Optimal edge detection and segmentation of SLC SAR images with spatially correlated speckle

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ABSTRACT

We have recently presented optimal criteria for edge detection and edge localization in Single Look Complex (SLC) Synthetic Aperture Radar (SAR) images. By working on complex data rather than intensity images, we can easily take the speckle autocorrelation into account, obtain more accurate estimates of local mean reflectivities, and thus achieve better edge detection and edge localization than with operators known from the literature. After a review of the theoretical aspects, we here propose solutions for the practical implementation. In SLC images the Maximum Likelihood (ML) estimator of reflectivity is the Spatial Whitening Filter (SWF), which is used in both tests. It necessitates precise knowledge of the speckle correlation. We describe how it can be determined and discuss the consequences of inaccuracies. Two-dimensional edge detection can be realized with multidirectional sliding windows. The watershed algorithm permits the extraction of closed, skeleton boundaries, and thresholding of the basin dynamics efficiently reduces the number false edges. The optimal estimator of the edge position is computationally intensive, so we examine suboptimal methods which require far less multiplications. The edge localization stage can be implemented with active contours or Gibbs random field techniques. Some segmentation results are shown.

Keywords: Edge detection, edge localization, complex SAR images, speckle correlation, spatial whitening filter, segmentation.

1. INTRODUCTION

In SAR image scenes with no texture, an edge can be defined as an abrupt change in the reflectivity. The presence of speckle, which can be modeled as a strong, multiplicative noise, makes the usual differential edge detectors inefficient. In particular, the false alarm rate varies with the reflectivity. Several edge detectors with constant false alarm rate have therefore been developed specifically for SAR images, of which the Normalized Ratio (NR)\(^1,2\) and the Likelihood Ratio (LR)\(^3\) operators are well-known examples. The latter can be considered as the optimal solution, and is shown to yield better edge detection performance than other criteria proposed in literature. Both operators work on detected (intensity) data, and the speckle is supposed to be Gamma distributed and spatially uncorrelated. They rely on the ML estimator of the local mean reflectivity, which is the Arithmetic Mean Intensity (AMI) in this case. Oliver et al. propose a two-stage algorithm, which first detects step edges optimally using the LR operator, and then determines the most probable edge positions. In fact, the same radiometric criterion is used in both stages, but the window configuration is different: a Scanning-Wind Central-Edge (SWCE) configuration is used for detection, whereas a Fixed-Window Scanning-Edge (FWSE) configuration is used to improve the edge localization.

SAR data generally has correlated speckle, in which case an underlying hypothesis of the NR and LR criteria described above is violated. As a consequence, the statistics of the operators will not be exact, and the performance becomes suboptimal. We have developed similar criteria for edge detection\(^4\) and edge localization\(^5\) in SLC images, which give optimal results even when the speckle is spatially correlated. The theoretical study is restricted to the monodoppler case, which means that we suppose that there is never more than one edge within the analyzing window. A certain edge direction is also assumed. The derivation of the optimal criteria is detailed in the first part of this article. We then address problems related to the two-dimensional implementation and propose suitable algorithms. We finally present theoretical and empirical curves which illustrate the supremacy of the new criteria, as well as some segmentation results.

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2. EDGE DETECTION

2.1. Vectorial Probability Distributions

Let us consider a window centered on a given pixel in a SLC image. The window is split in two parts, containing \( N_1 \) and \( N_2 \) pixels, respectively. Let \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \) be the complex signal vectors corresponding to the two half windows, and let \( \mathbf{X}_0 \) be the signal vector containing the \( N_0 \) pixels of the entire window. If the speckle is fully developed, the Probability Density Functions (PDF’s) of the different signal vectors are circular complex Gaussian distributions:

\[
p(\mathbf{X}_i) = \frac{1}{\pi^{N_i} |C_{X_i}|} \exp \left( -\mathbf{X}_i^* C_{X_i}^{-1} \mathbf{X}_i \right) \quad i = 0, 1, 2,
\]

where \( C_{X_i} \) is the \( N_i \times N_i \) complex spatial covariance matrix corresponding to signal vector \( \mathbf{X}_i, i = 0, 1, 2 \).

2.2. Neyman-Pearson Test

We want to test the hypothesis \( H_1 \), saying that \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \) are separated by an edge and cover homogeneous regions with different reflectivities \( R_1 \) and \( R_2 \), against the null hypothesis \( H_0 \), which says that \( \mathbf{X}_0 \) corresponds to a zone of constant reflectivity \( R_0 \). Hence the LR for edge detection is

\[
\Lambda = \frac{p(\mathbf{X}_0|H_0)}{p(\mathbf{X}_0|H_1)}.
\]

The null hypothesis \( H_0 \) is rejected if \( \Lambda \) is superior to a threshold \( t_\Lambda \), which corresponds to a certain Probability of False Alarm (PFA). The PFA is the probability of detecting an edge in a zone of constant reflectivity.

Obviously, \( p(\mathbf{X}_0|H_0) = p(\mathbf{X}_0|R_0) \). If \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \) are independent, \( p(\mathbf{X}_0|H_1) = p(\mathbf{X}_1|R_1) \cdot p(\mathbf{X}_2|R_2) \), and Eq. (2) becomes

\[
\Lambda = \frac{p(\mathbf{X}_1|R_1) \cdot p(\mathbf{X}_2|R_2)}{p(\mathbf{X}_0|R_0)}.
\]

As the speckle is correlated, \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \) must be separated spatially by a distance which is greater than the correlation length, in order to be independent. For most sensors the speckle correlation becomes insignificant for lags of more than one or two pixels. Hence this requirement is not very restrictive in the case of edge detection with the SWCE approach, where a window configuration like the one shown in Fig. 1 (a) can be used.*

Substituting the probabilities in Eq. (1) into Eq. (3) and taking the logarithm yields

\[
\log \Lambda = (N_0 - N_1 - N_2) \log \pi + \log |C_{X_0}| - \log |C_{X_1}| - \log |C_{X_2}| + \mathbf{X}_0^* C_{X_0}^{-1} \mathbf{X}_0 - \mathbf{X}_1^* C_{X_1}^{-1} \mathbf{X}_1 - \mathbf{X}_2^* C_{X_2}^{-1} \mathbf{X}_2,
\]

where \( \mathbf{X}_i^* \) denotes the complex conjugate transpose of \( \mathbf{X}_i \).

![Figure 1. 7 × 7 (a) SWCE configuration and (b) FWSE configuration for vertical edges.](image)
2.3. Complex Speckle Covariance

For each of the signal vectors \( \mathbf{X}_i \) we suppose that the underlying reflectivity \( R_i \) is constant. If this is true, the multiplicative speckle model allows us to express the covariance matrix of signal vector \( \mathbf{X}_i \) as

\[
\mathbf{C}_X = R_i \cdot \mathbf{C}_S ,
\]

where \( \mathbf{C}_S \) represents the spatial covariance due to speckle.

The spatial covariance matrix \( \mathbf{C}_S \) of the SLC speckle only depends on sensor and SAR processor parameters. Principally, it should therefore be possible to obtain exact values from the data provider. If \( \mathbf{C}_S \) is unknown, it can be estimated on any part of the SLC image where the speckle is fully developed and where the mean reflectivity is not so low that the thermal noise becomes dominant. The underlying reflectivity does not necessarily need to be constant.\(^7\) However, the correlations are not exactly the same in near range and in far range. This can be taken into account by using several different matrices when processing over the full swath. To produce the matrices \( \mathbf{C}_S \), \( i = 0, 1, 2 \), we simply rearrange the terms of \( \mathbf{C}_S \) in accordance with the corresponding signal vectors.

2.4. Spatial Whitening Filter

The reflectivities \( R_i \) are in our case unknown and must be estimated. From Eqs. (1) and (5) it can easily be shown that the ML estimator of reflectivity for SLC images is the Spatial Whitening Filter (SWF), which is given by

\[
\hat{R}_i = \frac{1}{N_i} \mathbf{X}_i^* \mathbf{C}_S^{-1} \mathbf{X}_i .
\]

The Whitening Filter (WF) was used\(^6\) to obtain an intensity image with minimal speckle variance from polarimetric data. A combined spatial and polarimetric WF was proposed\(^8\) for improved target detection. In\(^9,10\) it was shown that the WF is a ML estimator of texture.

Taking the AMI of \( N \) pixels does not reduce the variance by a factor \( N \), as in the case of uncorrelated speckle, but by a factor \( N^t < N \), and the computed mean values are only approximately Gamma distributed.\(^11\) However, if the data is available in SLC format, the optimal variance reduction factor \( N \) can be attained using the SWF. Moreover, the SWF output is truly Gamma distributed.

Let us now assume that the covariance matrix \( \mathbf{C}_S \), in Eq. (6) is not perfectly known, so that we have to use an estimated matrix \( \hat{\mathbf{C}}_S \). The expectation of the estimated reflectivity \( \hat{R}_i \) computed by the SWF is then given by\(^10\)

\[
E[\hat{R}_i] = \frac{1}{N_i} Tr(\mathbf{C}_S^{-1} \hat{\mathbf{C}}_S) R_i ,
\]

which means that the SWF is an unbiased estimator only when \( \hat{\mathbf{C}}_S = \mathbf{C}_S \). Hence any inaccuracy in \( \hat{\mathbf{C}}_S \) will introduce a bias on \( \hat{R}_i \). We have observed this bias when using estimated covariance matrices to compute the SWF on very large windows, especially when the speckle correlation is strong. The number of multiplications per pixel for the SWF is about \( N^2 + N \), so the computational cost becomes considerable for very large windows. A practical solution to both problems is to calculate the SWF on maximally overlapping smaller windows within the big window, and then average the results. The computational complexity of the hybrid filter is only slightly higher than that of the SWF for the small window. If the small window is big enough to contain all pixels with which the central pixel is strongly correlated, the performance loss will be modest. For the hybrid filters the bias will also be negligible.

2.5. Likelihood Ratio Operator

If the pixels in the band which assures the independence between \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \) are excluded from \( \mathbf{X}_0 \), \( N_0 = N_1 + N_2 \), and it can be shown that \( | \mathbf{C}_S | = | \mathbf{C}_{S_1} | \cdot | \mathbf{C}_{S_2} | \). If, furthermore, we replace \( R_i \) by \( \hat{R}_i \), \( i = 0, 1, 2 \), then

\[
\mathbf{X}_0^* \mathbf{C}_{X_0}^{-1} \mathbf{X}_0 - \mathbf{X}_1^* \mathbf{C}_{X_1}^{-1} \mathbf{X}_1 - \mathbf{X}_2^* \mathbf{C}_{X_2}^{-1} \mathbf{X}_2 = 0 .
\]

By substituting Eqs. (5), (6) and (8) into Eq. (4) we obtain the log-likelihood difference

\[
\log \Lambda = N_0 \log \hat{R}_0 - N_1 \log \hat{R}_1 - N_2 \log \hat{R}_2 .
\]
It is interesting to note that this expression is similar to the one that Oliver et al. derive for intensity images.\textsuperscript{3} The only difference resides in the way the local reflectivities are estimated.

The PDF of the LR operator\textsuperscript{1} given by Eq. (9) has not been developed analytically. In order to compute the theoretical PFA corresponding to a threshold $t_\lambda$ or \emph{vice versa}, a simple solution is to use the relation between the LR and the Double-sided Ratio (DR) operator\textsuperscript{1}

$$r = \frac{\hat{R}_1}{\hat{R}_2},$$  \hfill (10)

whose distribution for a single-look image is\textsuperscript{1,3}

$$p(r|R_1, R_2, N_1, N_2) = \frac{\Gamma(N_1 + N_2)}{r\Gamma(N_1)\Gamma(N_2)} \left( 1 + \frac{N_1 \frac{R_1}{R_2} \frac{N_2 R_2}{N_1 R_1}}{N_1+ N_2} \right)^{N_1}. \hfill (11)$$

For two DR thresholds $t_1 < 1$ and $t_2 > 1$, which are used when $\hat{R}_1 < \hat{R}_2$ and when $\hat{R}_1 > \hat{R}_2$, respectively, the Probability of Detection (PD) is given by

$$p_{\text{det}} = 1 - \int_{t_1}^{t_2} p(r|R_1, R_2, N_1, N_2) dr$$  \hfill (12)

which may be computed by numerical integration. However, it can be shown that\textsuperscript{3,5}

$$p_{\text{det}} = 1 - \frac{\Gamma(N_1 + N_2)}{N_1 \Gamma(N_1) \Gamma(N_2)} \left[ \left( \frac{N_1 R_2 t_2}{N_2 R_1 + N_1 R_2 t_2} \right)^{N_1} F \left( N_1 + N_2 + 1; N_1 + 1; \frac{N_1 R_2 t_2}{N_2 R_1 + N_1 R_2 t_2} \right) - \left( \frac{N_1 R_1 t_1}{N_2 R_1 + N_1 R_2 t_1} \right)^{N_1} F \left( N_1 + N_2 + 1; N_1 + 1; \frac{N_1 R_1 t_1}{N_2 R_1 + N_1 R_2 t_1} \right) \right], \hfill (13)$$

where $F$ is the hypergeometric function.\textsuperscript{12} The computation of the PFA ($R_1 = R_2$) or the PD ($R_1 \neq R_2$) corresponding to a pair of thresholds $t_1$ and $t_2$ can be effectuated very efficiently through Eq. (13), as the hypergeometric function here reduces to a finite series.

The relation between $\lambda$ and $r$ is found by rearranging the terms of Eq. (9):

$$\log \lambda = -N_1 \log r + N_0 \log \left( \frac{N_1 r + N_2}{N_0} \right)$$  \hfill (14)

In practice, we first fix the PFA that can be accepted. The next step is to find the appropriate threshold $t_\lambda$ for the LR, which will be used to decide whether or not an edge is present. We guess a first value for $t_\lambda$, solve Eq. (14) with respect to $r$ to obtain the two corresponding thresholds $t_1$ and $t_2$ for the DR, and compute the PFA by introducing $t_1$, $t_2$ and $R_1 = R_2$ into Eq. (13). The threshold $t_\lambda$ is adjusted and the procedure is repeated until the computed PFA is sufficiently close to the desired one. If we measure $\lambda > t_\lambda$ in a given pixel position, it indicates the presence of an edge with a risk of false detection equal to the specified PFA.

The Normalized Ratio (NR) proposed by Touzi et al.\textsuperscript{1} can also be mapped to $r$:

$$r_n = \min \left\{ \frac{\hat{R}_1}{\hat{R}_2}, \frac{\hat{R}_2}{\hat{R}_1} \right\} = \min \{r, 1/r\} \hfill (15)$$

Let us consider a new version of the NR operator, where $\hat{R}_1$ and $\hat{R}_2$ are computed by the SWF, so that problems due to speckle correlation are eliminated. A given threshold $t_n$ for the NR corresponds to the thresholds $t_1'$ and $t_2' = 1/t_1'$ of the DR, which in general are different from the optimal thresholds $t_1$ and $t_2$ found through Eq. (14). For the SWCE configuration, however, where $N_1 = N_2$, it can easily be shown that $t_2 = 1/t_1$, so that the LR and NR performances coincide.

\textsuperscript{1}As we have replaced the unknown parameters by their ML estimates in Eq. (9), $\lambda$ is here strictly speaking not the LR, but the \emph{generalised} LR. For simplicity, however, we keep the same notation and continue to call it the LR.
3. EDGE LOCALIZATION

3.1. ML Estimator

Edge localization is an estimation problem rather than a detection problem. Assuming that an edge is present within the window, we want to determine the most probable edge position. Edge localization can therefore be considered as a refinement stage following the edge detection procedure described above. The question is how to divide $\mathbf{X}_0$ into $\mathbf{X}_1$ and $\mathbf{X}_2$. ML estimation of the edge position consists in maximizing the probability density function $p(\mathbf{X}_0)$ given in Eq. (1) with respect to the edge position, or equivalently, by computing

$$\hat{\mathbf{X}}_{\text{edge}} = \left\{ \mathbf{X}_0 : \min_{\mathbf{X}_1, \mathbf{X}_2} U(\mathbf{X}_1, \mathbf{X}_2) \right\},$$

(16)

where the energy function $U$ is given by

$$U(\mathbf{X}_1, \mathbf{X}_2) = |C_{\mathbf{X}_0}| + \mathbf{X}_0^T \mathbf{C}_{\mathbf{X}_0}^{-1} \mathbf{X}_0.$$  

(17)

Using the FWSE configuration illustrated in Fig. 1(b) we center the window on the detected edge pixel and split the window in all possible edge positions. As the ML estimator must use the same data set for all positions, we cannot introduce any band of pixels separating the two parts of the window. The signal vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ will consequently be dependent. For each pair of signal vectors the SWF defined in Eq. (6) is used to compute $\hat{R}_1$ and $\hat{R}_2$. Let $\mathbf{Q}$ be a column vector where the $N_1$ elements which correspond to $\mathbf{X}_1$ are set to $\sqrt{\hat{R}_1}$ and the $N_2$ other elements are set to $\sqrt{\hat{R}_2}$. We can now write

$$C_{\mathbf{X}_0} = (\mathbf{QQ}^T) \odot C_{\mathbf{X}_0},$$

(18)

where $\odot$ denotes the element-by-element product of two equally sized matrices. To estimate the edge position using Eq. (16), the matrix $C_{\mathbf{X}_0}$, its determinant and its inverse must be computed for each new window position, and for all possible edge positions within the window. If we denote the number of possible edge positions by $M$, the total number of multiplications per edge pixel is about $M(N_0^3 + 2N_0^2 + N_1^2 + N_2^2 + 3N_0)$.

3.2. Approximate ML Solution

If the window is big and the speckle correlation is relatively weak, we may consider $\mathbf{X}_1$ and $\mathbf{X}_2$ as approximately independent. In this case $p(\mathbf{X}_0) \approx p(\mathbf{X}_1|\mathbf{R}_1) \cdot p(\mathbf{X}_2|\mathbf{R}_2)$. As $p(\mathbf{X}_0|\mathbf{R}_0)$ is constant for a fixed window position, $p(\mathbf{X}_0)$ is proportional to $\lambda$ in Eq. (3). Hence the most probable edge can be computed by Eq. (16) with the suboptimal energy function

$$U'(\mathbf{X}_1, \mathbf{X}_2) = N_1 \log \hat{R}_1 + N_2 \log \hat{R}_2.$$  

(19)

Computing Eq. (16) with the new cost function $U'$ requires about $M(N_1^2 + N_2^2 + N_0)$ multiplications per edge pixel, which is far less than for the true ML estimator based on Eq. (17).

4. TWO-DIMENSIONAL IMPLEMENTATION

The description so far is basically one-dimensional. The two-dimensional implementation poses some additional problems: Firstly, the edge orientation is in general unknown, so that a variety of edge directions must be considered in the edge detection stage. Moreover, the monoeche model is not always verified. Depending on the scene type, several edges may co-occur within the analyzing window, especially if a large window is used. Secondly, we must make sure that the edges that we extract are closed and skeleton if we want them to define a segmentation of the image. Finally, it is essential that the edges remain connected and relatively regular in the edge localization stage.

4.1. Two-dimensional Edge Detection

To detect edges with unknown orientation, a simple approach consists in splitting the analyzing window in several different directions. We have chosen to compute the SWF-based LR given by Eq. (9) across the vertical, horizontal, and diagonal axes. The maximum value is retained as the edge strength of the central pixel. However, the PD’s and PFA’s of the operator are not the same as in the unidirectional case. If the LR’s computed in the four directions were independent, the relation

$$p_4 = 1 - (1 - p_1)^4$$

(20)
could be used. Theoretical relations are difficult to establish when the LR’s are correlated. In the case of square windows, where the half-windows for the different directions are highly overlapping, the empirical relation

\[
p_h = 1 - (1 - p) \]

is a good approximation.\(^1\) For long and narrow windows, which give a lower degree of overlap between the masks for different directions, the observations tend towards Eq. (20).\(^13\)

On one hand large windows should be used to reduce the influence of the speckle on the estimates of local reflectivities. On the other hand smaller windows yield higher spatial resolution and the monoedge model is more easily verified. The window size constitutes a compromise between these two requirements.

Rather than using a fixed size window, a statistical multiresolution approach like the one described\(^14\) can be used. Weak edges separating large regions can then be detected with large windows, while strong edges between small regions are detected with small windows. The fact that LR’s computed on different resolution levels have different statistical significance is taken into account: Thresholds corresponding to the accepted PFA are first computed over the whole range of relevant window sizes. The maximum LR on each level is then divided by the appropriate threshold, and the maximum normalized LR across the resolution levels, which is the strongest indication of the presence of an edge, is retained as the edge strength of the pixel.

### 4.2. Two-dimensional Edge Extraction

Plain thresholding of the edge strength map computed by the multidirectional (multiresolution) LR operator generally produces several pixel wide, isolated edge segments. Closed, skeleton boundaries defining a segmentation of the image can be obtained by applying the watershed algorithm\(^15\) to the edge strength map. The watershed algorithm usually yields an over-segmented image. To reduce the number of false or irrelevant edges to an acceptable level, we can threshold the basin dynamics of the edge strength map.\(^16\) The concept of edge dynamics\(^17\) permits a compact, hierarchical representation of the segmentations obtained by applying different thresholds to the basin dynamics. The user can then choose the best threshold for his application interactively. More details on the use of basin and edge dynamics in SAR image segmentation are given in.\(^20\)

Another way of reducing the over-segmentation is to merge adjacent regions whose mean reflectivities are not significantly different.\(^21\),\(^22\)

### 4.3. Two-dimensional Edge Localization

We shall now suppose that the actual contours have been extracted with the methods described above, but that the detected edge pixels not necessarily are in the correct positions. The segmentation is initially represented by an edge map, but we also create a label image, in which every pixel carries the number \(\ell\) of the region it belongs to. From a radiometric point of view, the optimal edge position is the one that minimizes the cost function in Eq. (17). However, constraints on the regularity of the edges must be included in the energy function, as the radiometric criterion alone tends to create very irregular edges which follow local speckle patterns. We have considered to ways of introducing regularity constraints: a simple method based on Gibbs random fields, and a more sophisticated one based on active contours (snakes).

#### 4.3.1. Contextual regularization using Gibbs random fields

Gibbs random fields have been introduced in image processing to model regularity constraints in the entire image. Geman\(^23\) used it in the field of pattern recognition. Descombes\(^24\) and Sigelk\(^25\) studied their links to physical statistics. Potts model is a simple and frequently used model, which has constant and anisotropic potential functions. It permits to model geometric regularity efficiently without having to specify complicated local configurations.

Let the labels \(\ell\) within a \(3 \times 3\) window have indices as shown in Fig. 2 (a). The energy term \(V\) corresponding to Potts model is then given by:

\[
V(X_1, X_2) = \lambda \left( \frac{1}{T} \sum_{i=1}^{8} (2\delta(\ell_0, \ell_i) - 1) \right),
\]

where \(\lambda\) in our case is a negative constant which controls the weight given to the regularization constraint compared to the radiometric criterion. The parameter \(T\) diminish exponentially from the initial value \(T_0\) during the global optimization stage, similar to the temperature in simulated annealing. The sum in Eq. (22) simply calculates the
number of pixels in the 8-neighbourhood which have the same label as the central pixel, minus the number of pixels which have a different label.

The energy function, including the regularization term, becomes

$$U_V(X_1, X_2) = U(X_1, X_2) + V(X_1, X_2),$$

(23)

where the radiometric term $U$ is given by Eq. (17) or Eq. (19).

\begin{tabular}{|c|c|c|}
\hline
1 & 2 & 3 \\
\hline
8 & 0 & 4 \\
\hline
7 & 6 & 5 \\
\hline
\end{tabular}

(a)

\begin{tabular}{|c|c|c|}
\hline
94 & 94 & 94 \\
\hline
94 & 94 & 94 \\
\hline
37 & 37 & 37 \\
\hline
\end{tabular}

(b)

\begin{tabular}{|c|c|c|}
\hline
19 & 19 & 19 \\
\hline
19 & 67 & 19 \\
\hline
19 & 67 & 67 \\
\hline
\end{tabular}

(c)

\textbf{Figure 2.} Illustration of Eq. (22). (a) The indices $i$ of the labels $\ell_i$ within a $3 \times 3$ window. (b) A regular configuration of labels which yields $V = 2\lambda/T$. (c) An irregular spatial configuration for which $V = -4\lambda/T$.

We traverse the image several times and compute the minimum of Eq. (23) with respect to the edge position for each detected edge pixel, similar to what is done the Iterated Conditional Modes (ICM) algorithm. The role of the regularization term is to privilege regular shapes, which in most natural scenes are more likely than very irregular structures. The algorithm can be considered as contextual, even though it is iterative.

4.3.2. Active contours

The edge localization algorithm based on active contours is preceded by a vectorization of the edges, as shown in Fig. 3. The edges are represented by nodes and arcs. We have to set the maximum distance that can be accepted between an edge pixel and the corresponding arc. The higher the required precision is, the higher the number of nodes will be. The refinement of the edge position consists in traversing the list of nodes iteratively, and for each node calculate an energy function for a series of new node positions in its neighbourhood. When a new edge position is considered, we have to take the corresponding changes for the arcs and the label statistics into account to compute the energy function. If a position with a lower cost is found, the new position is accepted and the concerned labels and label attributes are updated before continuing.

\begin{tabular}{|c|c|c|}
\hline
\end{tabular}

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\begin{tabular}{|c|c|c|}
\hline
\end{tabular}

\textbf{Figure 3.} (a) Initial contours. (b) Vectorization process. (c) Vectorized contours. (d) Modifying the node position.

The radiometric term in Eq. (17) or Eq. (19) can again be used. If we use Eq. (19), the method is similar to the one proposed by Réfrégier et al. for intensity images. Nodes where several regions meet need a special treatment. The energy function in Eq. (19) can easily be generalized to more than two adjacent regions.

The fact that we work on vectorized edges constitutes a regularity constraint in itself. However, regularity constraints based on the angle between adjoining arcs or form parameters describing the complexity of the contours of entire regions can be introduced.
5. EXPERIMENTAL RESULTS

The methods were tested on simulated complex speckle whose spatial covariance properties are very close to those of the speckle in ERS SLC images. The covariance magnitudes up to a lag of 2 pixels in each direction are shown in Table 1 for the simulated speckle, and the speckle covariance estimated in an ERS SLC image is shown in Table 2.

Table 1. Spatial covariance of simulated SLC speckle.

| \(|\rho_s|\) | \(\Delta x = 0\) | \(\Delta x = 1\) | \(\Delta x = 2\) |
|---|---|---|---|
| \(\Delta y = 0\) | 1.00 | 0.48 | 0.07 |
| \(\Delta y = 1\) | 0.61 | 0.29 | 0.04 |
| \(\Delta y = 2\) | 0.12 | 0.06 | 0.01 |

Table 2. Estimated covariance of ERS SLC speckle.

| \(|\rho_s|\) | \(\Delta x = 0\) | \(\Delta x = 1\) | \(\Delta x = 2\) |
|---|---|---|---|
| \(\Delta y = 0\) | 1.00 | 0.48 | 0.03 |
| \(\Delta y = 1\) | 0.61 | 0.29 | 0.02 |
| \(\Delta y = 2\) | 0.14 | 0.07 | 0.00 |

Fig. 4 (a) illustrates the speckle reduction capacity of the SWF and the AMI in terms of the equivalent number of independent looks, which is proportional to the inverse of the normalized speckle intensity variance. As predicted, the speckle variance is reduced by a factor \(N\) when the SWF is computed on a sliding window covering \(N\) pixels of the simulated SLC image, whereas the AMI applied to the corresponding single-look intensity image reduces the speckle variance by a factor \(N'\), which is here about 60% lower than \(N\), due to the spatial speckle correlation. While the SWF becomes quite time-consuming for large windows, hybrid filters, which first compute the SWF on maximally overlapping small windows and then combine the results to estimate the reflectivity within a larger window, are very rapid. We see from Fig. 4 (a) that averaging the results of the 3 × 3 SWF actually doubles the speckle reduction compared to the AMI. Starting from the result of the 7 × 7 SWF, we obtain a speckle reduction factor that is only slightly below the optimum, and the computational complexity of the hybrid filter is still fair.

For the subsequent tests we used analyzing windows of size 7 × 7. This window size yields a reasonable compromise between speckle reduction and spatial resolution for single-look images.

We computed the threshold of the LR operator with the window configuration shown in Fig. 1 (a) for a series of PFA’s, as described in section 2.5. Experimental PFA’s were measured on the simulated speckle image for the SWF-based LR in Eq. (9) and for the AMI-based LR.\(^3\) We also tested another AMI-based LR, where \(N_1\) and \(N_2\) in Eqs. (9) and (13) were replaced by the equivalent numbers of independent pixels,\(^2\) \(N_1^0\) and \(N_2^0\), to compensate for the speckle correlation. Fig. 4 (b) shows that the correspondence between theoretical and measured PFA is excellent.
for the SWF-based LR, but that the error is considerable for the AMI-based LR when no compensation is made for the speckle correlation. At a theoretical PFA of 0.1%, for example, the measured PFA for the AMI-based LR is more than 10 times too high. A much better fit is obtained when using the equivalent number of pixels in each half window rather than the actual number of pixels. However, the measured PFA of the correlation compensated AMI-based LR deviates somewhat from the theoretical value when it becomes very low. This is probably due to the fact that the correlated speckle is not truly Gamma distributed. The difference from the true multilook intensity distribution of SAR images with spatially correlated speckle\cite{11,28} becomes visible when integrating near the extremities of Eq. (11), which is derived from the Gamma distribution. It should be stressed that Fig. 4 (b) only illustrates the correspondence between predicted and measured PFA’s, and that this has no influence on the performance of the detectors.

Fig. 5 (a) shows the experimental Receiver Operating Curves (ROC) of the two operators. All edges in the test image were vertical, and the window configuration in Fig. 1 (a) was used. The measured PD is plotted against the measured PFA, so that the problem with the fit of the PFA for the AMI-based LR has no impact on the results. The SWF-based LR generally yields a substantially higher PD than the AMI-based LR for a given PFA. We see that the PD of the SWF-based LR for a 2dB edge is close to the PD of the AMI-based LR for a 3dB edge. For the weakest contrast ratio (1 dB), the difference is more modest, but this is an extreme case, where the PD is only slightly higher than the PFA. If we use thresholds corresponding to a low PFA (1%), only edges with very strong contrast (6 dB or more) have a high PD when using a 7×7 window. A larger window must be used if we want to detect weaker edges while keeping a low PFA and a high PD.

The theoretical PD as a function of the edge position for the SWF-based versions of the LR and NR operators is shown in Fig. 5 (b). The total number of pixels $N_0$ was set to 42, like in Fig. 1 (a), but other ways of splitting the window were considered. For all possible combinations of $N_1$ and $N_2$ the NR and LR thresholds were calculated for a PFA of 1%, and theoretical PD’s were computed for edges in the corresponding positions. The edge position is denoted by the number of pixels on the left hand side, $N_1$, and the number of pixels on the right hand side, $N_2$. The highest PD is obtained in the central edge position ($N_1 = N_2 = 0$), where the NR as predicted has the same performance as the LR. However, the NR rapidly becomes suboptimal when the edge position moves away from the center, especially for weak edges. Symmetry is obtained by combining the results for $R_1 > R_2$ and for $R_1 < R_2$.

Fig. 6 shows the bias and the Mean Square Error (MSE) of the ML estimator and the approximate ML estimator of the edge position, computed for the 6 possible vertical edge positions of a 7×7 window, as illustrated in Fig. 1 (b). Results are given for edge contrasts of 3 dB and 6 dB, respectively, and the higher reflectivity is on the left hand side ($R_1 > R_2$). The true ML estimator yields the better result, as it takes speckle correlation across the edge into account, but its computational complexity is prohibitive. We note that the estimates are biased towards the middle.
of the window, and also to the side of higher reflectivity for the suboptimal estimator. Experiments with other window configurations indicate that the latter phenomenon is related to the speckle correlation across the edge. This is confirmed by tests on uncorrelated speckle, where no bias towards the side of higher reflectivity is observed. The MSE increases dramatically towards the extremities, which suggests that one should exclude the extreme positions from the test, and rather concentrate on positions closer to the center.

Fig. 7 shows some results obtained with the two-dimensional methods. The ideal reflectivity image in Fig. 7 (a) is composed of regions of constant reflectivity, and the edge contrast is 6dB. This synthetic image was multiplied by simulated speckle with the correlation characteristics shown in Table 1, to obtain the speckled image in Fig. 7 (b). The SWF-based LR operator was applied to this image using an 11 x 11 window split along the horizontal, vertical and diagonal axes. The resulting edge strength map is shown in Fig. 7 (c). The edge dynamics were calculated on the edge strength map. In Fig. 7 (d) the edge dynamics have been inverted, so that a dark edge segment indicates a strong edge. Using an interactive visualization tool, we observed the segmentation result for a wide range of thresholds. With the most suitable one, all real edges and only two false ones were detected. This is mainly due to the fact that we used a relatively big analyzing window when computing the edge strength map. However, the edge localization is not always perfect, as can be seen from Fig. 7 (e), where the detected edges have been superposed on the ideal image. When the segmentation is superposed on the speckled image, as in Fig. 7 (f), it is much more difficult to assess such small deviations from the correct edge positions by eye. The edge localization was effectuated with the approximate ML criterion given by Eqs. (16) and (19) and the contextual regularization term based on Potts model. Several combinations of parameters were used in Eq. (22). The most satisfactory result, obtained with $\lambda = -0.8$, $T_0 = 6$ and 5 iterations, is shown in Fig. 7 (g) and (h). The fit to the correct edge positions has generally become better, but the contours are slightly more irregular. Stronger regularization created problems at the extremities of narrow structures. Preliminary tests of the method based on active contours indicate the necessity of including a regularization term in the energy function here too.

6. CONCLUSION

The optimal LR criterion for edge detection in SLC images with correlated speckle is shown to be similar to the LR criterion for intensity images with uncorrelated speckle. The difference is in the estimation of local mean reflectivities. The SWF decorrelates the speckle in SLC images and yields optimal speckle reduction. For intensity images this optimum can only be attained by the AMI in the case of uncorrelated speckle. Experiments on simulated images show that the edge detection performance can be improved by working on SLC images rather than intensity images. We also introduce the ML estimator of the edge position, as well as an approximate ML estimator which requires less computation. A two-dimensional implementation is proposed, where the edge detection stage is effectuated with
Figure 7. (a) Ideal “cartoon model” reflectivity image. (b) Corresponding image with single-look correlated speckle. (c) Edge strength map obtained with the $11 \times 11$ LR operator. (d) Inverted edge dynamics. (e) Edges obtained by thresholding the edge dynamics superposed on the ideal image and (f) on the speckled image. (g) Edges after the localization refinement stage superposed on the ideal image and (h) on the speckled image.

Multidirectional windows and thresholding of basin dynamics, and the edge localization stage is based on Gibbs random fields or active contours.

The spatial correlation coefficients of our simulated speckle are close to those of the speckle in ERS images. In images with even stronger spatial correlation, the superiority of the SWF-based methods will be even more striking. Working on complex data combines the advantages of full spatial resolution and optimal radiometric estimators, which permit improved statistical tests.

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