MSE criterion [4].

CLDFSE-NEW A 50-tap FIR pre-filter is used before DFSE.

The filter is designed by minimizing Φ defined in (16).

CLDFSE-Allpass An IIR allpass pre-filter is used before DFSE. The aim of the filter is to convert nonminimum phase channels into minimum phase ones [4].

FBSSD This technique is not based on any pre-filtering.

Instead, it cascades two DFSE algorithms, one is running in the reverse-time while the other is running in the forward-time [5].

Note that since the FBSSD technique implements two passes of the DFSE algorithms, its complexity is twice that of other techniques when the same number of states is used. For this reason, we have reduced the number of states of the FBSSD technique to only 2-states (which is half that of other techniques) in order to insure fair comparison with other techniques.

The simulation results of all techniques considered are shown in Figure 3. The first thing to notice is that the DFSE approach (without any pre-filtering) fails because the channel is a nonminimum phase channel having most of its energy concentrated in the middle of the impulse response and, therefore, unsuitable for DFSE. All other techniques for improving DFSE failed to achieve acceptable BER performance as can be seen from Figure 3 except the CLDFSE-NEW and the CLDFSE-Allpass techniques with an advantage of about 1.25 dB for the CLDFSE-NEW technique over the CLDFSE-Allpass technique.

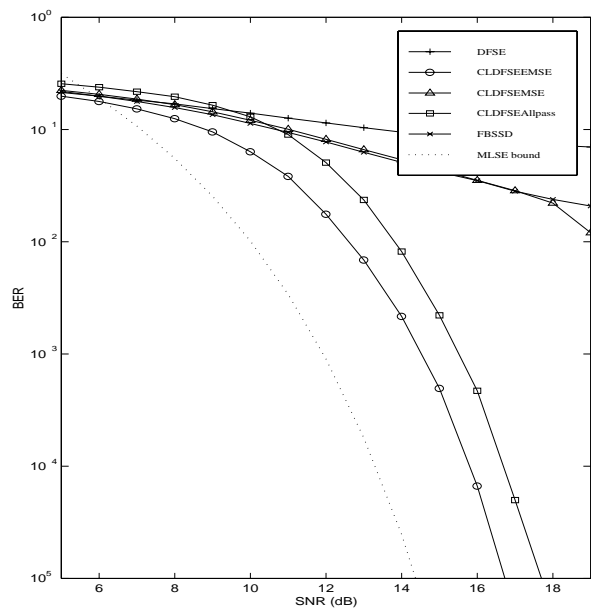


Figure 3: BER performance of all DFSE-based techniques.

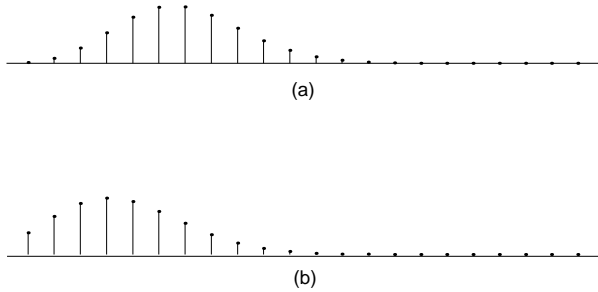


Figure 4: (a) Channel impulse response of an arbitrary minimum phase channel and (b) the overall impulse response after shaping.

The amount of improvement of the proposed CLDFSE-NEW technique over that of other techniques depends on the channel characteristics. Although the performance difference between the CLDFSE-NEW and the CLDFSE-Allpass techniques for the simulated channel may be considered small, there may be other cases in which this difference becomes considerable. As an example, consider the channel whose impulse response is shown in Figure 4a. This channel is a minimum phase channel despite the fact that most of its energy is not concentrated at the beginning of the impulse response. Since this channel is already minimum phase, the CLDFSE-Allpass approach will not be able to introduce any improvement and, hence, the performance of the CLDFSE-Allpass technique will be equal to that of a regular DFSE. On the other hand, the CLDFSE-NEW technique tries to optimize the FIR pre-filter in a way that concentrates the energy more to the left as can be seen from Figure 4b. As a result, the CLDFSE-NEW technique outperforms the CLDFSE-Allpass technique by about 5 dB at BER of 10^{-5} for this channel shown in Figure 5.

6. REFERENCES

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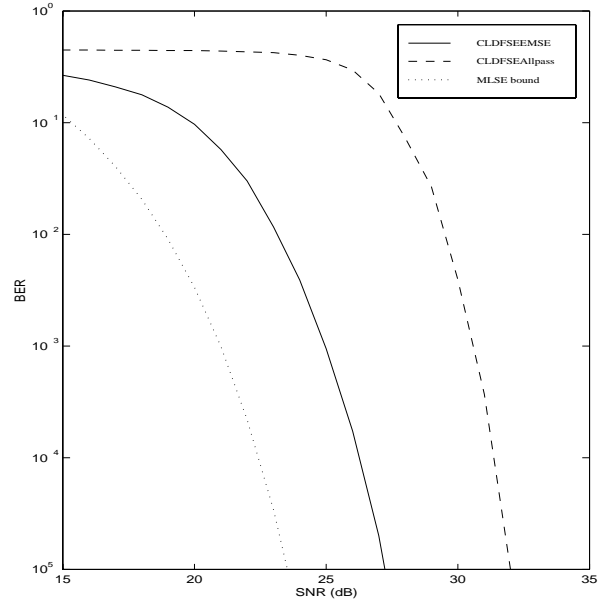


Figure 5: BER performance of the CLDFSE-NEW and CLDFSE-Allpass techniques for the arbitrary minimum phase channel.

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