Name of Student ____________________ **Univ. Number** __________________

King Saud University

College of Engineering

Mechanical Engineering Department

First Semester Final Examination (1428/1429)

Subject: TURBOMACHINES-ME 485

QUESTION # (1) **Marks (9)**

Draw the following velocity diagrams of turbomachines showing the directions of the velocity vectors and their angles.

1. Velocity diagram at the exit of a backward bladed impeller of a centrifugal pump showing the effect of slip.
2. Velocity diagram for the rotor of an axial flow turbine with flow leaving the rotor in a purely axial direction.
3. Velocity diagram for the rotor and the stator of an axial flow compressor.
4. Enthalpy change diagram through an axial compressor stage.
5. h-s diagram for an impulse turbine.
6. h-s diagram for a 100% reaction axial turbine.

QUESTION # (2) **Marks (10)**

1. Define total to total and total to static efficiency of a gas turbine stage.
2. The inlet and exit conditions of a gas turbine are
    - Inlet total pressure =12 bar;
    - Exit total pressure = 3 bar;
    - Inlet total temperature = 1000 K,
    - Exit total temperature = 720 K,
    - Absolute velocity at the exit = 150 m/s.
    - C_p=1.148, k=1.33

Determine the following

1. Work done per unit mass by the turbine,
2. Total to total efficiency of the turbine,
3. Total to static efficiency of the turbine,
4. Polytropic efficiency of the turbine stage.

QUESTION # (3) **Marks (9)**

The head characteristics of a centrifugal water pump running at 2000 rpm is described as H_{pump}= 35 – 62250 Q^2, where H_{pump} and Q are measured in meters and m^3/s respectively. The system consists of a piping system having internal diameter of 100 mm, total length of 100 meters and having a pipe friction coefficient of 0.025. The elevation difference between the inlet of suction side of the pump and the exit of delivery side of the pump is 25 meters. Calculate,

- Pipe system Characteristics;
- Operating point;
- Specific speed of the pump;
- Speed of rotation (rpm) at which the flow rate would decrease by 25 percent;
- Pump head at that speed.
- Head characteristics of pump at new speed.

QUESTION # (4) **Marks (12)**

A centrifugal compressor having an impeller with radially tipped blades compresses air from a stagnation state of 100 kPa and 288 K. The following data is available for the compressor:

Stagnation pressure at the exit of impeller P_{02} =300 kPa,
Radial velocity at the exit of the impeller C_{r2}= 30 m/s,
Slip factor (Stanitz) =0.9
Total to total efficiency of impeller= 0.9,
Flow area at the exit of the impeller A_2=0.09 m^2.

Assuming zero inlet swirl draw velocity triangle at the exit of impeller and calculate

(1)The number of impeller blades
(2) The blade tip speed.
(3) The specific work input.
(4) The stagnation temperature at the exit of the impeller.
(5) The absolute velocity at the exit of the impeller.
(6) The Mach number at the exit of the impeller.
(7) The mass flow rate through the compressor.
(8) Absolute flow angle and relative flow angle at impeller exit.

QUESTION # (5) **Marks (10)**

The stator blades of a normal stage of a gas turbine has an exit flow angle of 72^0C and produce an exit velocity 610 m/s. The rotor hub and tip diameters are 500 mm and 700 mm respectively. The rotor runs at 10000 rpm. The static conditions at the rotor inlet are 330 kPa and 340K. The fluid leaves the stage in a pure axial direction. Assuming similar condition along the height of the rotor, R=0.287 kJ/kg-K and C_p= 1.014 kJ/kg-K, calculate

(i) The mass flow rate through the turbine. (ii) The flow coefficient.
(iii) The relative flow angles at the inlet and exit of the rotor. (iv)The stage reaction
(v) The stage loading coefficient. (vi)The power produced by the stage

Some Formulae

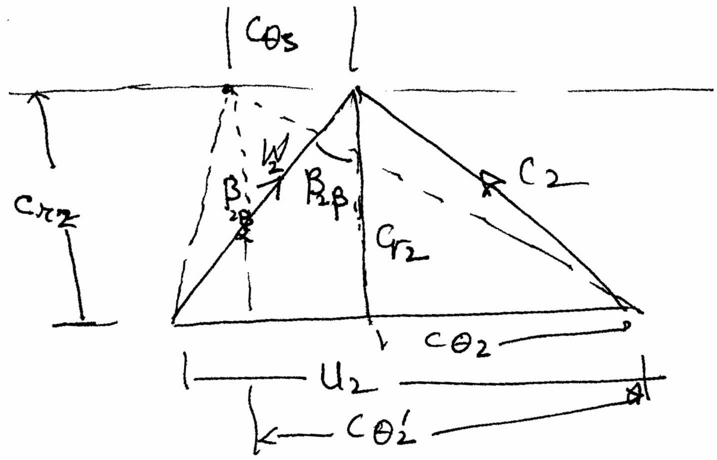
$\Psi = \varphi(\tan\beta_2 - \tan\beta_3)$	$^0R = -\frac{\varphi}{2}(\tan\beta_2 + \tan\beta_3)$
$\Psi = \varphi(\tan\alpha_2 - \tan\alpha_3)$	$^0R = 1 - \frac{\phi}{2}(\tan\alpha_2 + \tan\alpha_3)$
$\frac{k-1}{k} * \eta_p = \frac{n-1}{n}$	$^0R = \frac{1}{2} - \frac{\phi}{2}(\tan\alpha_2 + \tan\alpha_3)$

1428/1429

FIRST Semester Final

Question 1

i

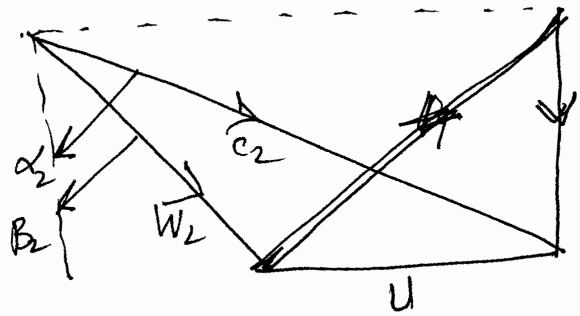


$$\sigma = \frac{C_{\theta 2}}{C_{\theta 2}'}$$

$$\sigma = \frac{C_{\theta 2} - C_{\theta 3}}{C_{\theta 2}'}$$

$$\sigma = 1 - \frac{C_{\theta 3}}{C_{\theta 2}'}$$

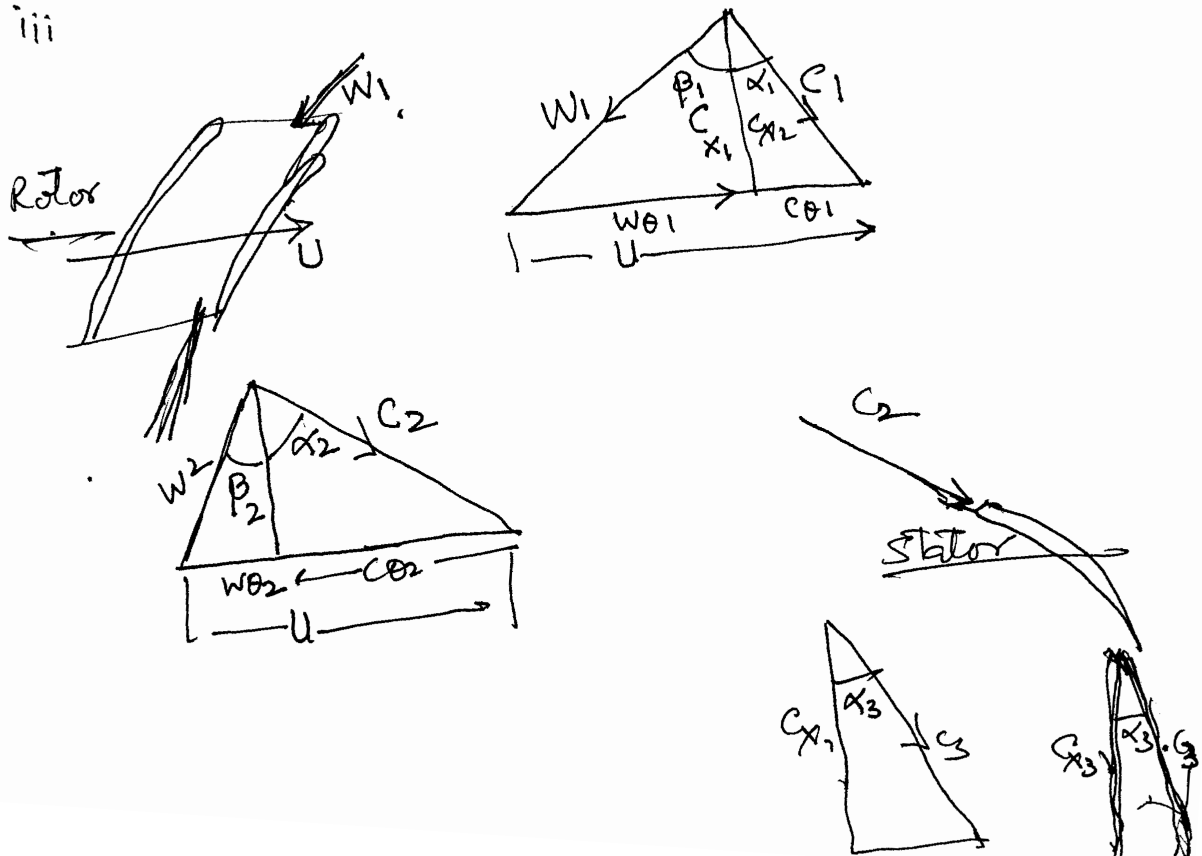
ii



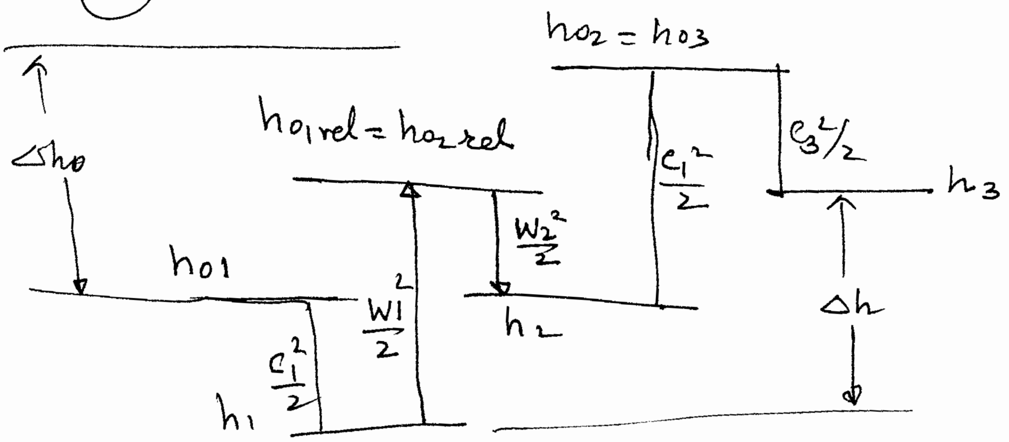
$\alpha_1 = \alpha_3 = 0$

$C_{x_2} = C_1 = C_3$

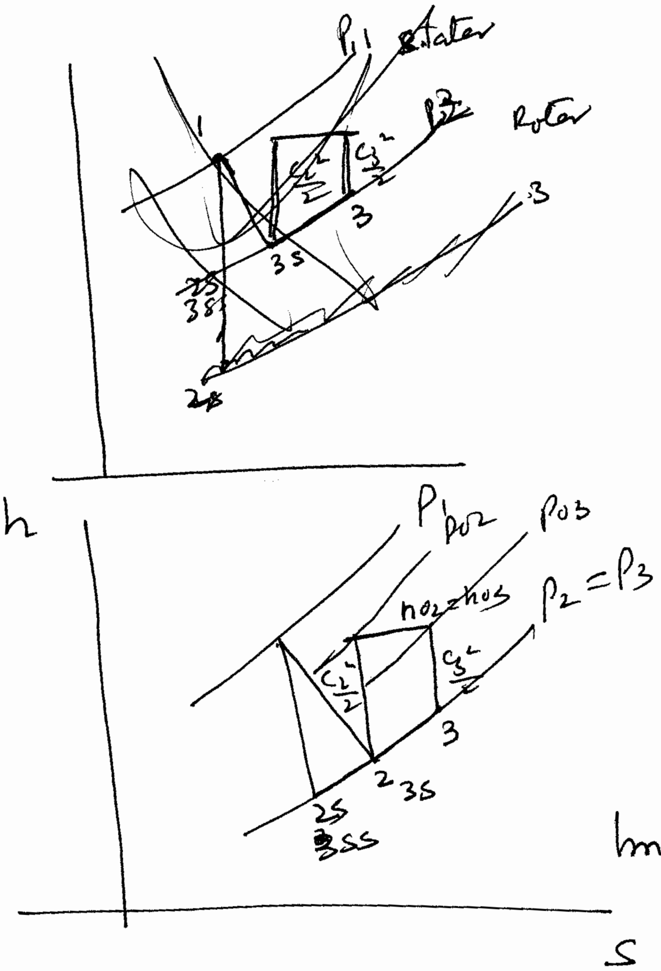
iii



(IV)

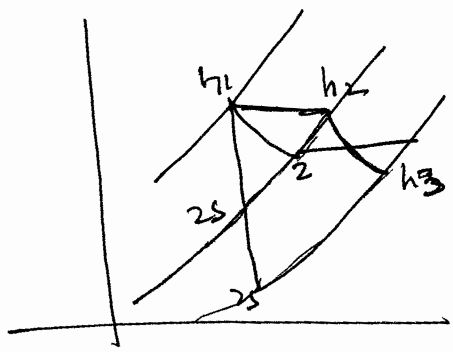


(V)



Impulse turbine

(VI)



$$\frac{h_2 - h_3}{h_1 - h_3} = 1$$

$h_1 = h_2$

100% Reaction

Q2

$$\Delta W_t = \Delta h_0 = C_p (T_{01} - T_{02})$$

$$= 1.148 (1000 - 720) = 321.44 \text{ kJ}$$

$$\eta_{tt} = \frac{\Delta W}{\Delta W_s} = \frac{C_p (T_{01} - T_{02})}{C_p (T_{01} - T_{02s})}$$

$$\eta_{tt} = \frac{\Delta W_t}{C_p T_{01} \left(1 - \frac{T_{02s}}{T_{01}}\right)} = \frac{\Delta W_t}{C_p T_{01} \left(1 - \left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}}\right)} = \frac{321.44}{1.049 \times 1000 \left[1 - \left(\frac{300}{1000}\right)\right.}$$

$$\eta_{tt} = 96.16\%$$

$$\eta_{ts} = \frac{h_{01} - h_{02}}{h_{01} - h_{2s}} = \frac{C_p (T_{01} - T_{02})}{C_p (T_{01} - T_{2s})} = \frac{(T_{01} - T_{02}) C_p}{C_p \left(T_{01} - \left(T_{02s} - \frac{C_2^2}{2C_p}\right)\right)}$$

$$= \frac{(T_{01} - T_{02}) C_p}{C_p \left(T_{01} - T_{02s} - \frac{C_2^2}{2C_p}\right)} = \frac{(T_{01} - T_{02}) C_p}{C_p T_{01} \left(1 - \frac{T_{2s}}{T_{01}} - \frac{C_2^2}{2 C_p T_{01}}\right)}$$

$$\frac{\Delta W_t}{C_p T_{01} \left[1 - \left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}} + \frac{C_2^2}{2 C_p T_{01}}\right]} = \frac{321.44}{1.005 \times 1000 \left(1 + \left(\frac{300}{1200}\right)^{0.248} + \frac{150^2}{2 \times 1005 \times 10}\right.}$$

$$\eta_{ts} = \frac{T_{01} - T_{02}}{T_{01} - T_{02s} + \frac{C_2^2}{2C_p}} = \frac{1000 - 720}{1000 - 709.1 + \frac{C_2^2}{2C_p}} \quad 291.$$

$$\frac{T_{02s}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}} = 0.25^{0.248} \qquad T_{2s} = 709.1 \text{ K}$$

$$\eta_{ts} = \frac{280}{290.93 + \frac{150^2}{2 \times 1333}} = 93.53\%$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{n-1}{n}} \qquad (0.25)^{\frac{n-1}{n}} = 0.72$$

$$\frac{n-1}{n} \log 0.25 = \log 0.72$$

$$\frac{n-1}{n} = \log \frac{72}{25} = ~~0.4594~~ \; 0.2359$$

$$~~1 - \frac{1}{n} = 0.4594 \quad \frac{1}{n} = 0.54 \quad n =~~$$

$$\eta_{p2} \quad \frac{n-1}{n} \Big/ \frac{k-1}{k} = \frac{0.2359}{0.248} = 0.95 \qquad \eta_p = 95\%$$

$\underline{Q_3}$

$H_{pump} = 35 - 62250 Q^2$

$N = 2000$ rpm

$D_p = 100$ mm

$L_p = 100$ m

$f = 0.025$

$H_s = 25$ m.

$$H_{pipe} = H_s + \frac{8 f L Q^2}{\pi^2 g D^5}$$

$$\underline{H_{pipe} = 25 + 20677.7 Q^2}$$

O/p

$H_{pipe} = H_{pump}$

$35 - 62250 Q^2 = 25 + 20677.7 Q^2$

$82927.7 Q^2 = 10$

$$\underline{Q = 0.011 \text{ m}^3/\text{s}}$$

$H = 25 + 20677.7 \times Q_{op}^2$

$$\underline{H = 27.4935 \text{ m}}$$

$$N_s = \frac{N \sqrt{Q}}{(gH)^{3/4}} = \frac{2000 \sqrt{0.011}}{(9.81 \times 27.49)^{0.75}}$$

$= 3.15$

$Q_2 = 0.75 Q_1 = 0.00825 \text{ m}^3/\text{s}$

$Q_1 = 1.33 Q_2$,

$\phi_1 = \phi_2$

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \qquad \underline{N_2 = \frac{Q_2}{Q_1} N_1 = 1500 \text{ rpm}}$$

$$\psi_1 = \psi_2$$

$$\frac{H_1}{N_1^2 \cancel{D_1^2}} = \frac{H_2}{N_2^2 \cancel{D_2^2}}$$

$$H_2 = \frac{N_2^2}{N_1^2} H_1 = \underline{15.465\ m}$$

$$H_1 = 1.778\ H_2$$

$$H_{pump} = H_1 = 35 - 62250 Q_1^2$$

$$1.778 H_2 = 35 - 62250\ (1.33)^2 Q_2^2$$

$$H_2 = 19.685 - 61931.4\ Q_2^2$$

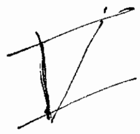
$$\boxed{H = 19.685 - 61931.4\ Q^2}$$

Q4

$P_{01} = 100\ kPa \qquad T_{01} = 288K$

Stanitz formula

$\sigma_s = 1 - \frac{2}{Z} = 0.9 \qquad Z = 20$ blades.



$$\eta_{tt1} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}$$

$$\Delta W_c = U_2 C_{2\theta} - U_1 C_{1\theta} \qquad C_{1\theta} = 0$$

$$\Delta W_c = U_2 C_{2\theta} = U_2 \sigma C_{2\theta\infty}$$

$$= U_2 \sigma (U_2 + C_{r2} \tan\beta_{B2})$$

$$U_2^2 \sigma \left(1 + \frac{C_{r2}}{U_2} \tan\beta_{2B}\right) \qquad \beta_{2B} = 0$$

$$\Delta W_c = U_2^2 \sigma = h_{02} - h_{01} = h_{02} - h_{01}$$

$$\eta_{tt1} = \frac{h_{02s} - h_{01}}{U_2^2 \sigma} = \frac{C_p (T_{02s} - T_{01})}{U_2^2 \sigma}$$

$$U_2^2 = \frac{T_{01} C_p \left(\frac{T_{02s}}{T_{01}} - 1\right)}{\eta_{tt} \sigma} = \frac{T_{01} C_p \left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}} - 1\right]}{\eta_{tt} \sigma}$$

$$= \frac{288 \times 1005 \left[\left(\frac{300}{100}\right)^{\frac{0.4}{1.4}} - 1\right]}{0.9 \times 0.9} \qquad \frac{k-1}{k} = 0.$$

$$U_2 = 363.2\ m/s$$

$$\Delta W_c = \sigma U_2^2 = 118.724\ kJ/kg$$

$$\Delta W_c = h_{02} - h_{01} = C_p (T_{02} - T_{01})$$

$$T_{02} = \frac{\Delta W_c}{C_p} + T_{01} = 406.13K$$

$$\boxed{T_{02} = 406.13K}$$

~~T_{0X}~~ = ~~T_0~~

$$C_2 = \sqrt{C_{2r}^2 + C_{\theta 2}^2}$$

$C_{2r} = 30$ m/s

$\Delta W_c = U_2 C_{\theta 2}$ $\quad C_{\theta 2} = \frac{\Delta W_c}{U_2}$

$C_{\theta 2} = 326.88$ m/s

$C_2 = 328.26$ m/s

$$M_2 = \frac{C_2}{a_2} = \frac{C_2}{\sqrt{kRT_2}} = ~~328.26~~ =$$

$$T_2 = T_{02} - \frac{C_2^2}{2c_p} = ~~406.18~~$$

$T_2 = 352.52$

$M_2 = 0.872$

$\dot{m} = \rho_2 A_2 C_{2r}$

$$\frac{\rho_2}{\rho_{02}} = \left(\frac{T_2}{T_{02}}\right)^{\frac{1}{k-1}} = \left(\frac{352.52}{406.31}\right)^{\frac{1}{0.44}}$$

$\rho_2 = 0.965 \rho_0$

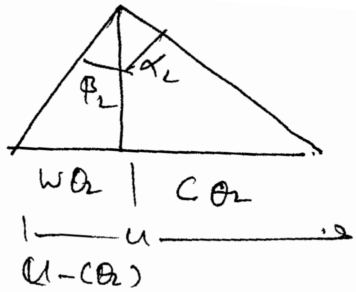
$$\rho_0 = \frac{P_{02}}{RT_{02}} = \frac{300}{0.287 \times 406.13} = 2.574$$

$\rho_2 = 2.484$ $\quad A_2 = 0.09$ $\quad C_{2r} = 30$

$\dot{m} = 6.707$ kg/s

$\alpha_1 = \tan^{-1} \frac{C\theta_2}{C_{r2}}$

$= \tan^{-1} \frac{326.88}{30}$



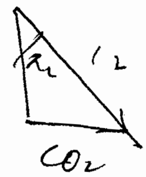
$\alpha_1 = 84.76^\circ$

$\beta_2 = \tan^{-1} \frac{W\theta_2}{C_{r2}} = \tan^{-1}\left(\frac{U_2 - C\theta_2}{C_{r2}}\right)$

$= \tan^{-1}\left(\frac{363.2 - 326.88}{30}\right)$

$\beta_2 = 50.44^\circ$

Q5



$C_{2\theta} = C_2 \sin\alpha_2$

$= 610 \sin 72^\circ = 580.1 \text{ m/s}$

$C_x = C_2 \cos\alpha_2 = 610 \cos 72 = 188.5 \text{ m/s}$

$\rho_2 = \frac{P_2}{RT_2} = \frac{330}{0.287 \times 340} = 3.382 \text{ kg/m}^3$

$A_2 = \frac{\pi}{4}(D_t^2 - D_h^2) = \frac{\pi}{4}(0.7^2 - 0.5^2) = 0.1885 \text{ m}^2$

$\dot{m} = \rho A C_x = 3.382 \times 0.1885 \times 188.5$

$\dot{m} = 120.2 \text{ kg/s}$

~~$\phi = \frac{C_x}{U} = 188.5$~~

$D_m = \frac{D_t + D_h}{2} = 600 \text{ mm}$

$U = \frac{\pi D N}{60} = \frac{\pi \times 0.6 \times 10000}{60}$ 314.2 m/s

$\phi = \frac{C_x}{U} = \frac{188.5}{314.2} = 0.6$

$\phi = 0.6$

$$\tan\beta_2 = \tan\alpha_2 - \frac{1}{\phi}$$

$$= \tan 72^\circ - \frac{1}{0.6}$$

$$\beta_2 = +54.07^\circ$$

$$\tan\beta_3 = \tan\alpha_3 - \frac{1}{\phi} = 0 - \frac{1}{0.6}$$

$$\beta_3 = -59.04^\circ$$

$$R = -\frac{\phi}{2}(\tan\beta_2 + \tan\beta_3) = 0.077$$

$$\psi = \phi(\tan\beta_2 + \tan\beta_3) = 1.8466$$

$$\Delta W_t = \psi U^2 = 182.3 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}\Delta W_t = 21.9 \text{ MW}$$