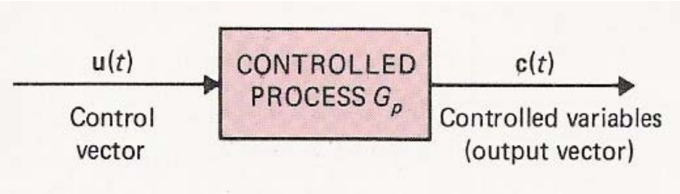
Suppose a physical system has the

D.E $y + 0.5 + y(t) + y(t) = u(t)$



How are you going to analyze this system?

Stability and system specification: speed, bandwidth, overshoot, rise time and setting time.

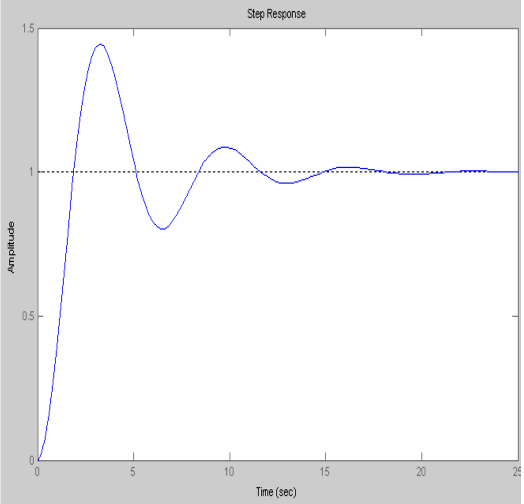
$$G(s) = \frac{1}{s^2 + 0.5s + 1}$$

poles at -0.25 ± J 0.968

$$y(t) = \frac{wn}{\sqrt{1-\gamma^2}} e^{-jwnt} \sin\left(wn\sqrt{1-\gamma^2 t}\right)$$

open loop natural frequency $w_n = 1$

open loop damping ratio $\gamma = 0.25$

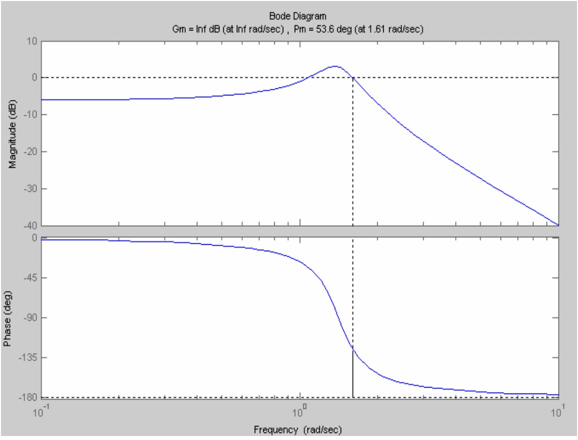


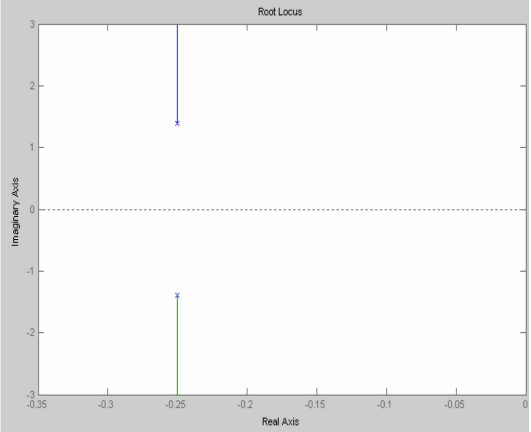
for closed loop k = 5

$1 + G = s^2 + 0.5\ S + 1 + k$

$$w_n = \sqrt{6} \cong 2.45$$

$$\gamma = \frac{0.5}{2\sqrt{6}} \simeq 0.1021$$





Control Systems Design:

In continuous time system, the design of control system is usually carried out either in the time-domain and root locus plane or in the frequency domain. For each approach there are many ways of changing the system specifications in order to achieve acceptable performance. These design methods which are called compensators, may include: gain adjustment PI, PD, PID, lag, lead or lag-lead compensators.

Depending on the application and the derived properties the added compensator can be inserted in the formed path (cascade compensation) (low energy point) or the feedback loop (feedback compensation) which is usually used to improve system tracking of desired input or to improve rejection of a disturbance.

Linear/ nonlinear

Time varying/ time invariant

Continuous system discrete system

Classical modern

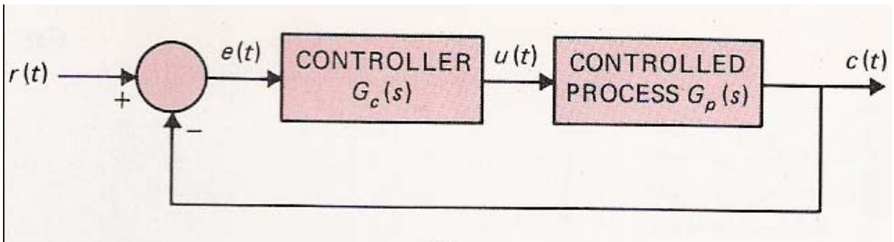
Time domain frequency domain optimal cont. auto. cont

Lag, lead PID

Cascade comp. feed back comp.

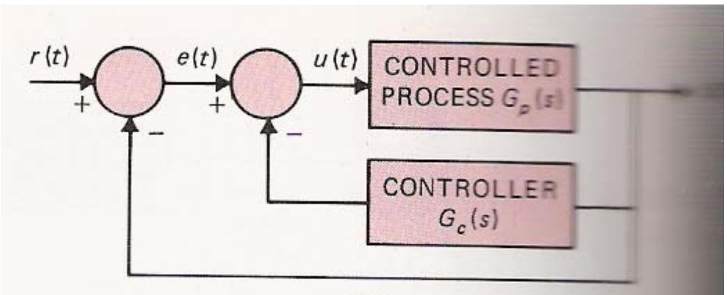
Different configurtion of compensators:

* Cascade compensator (most common)



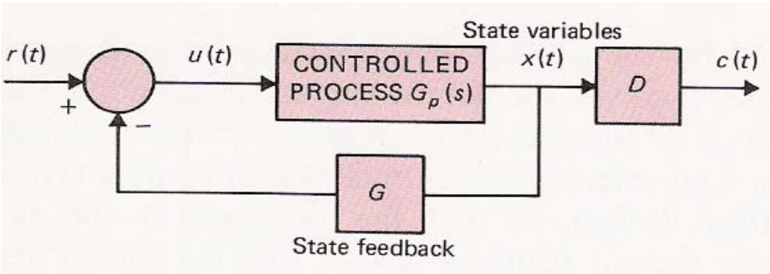
(a)

* Minor loop feedback comp.



(b)

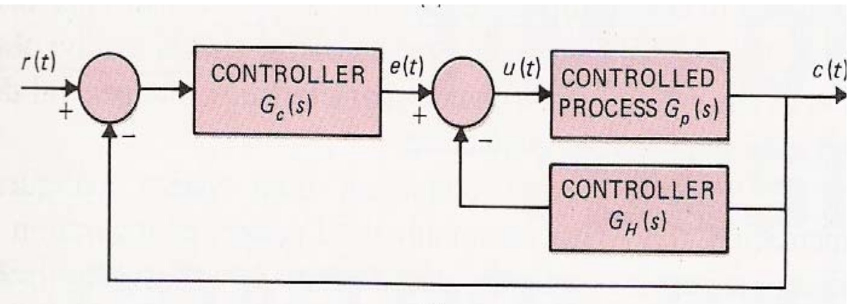
* Output feedback
    - State feedback



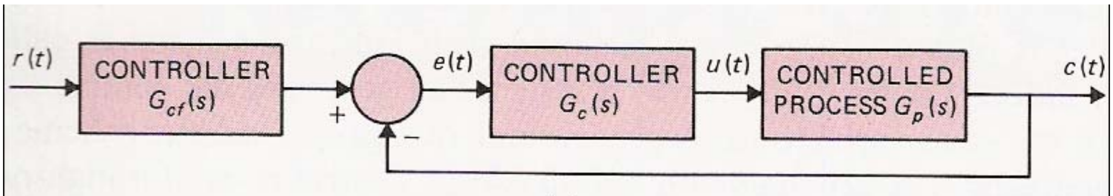
(c)

* Series & forward comp.

(2 deg of freedom)



(d)

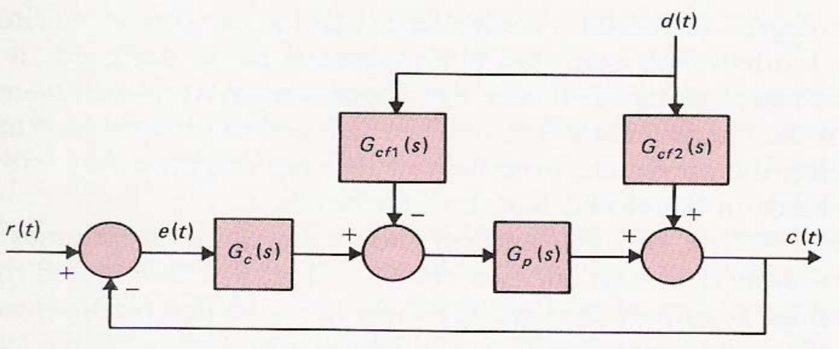


(e)

Note G_{c1} not in the loop not affecting the CE of the system and hence can be used to cancel pole or zero.

* Forward load dist & series

(2 deg of freedom)



(g)

General Notes:

- Usually feedback compensators are more difficult to design than those of cascade compensator.

- The cost, the size, and weight of a components and amplifiers (some time needed when the compensator is inserted in the forward path i,e the low energy level so as to keep power loss small and to set the input impendence high whereas when using feedback compensation it is inserted at a high energy level and amplifiers may not be necessary).
- For some application high degree of accuracy is needed or rapid changing in the system parameters occur. In these case, output or state feedback may give better performance.
- The presence of noise may influence the choice of compensators.

Dominant Complex Poles:

Most of real systems are or can be designed so that the time step response looks like an under damped system Fig (1).

The time response in this figure is dominated by the response of two complex poles. the other poles must be located far to the left of the dominant poles so that the transient response due to these insignificant poles will have small magnitudes and vanish quickly. However, if a non-dominant pole is not far then a zero must be close to it so as to cancel the large effect of this pole.

To study the effect of the existence of significant poles or zeros, consider a system which has two complex dominant poles. A third real pole is added to the system at different location in the real axis. (see Figure).

Notes:

The transfer functions:

$$\frac{C(S)}{R(s)} = \frac{K}{(s^2 + 2\gamma\,\omega_n\,S + \omega_n^2)\,(S - P_3)} \qquad P3 < 0$$

In each case K is chosen so that K = - $\omega_n^2 P_3$

to keep $\frac{\mathrm{C(o)}}{\mathrm{R(o)}} = 1,$ constant

The unit step time response is given by

$$C(t) = 1 + 2\,|A_1|\; e^{-\gamma\omega_n t} \sin\left(\omega_n \sqrt{1-\gamma^2}\, t + \phi\right) + A_3 e^{P_3 t}$$

* the contribution of the third pole is $A3\ e^{P_3 t}$

* A_3 is always negative (see Laplace tables) and its magnitude depends on the magnitude of P_3 and its relative distance from the complex poles. Since A_3 is negative the overshoot is always reduced.

* When P_3 is located to the left of the complex poles there is overshoot but decreasing as the pole moves to the right. When the added pole is in line with the complex poles no overshoot. When the added pole moves further to the right of the complex pole it becomes dominant pole itself and the response is over damped.

Now consider that a zero is added so that the system will have two complex poles, one real pole and a real zero.

The time response equation will have the same form as before but A_3 now is not always negative. The magnitude and sign of A_3 now is not always negative.

The magnitude and sign of A_3 depends on the relative location of the real pole and zero.

Notes:

- When the added zero is close to the real pole $|A_3|$ becomes small (like pole zero cancellation).
- When the zero is to the left of the real pole the response is similar to complex-pole-only system but with smaller overshoot. (T_p is increased).
- When the zero is to the right of the pole the overshoot is greater than that of complex-poles-only system (T_p is decreased).

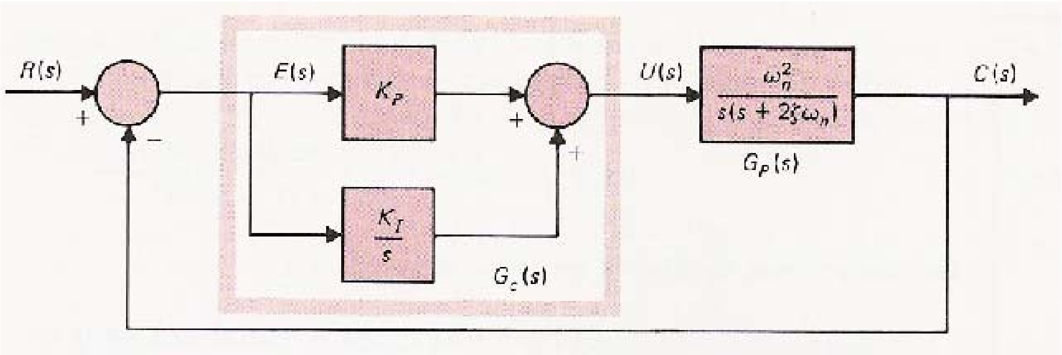
Figure

Proportional-Plus-Integral Cascade Compensator:

The PI controller has two effects which are, changing the forward system gain and integrating the input signal so a general PI takes the form.

$$G_c(s) = Kp + \frac{K_I}{S} = \frac{K_P S + K_I}{S} = Kp\frac{\left(S + \frac{K_I}{K_P}\right)}{S}$$

Figure



Clearly, as an ‘S’ appears in the denominator of the transfer function $G_c(s)$, the effect of this compensator will be an increase in the total system type which in the end will help to reduce the steady state error.

The key issue in the design is how to choose the zero S = -$\frac{K_I}{K_P}$ so that the system stability is meintained.

A general guideline for the design of PI integrator is to place the zero $\left(S = \frac{-k_I}{k_P}\right)$ at a location which is relatively close to the origin and far a way from the significant pole of the plant (G(s)).

Example (PI):

Let $G(s) = \frac{815000}{S(S+360)}$

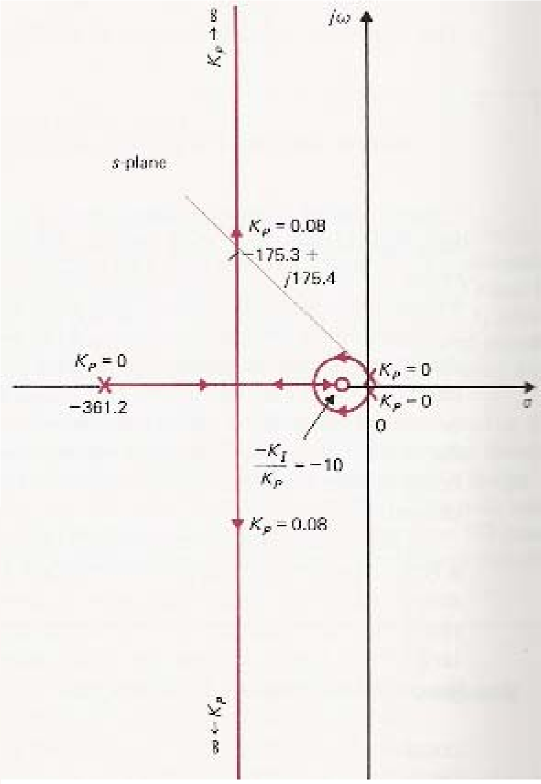
In this second order system (Type 1), the significant pole is (S = -360) therefore to keep the compensator zero away from this pole the condition K_I/K_p, << 360 (note that the condition for stability is (0 < K_I < 360 K_p).

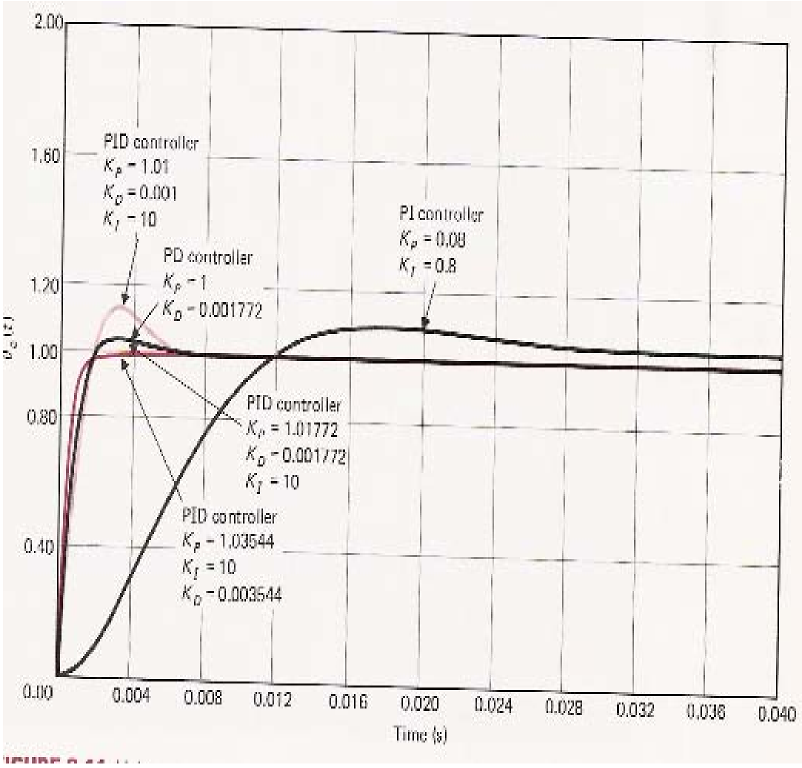
$$G_c(s)\ G(s) = \frac{815000\,K_p(S + K_I/K_P)}{S^2(S+360)}$$

Therefore, Choose $\frac{K_I}{K_P} = 10 \ll 360$

$\frac{K_I}{K_P} = 10$, $K_p = 0.08$, $K_I = 0.8$

(notice the effect of increasing or decreasing $\frac{K_I}{K_P}$)





Comments on PI Compensators:

1. The system type is increased ⇒ reduce study state error.
2. The speed is decreased & settling time is increased.
3. Bandwidth is decreased.
4. Depending on $\frac{K_I}{K_P}$, the overshoot and hence stability is improved.