Numerical Summary Measures

With thanks to Dr. Rino Bellocco and Prof. Marcello Pagano

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Let x represent FEV_1 in liters

$x_1 = 2.30$
$x_2 = 2.15$
$x_3 = 3.50$
$x_4 = 2.60$
$x_5 = 2.75$
$x_6 = 2.82$
$x_7 = 4.05$
$x_8 = 2.25$
$x_9 = 2.68$
$x_{10} = 3.00$
$x_{11} = 4.02$
$x_{12} = 2.85$
$x_{13} = 3.38$

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Measures of central tendency

Numbers: $x_1, x_2, \ldots, x_n$

$$\text{Mean} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

2.3, 2.15, 3.50, 2.60, 2.75, 2.82, 4.05, 2.25, 2.68, 3.00, 4.02, 2.85 (n=13)

$$\text{Sum} = \sum_{i=1}^{n} x_i = 38.5$$

Mean = $\bar{x} = \frac{38.35}{13} = 2.95$

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Approx 4 million singleton births, 1991 :

Variable	Mean
Mother's age	26.4 years
Gestational age	39.15 weeks
Birth weight	3358.6 grams
Weight gain*	30.4 lbs

Among those who died within 1 year:

Survival	49.4 days

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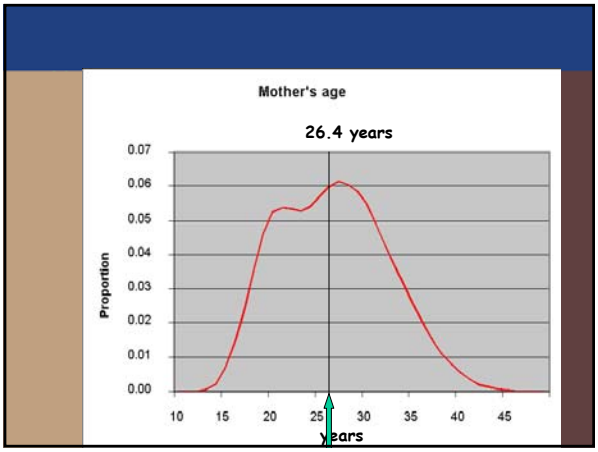

Note what happens when one number, 4.02 say, becomes large, say 40.2 :

2.3, 2.15, 3.50, 2.60, 2.75, 2.82, 4.05, 2.25, 2.68, 3.00, 40.2, 2.85

Mean = $\bar{x}$ = 5.73

(versus 2.95, from before)

Mean is sensitive to every observation, it is not robust.

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Median

More robust, but not sensitive enough.

Definition: At least 50% of the observations are greater than or equal to the *median*, and at least 50% of the observations are less than or equal to the *median*.

2.15, 2.25, 2.30 --- median = 2.25

2.15, 2.25, 2.30, 2.60 ---

median = $\frac{1}{2}$ (2.25 + 2.30) = 2.275

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Singleton births, 1991 :

Variable	Mean	Median
Mom's age (yrs)	26.4	25
Gest. Age (wks)	39.2	39
Birth weight (gms)	3359	3374
Weight gain (lbs)	30.4	30
Survival (days)	49.4	7

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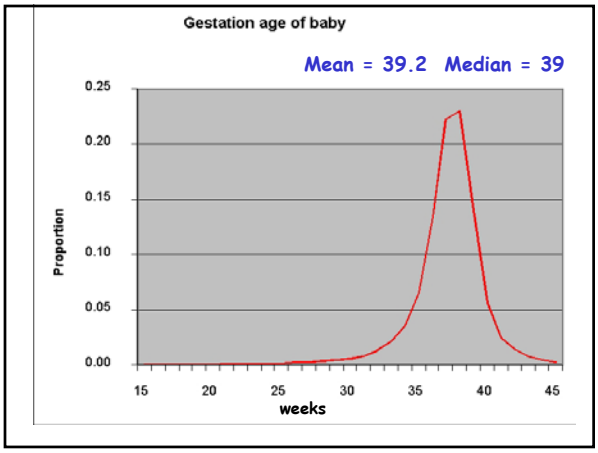
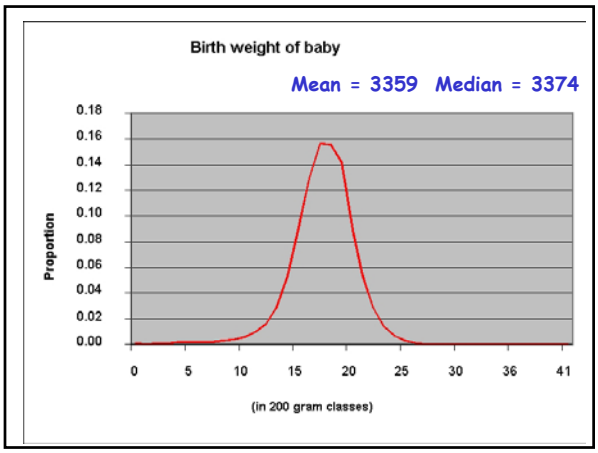
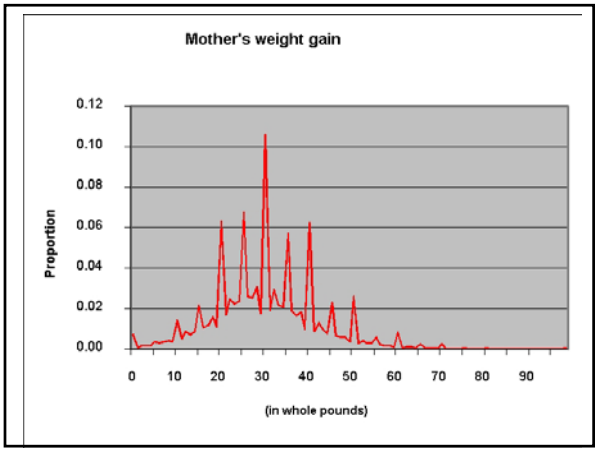
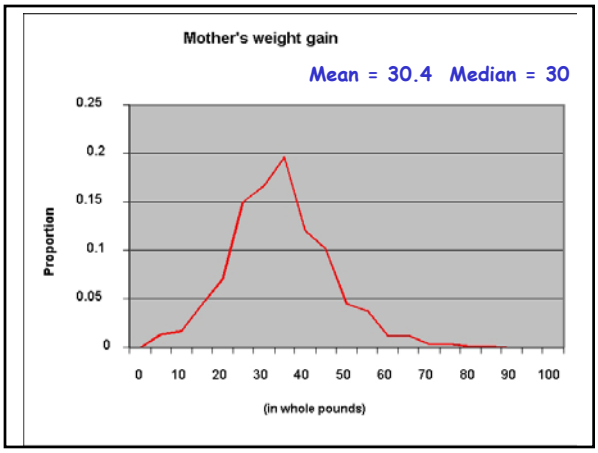



Birth weight of baby
Mean = 3359 Median = 3374
Proportion
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0 5 10 15 20 25 30 36 41
(in 200 gram classes)


Mother's weight gain
Proportion
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0 10 20 30 40 50 60 70 80 90
(in whole pounds)


Mother's weight gain
Mean = 30.4 Median = 30
Proportion
0.25
0.2
0.15
0.1
0.05
0
0 10 20 30 40 50 60 70 80 90 100
(in whole pounds)

Mean or median?

Mean performs best when we have a symmetric distribution with thin tails.

If data are skew, use the median.

Can also report both the mean and median if space permits.

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Spread:

Singleton births, 1991 :

Variable	Min	Max	Range
Mom's age	10	49	39
Gest. Age	17	47	30
Birth weight	227	8164	7937
Weight gain	0	98	98
Survival	0	363	363

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FEV_1	$(x_i - \bar{x})$
2.30	-0.65
2.15	-0.80
3.50	0.55
2.60	-0.35
2.75	-0.20
2.82	-0.13
4.05	1.10
2.25	-0.70
2.68	-0.27
3.00	0.05
4.02	1.07
2.85	-0.10
3.38	0.43

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FEV_1	$(x_i - \bar{x})$
2.30	-0.65
2.15	-0.80
3.50	0.55
2.60	-0.35
2.75	-0.20
2.82	-0.13
4.05	1.10
2.25	-0.70
2.68	-0.27
3.00	0.05
4.02	1.07
2.85	-0.10
3.38	0.43
Total	0.00

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FEV_1	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
2.30	-0.65	0.423
2.15	-0.80	0.640
3.50	0.55	0.303
2.60	-0.35	0.123
2.75	-0.20	0.040
2.82	-0.13	0.169
4.05	1.10	1.210
2.25	-0.70	0.490
2.68	-0.27	0.073
3.00	0.05	0.003
4.02	1.07	1.145
2.85	-0.10	0.010
3.38	0.43	0.185
Total	0.00	4.66

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$$Variance = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

e.g.

$$= \frac{4.66}{12} = 0.39 \text{ liters}^2$$

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Standard deviation = $+\sqrt{\text{Variance}}$

e.g.

$= \sqrt{0.39}$

$= 0.62$ liters

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Singleton births, 1991 :

Variable	Mean	Std dev
Mom's age (yrs)	26.4	5.84
Gest. Age (wks)	39.2	2.61
Birth weight (gms)	3359	227
Weight gain (lbs)	30.4	12.13
Survival (days)	49.4	76.1

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Empirical Rule:

If dealing with a *unimodal* and *symmetric* distribution, then

Mean ± 1 sd covers approx 67% obs.

Mean ± 2 sd covers approx 95% obs

Mean ± 3 sd covers approx all obs

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Mother's age: mean = 26.4 yrs
s.d. = 5.84 yrs

Table of $\bar{x}$ ± k s.d.s

k	left limit	right limit	Emp.
1	20.56	32.24	67%
2	14.72	38.08	95%
3	8.88	43.92	all

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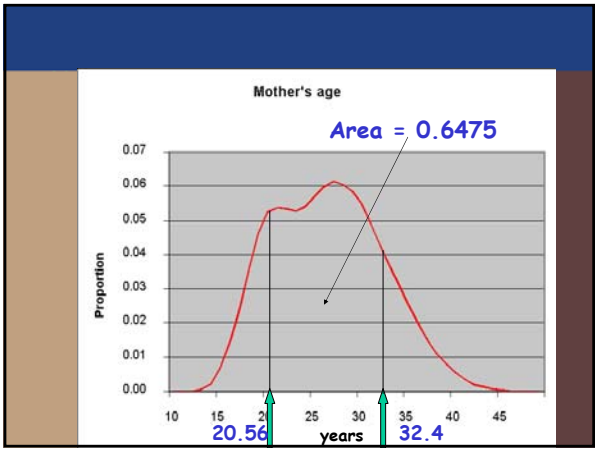

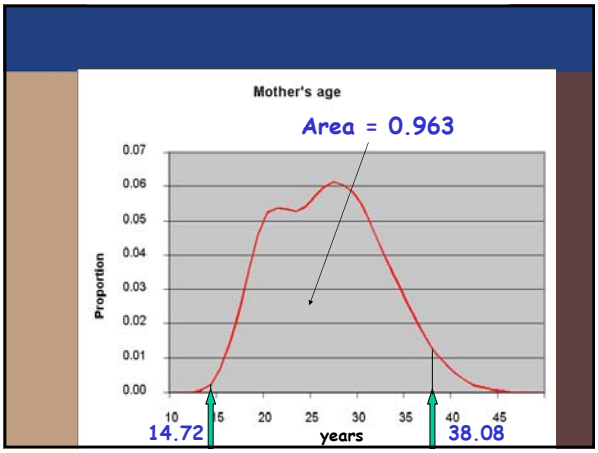

Mother's age: mean = 26.4 yrs
s.d. = 5.84 yrs

Table of $\bar{x}$ ± k s.d.s

k	left limit	right limit	Emp.	Actual
1	20.56	32.24	67%	64.75%
2	14.72	38.08	95%	96.3%
3	8.88	43.92	all	99.89%

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InterQuantile Range

IQR= Q-q

Q = 75th percentile

q = 25th percentile

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Table 1. Baseline Descriptive Statistics*

Variable	CHA Cohort (Aged 18-39 y)	PG Cohort (Aged 25-39 y)	MRFIT Cohort (Aged 35-39 y)
No. of men	11 017	1266	69 205
Age, y	29.7 (5.5)	32.5 (3.9)	37.0 (1.4)
Serum cholesterol level, mg/dL†	189.9 (36.2)	209.0 (38.7)	205.7 (38.7)
Systolic blood pressure, mm Hg	134.6 (15.4)	125.3 (14.6)	126.4 (13.3)
Diastolic blood pressure, mm Hg	78.2 (10.5)	77.8 (8.8)	81.7 (10.1)
Cigarette smokers, %	47.4	65.6	39.6
Cigarettes/d, No. (smokers)	21.2 (10.5)	21.7 (7.5)	25.4 (13.1)
Cigarettes/d, No. (all)	10.0 (12.8)	14.3 (12.0)	10.0 (14.9)
African American, %	8.1	8.0	6.7
Education, y	13.8 (2.6)	11.8 (2.4)	14.2 (0.8)
ECG abnormality			
Major, %	3.3	1.8‡	NA§
Minor, %	5.5	4.1‡	NA§
Body mass index, kg/m²	26.0 (3.6)	26.2 (3.4)	NA§

*CHA indicates Chicago Heart Association Detection Project in Industry; PG, Peoples Gas Company Study; MRFIT, Multiple Risk Factor Intervention Trial; ECG, electrocardiogram; and NA, not available. Data are presented as mean (SD) unless otherwise indicated.
†To convert mg/dL to mmol/L, multiply by 0.0259.
‡Data are not available for 428 men.
§Data were not collected on these variables at first screening for recruitment for MRFIT.