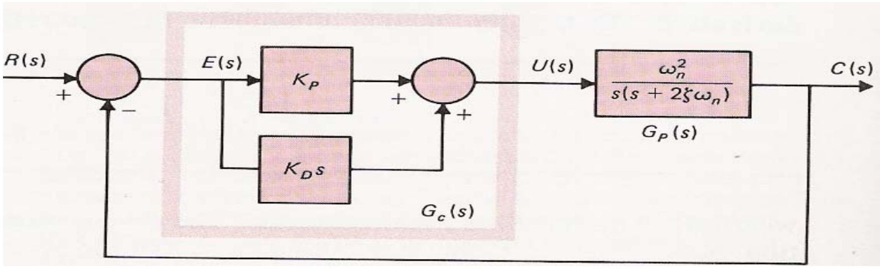
PD cascade compensator:



The proportional plus derivative (PD) controller which is shown in the figure has the form:

$G_c = Kp + K_D S$

Which has the effect of adding a zero to the system (Note that in this TF the no. of zeros is greater than the no. of poles and this may pose some implementation problem if designed using passive elements and when amplifiers are incorporated in the design in addition to other factors, high frequency noise components in the input are usually magnified.

In this design the system output is effected not only by the magnitude of the error signal e(t) but also by its derivative (rate of change).

Therefore if the error is large or changing rapidly, the effect of the control will be an increase in the system response.

The introduction of a zero may serve as a stabilizing factor by shifting the root locus to the left: e.g.

Example: Consider again the system

$$G(S) = \frac{815000}{S(S+360)}$$

It is requited that the overshoot & damping are improved while the steady state error due to a ramp is unchanged.

Then

$$G_c(s)\ \ G(s) = \frac{815000(Kp + K_D S)}{S(S+360)}$$

The direct application of root-locus design approach may not be appropriate since the zero location depends on both design factors gains K_P & K_D.

The CE of the CL system is given by

$$S^2 + (360 + 815000\ K_D)S + 815000\ Kp = 0$$

$$y = \frac{0.2 + 450 K_D}{\sqrt{K_p}}$$

Suppose Kp is fixed (say Kp = 1) then from the C.E.

$$y = 0.2 + 450\ K_D$$

Which shows that the damping ratio increases when K_D increase and rise-versa. Also it should be rated that increasing K_D makes the zero ($S = -\frac{K_p}{K_D}$) closer to the origin and this may increase the overshoot. However, in this example since a system pole is located at the origin the net result will be a pole/zero cancellation and no major change in the overshoot will occur.

For critical damping $\gamma = 1$, Kp = 1, K_D = 0.00177

Another way of looking at the problem is by using root-locus techniques. The CE of the closed loop system can be reformulated so that the root locus can be drawn as one gain varies while the other is constant.

$$CE = 1 + \frac{815000\,K_D S}{S^2 + 360\,S + 815000\,Kp} = 0$$

(In this case Kp constant and K_D varies).

For Kp = 1, the root locus when K_D varies is shown

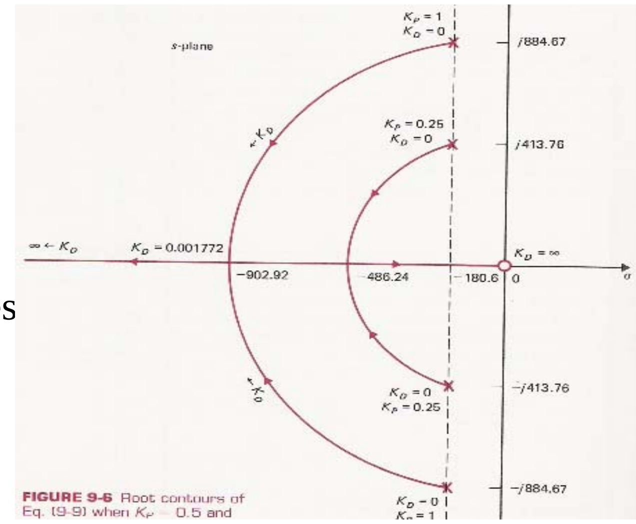
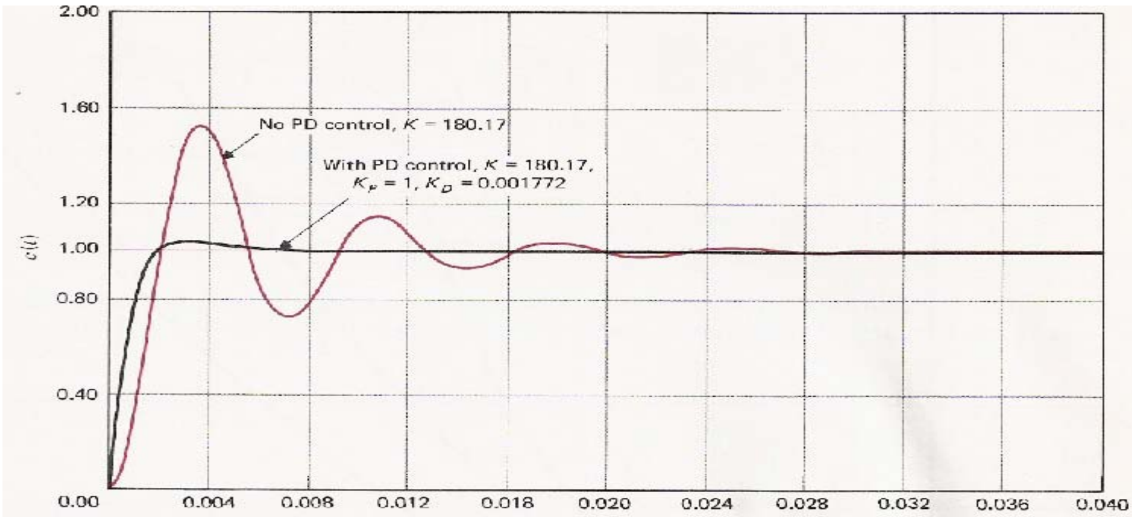


FIGURE 9-6 Root contours of Eq. (9-9) when Kp = 0.5 and



The steady state error due to a ramp will not be affected since the uncompensated system has constant e_{ss} (type 1 system) and therefore the derivative at SS will be zero and no effect will result from this branch, the other branch has a unity gain.

Notes:

PD controllers (in general)

- Fast rise time and increased speed.
- Less overshoot
- As to the steady state error, this depends on the $e_{ss}(t)$ of the uncompensated system If e_{ss} is not changing with time then derivative part of the PD has no rule to play.

PID cascade controller:

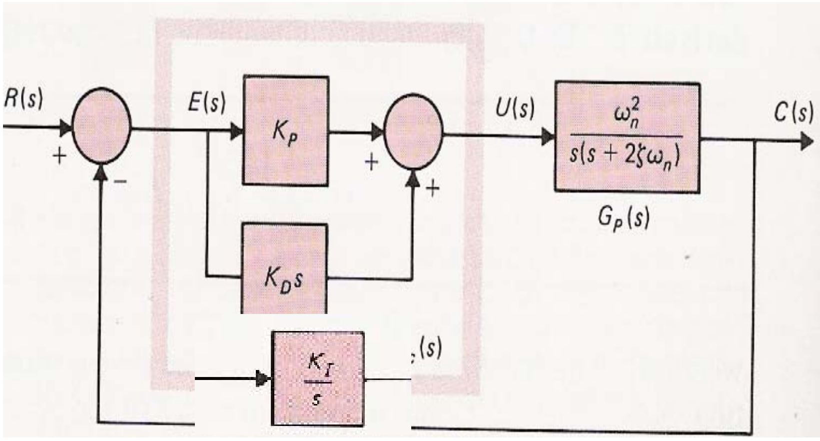
If properly designed, the PID controller should have the advantages of both the P1 & PD controllers. The transfer function of the PID controller is

$$G_c = K_p + K_D S + \frac{K_I}{S} = (1 + K_{D}, S)\,(Kp_2 + K_{I2}/S)$$

Where $Kp = Kp_2 + K_{D1} K_{I2}$

$K_D = K_{D1} KP_2$

$K_I = K_{I2}$



Design Procedure:

1. To start with the design of the PI section only (K_{I2} & Kp_2) So that the system will have a good rise time.

This step improve steady state error as the type is increased by one. At this step the overshoot may be high.

2. Introduce the PD section to reduce the maximum overshoot by selecting k_{D1} that gives appropriate damping ratios.
3. Calculate k_p , K_D & K_I and fit?

Another way is to first design the PD controller section (K_{D1}) then the PI is designed to satisfy the desired relative stability

Ex. $G(s) = \dfrac{815000}{S(S+360)}$

Starting with the P1 controller design:

$$G_{c_1}(S)\, G(S) = \frac{81500\; Kp_2 (S + KI_2 / Kp_2)}{S^2 (S + 360)}$$

As before take $\frac{K_{I2}}{Kp_2} = 10$. choose Kp_2 & K_{I2} for a damping ratio that gives short rise time but does not account for the overshoot as it can be dealt with by the PD. Setting $Kp_2 = 1 \Rightarrow \gamma = 0.2$.

Next (possibly using the root-locus technique) design the PD (choosing K_{D1}) to improve the damping ratio which will give acceptable overshoot.

Phase-lag & phase-lead compensators:

These approximate the PI & PD compensator.

Pole/zero cancellation compensation:

In some process, these are complex poles that are very close to the imaginary axis which could case the system to have undesirable characteristics such as high-damping or even unstability.

One suggestion to eliminate this problem is to use a compensation that has zeros which match these undesirable poles so that their effect is cancelled out. However, this method is not always applicable due to practical problems that arise from:

* Error in system modeling.
* Presense of non-linear components
* Approximation of complex models (high-order-model are usually approximated by reduced order models to simplify analysis and design.
* Parameter Variation
* Presence of noise & disturbance

For those reasons and may be others, exact pole/zero cancellation is difficult to achieve. This fact highlights the danger of inexact cancellation of unstable poles. For example let

$$G(S) = \frac{S+5}{S-1}$$

and it is suggested that the pole at (-1) in cancelled by a zero at (-1). In practice the T.F. may not take the above form exactly and may take the form:

$$G_e(S) = \frac{S+5\pm\Delta_1}{(S-1\pm\Delta_2)}$$

Where Δ_1 & Δ_2 are positive real values.

When a controller having a zero at (-1) is used

e.g. $G_c(S) = \dfrac{S-1}{S+2}$

The compensated system becomes

$$G_e(S)G_C(S) = \frac{S-1}{S+2}\,\frac{S+5\pm\Delta_1}{(S-1\pm\Delta_2)}$$

Now due to the presense of Δ_2 the zero (S-1) will not totally cancel the unstable pole and another unstable pole will result leading to instability.

Multiple Controllers:

The use of a forward controller may not prove to be effective all the time since:

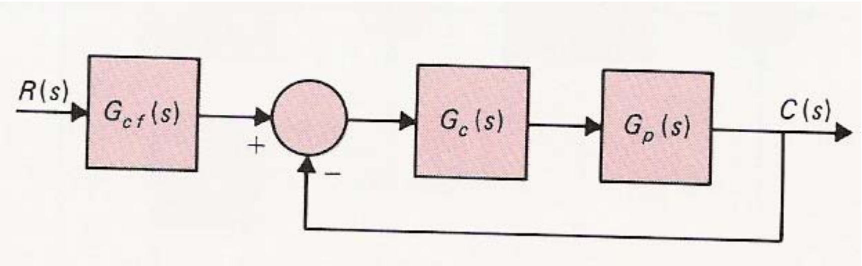
(1) for some systems if the controller is designed to meet some amount of relative stability it might be very sensitive to parameter-variation.

(2) Due to the presence of some zeros, high overshoot is affecting the response of the C/L system.

(3) It is needed to reduce the effect of disturbance.

(4) Sometimes the value of gain can't be increased (causes saturation) whereas another flexibility is needed.

In general multiple design objectives (high speed & low overshoot) are achieved using multiple compensator (Forward & feed forward compensators).



The C/L TF of the above diagram is

$$\frac{C(S)}{R(S)} = \frac{G_{C2}(S)\,G_{C1}(S)\,G_P(S)}{1 + G_{C1}G_P}$$

While the error T.F is

$$E(S) = \frac{1}{1 + G_{C1}(S)\,G_P(S)}$$

Therefore, the forward compensator $G_{C1}(S)$ can be designed so that the error will have some desirable specifications (fast response & certain damping ratio) and the feed forward compensator $G_{C2}(S)$ is chosen so that the i/o characteristic will meet other objectives (e.g canceling a system zero that corresponds to high overshoot).