Recursion

CSC212

Recursion

- **Sometimes, certain statements in an algorithm are repeated on different sizes of an input instance.**
- **Repetition can be achieved in two different ways.**
  - **Iteration: uses for and while loops**
  - **Recursion: function calls itself**

Example -1:

– Factorial Function

–Factorial function of any integer n is defined as

$$n! = \begin{cases} 1 \text{ if } n = 0 & \leftarrow \text{ Base Case} \\ n(n-1)! \text{ if } n \geq 1 & \leftarrow \text{ Recusion Case} \end{cases}$$

– This is recursive definition. It consists of two parts:

i. **Base case**

ii. **Recursive case**

Recursion

Example -1 (Continue) :

- It can be written as:

$$\text{fact}(n) = \begin{cases} 1 \text{ if } n = 0 & \leftarrow \text{ Base Case} \\ n\text{fact}(n-1)! \text{ if } n \geq 1 & \leftarrow \text{ Recusion Case} \end{cases}$$

where fact(n) is the function that calculates n!.

Recursion

Example -1 (Continue) :

- Implementation

Recursive:

```
public static int recursiveFact(int n)
{
     if(n==0)return 1;
     else
        return n*recursiveFact(n-1);
}
```

Iterative:

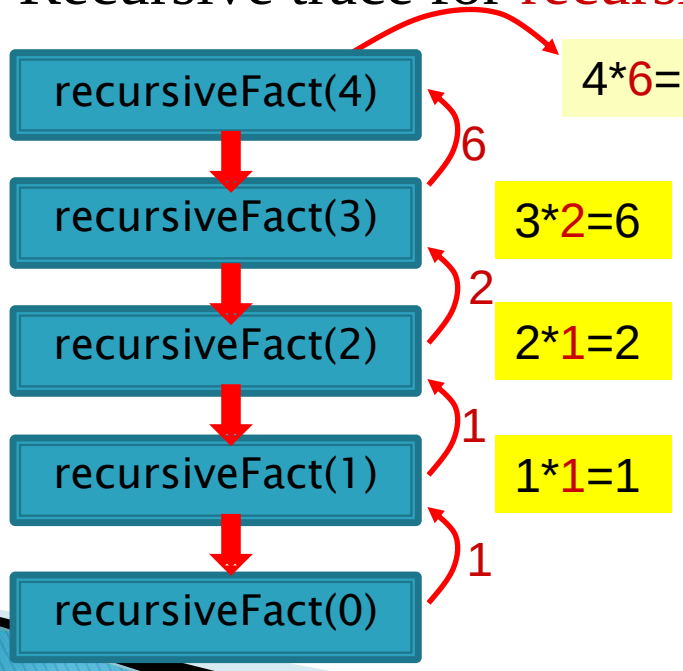
```
public static int iterativeFact(int n)
{
     int fact = 1;
     for(i = 1; i <= n; i++)
        fact=fact*I;
     return fact;
}
```

Recursive Trace

- A graphical representation of recursive calls.
- It is used to analyze the algorithm.

Example -2:

- Recursive trace for recursiveFact(4)




```
public static int recursiveFact(int n)
{
        if(n==0)
          return 1;
        else
          return n*recursiveFact(n-1);
}
```

Main Types of Recursion

- Linear Recursion
- Binary Recursion

Linear Recursion

In this case a recursive method makes at most one recursive call each time it is invoked.

Example – 3:

- **Problem:** Given an array A of *n* integers, find the sum of first *n* integers.
- **Observation:** Sum can be defined recursively as follows:

$$\text{Sum}(n) = \begin{cases} \text{A}[0] \text{ if } n = 0 & \leftarrow \text{ Base Case} \\ \text{Sum}(n-1) + \text{A}[n-1] \text{ if } n \geq 1 & \leftarrow \text{ Recusive Case} \end{cases}$$

Linear Recursion

Example – 3 (Continued):

- **Algorithm**

Sum(A, n)

Input: An integer array A and an integer $n \geq 1$, such that A has at least n elements

Output: The sum of the first n integers in A.

Processing:

```
if n = 1;
    return A[0];                          → base case
else
    return Sum(A, n-1) + A[n-1];          → recursive case.
```

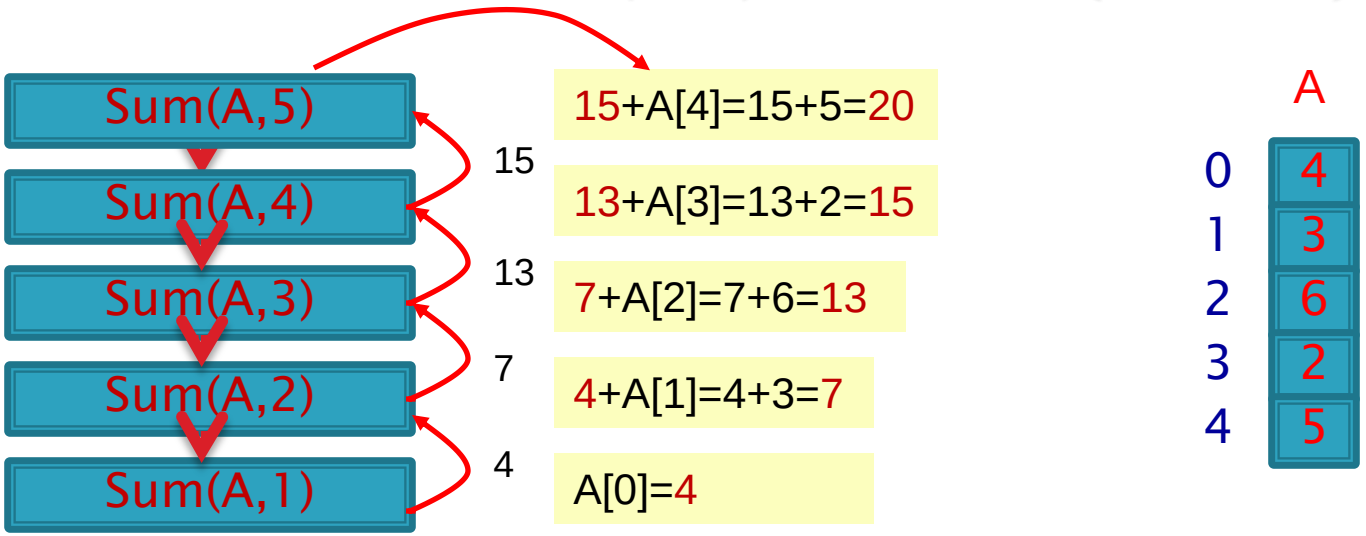
Note:

- Base case should be defined so that every possible chain of recursive calls eventually reach a base case.
- Algorithm must start by testing a set of base cases.
- After testing for base cases perform a single recursive call.

Linear Recursion

Example – 3 (Continued):

- Recursive trace for sum(A ,n), where A ={4,3,6,2,5}, n=5



Note:

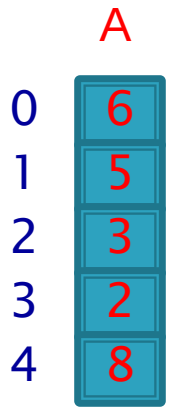
- For an array of size *n*, Sum(A, *n*) makes n calls.
- Each spends a constant amount of time.
- So time complexity is O(*n*).

Binary Recursion

In this case, a recursive algorithm makes two recursive calls.

Example – 4

- **Problem:** Find the sum of n elements of an integer array A.
- **Algorithm:**
  - Recursively find the sum of elements in the first half of A.
  - Recursively find the sum of elements in the second half of A.
  - Add these two values



BinarySum(A ,i ,n)

Input: An integer array A and an integer $n \geq 1$, such that A has at least n elements

Output: The sum of the first n integers in A.

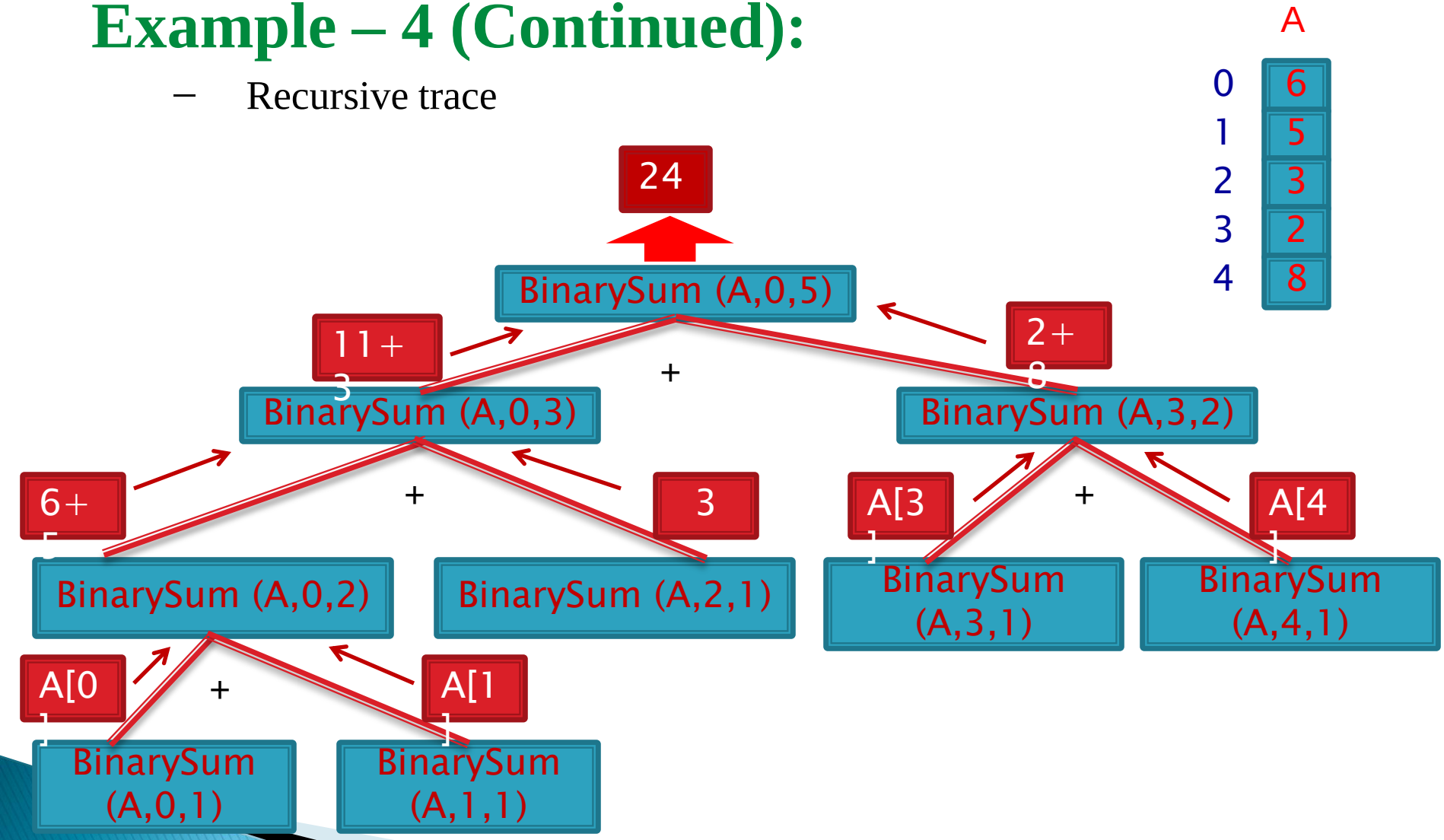
Processing:

```
if n =1
        return A[i];
Else
return BinarySum(A ,i, ⌈n/2⌉)+BinarySum(A ,i+⌈n/2⌉, ⌊n/2⌋);
```

Binary Recursion

Example – 4 (Continued):

- Recursive trace



Binary Recursion

Example – 5

- **The Fibonacci Number**
  1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
- Each number after the second number is the sum of the two preceding numbers.
- These numbers can naturally be defined recursively :

$$F(n) = \begin{cases} 1 \text{ if } n = 0 & \leftarrow \text{ Base Case-1} \\ 1 \text{ if } n = 1 & \leftarrow \text{ Base Case-2} \\ F(n-1) + F(n-2) \text{ if } n > 1 & \leftarrow \text{ Recursive Case} \end{cases}$$

Binary Recursion

Example – 5 (Continued)

- Recursive Implementation of Fibonacci Function

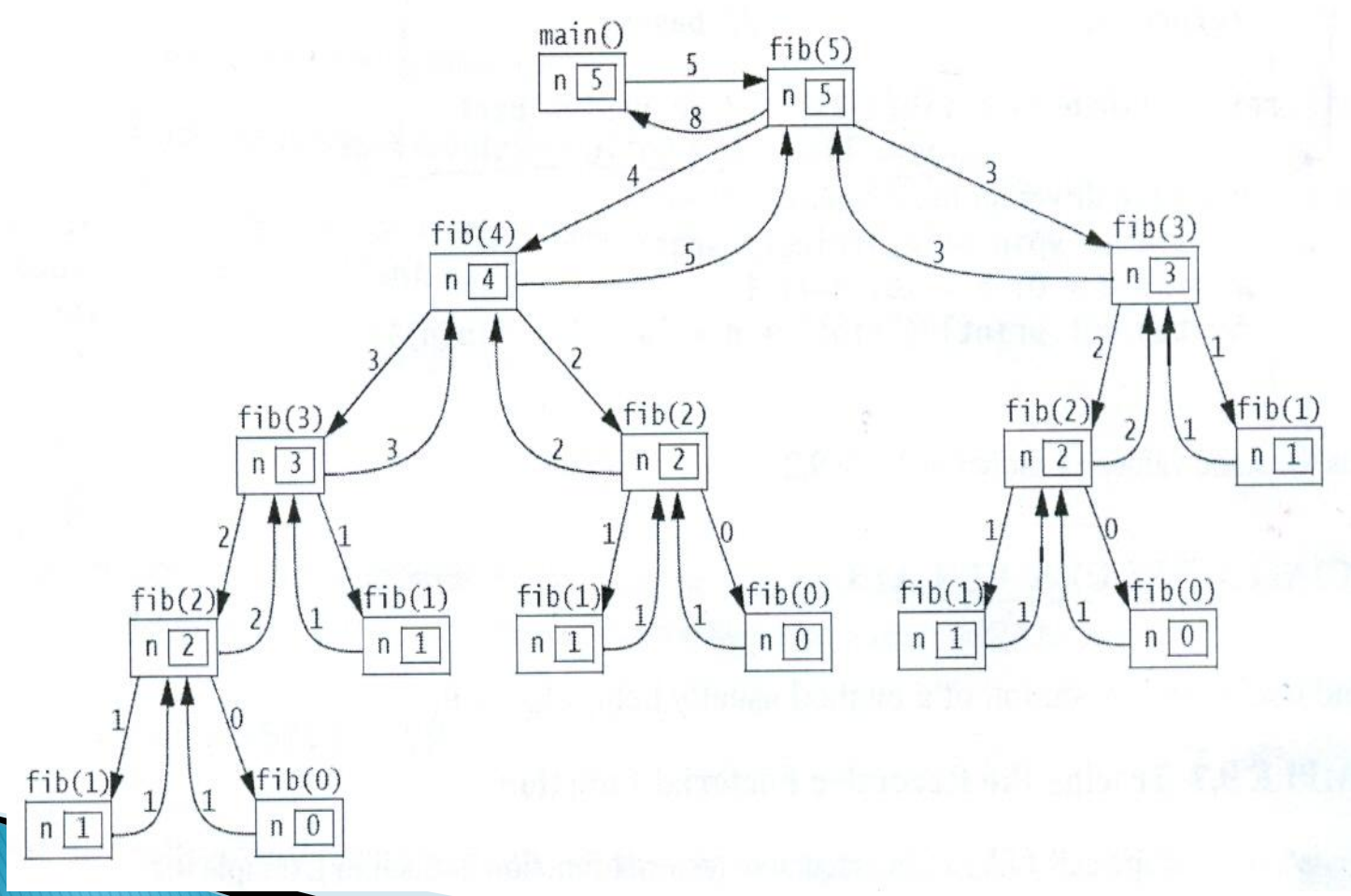
```
public static int fib(int n)
{
        if (n < 2)
                return 1;                       // base cases
        else
                return  fib(n-1)+fib(n-2);   // recursive part

}
```

Binary Recursion

Example – 5 (Continued)

- Recursive Trace of Fibonacci Function: fib(5)



Linear Recursion

Example – 6: Binary Search

- **Problem**: Given $S=\{s_0, s_1, \square\ s_{n-1}\}$ is a sorted sequence of *n* integers, and an integer x. Search weather x is in S.
- **Binary Search Algorithm:**
  - If the sequence is empty, return -1.
  - Let s_i be the middle element of the sequence.
    - If $s_i = x$, return its index i.
    - If $s_i < x$, apply the algorithm on the subsequence that lies above s_i.
    - Otherwise, apply the algorithm on the subsequence of S that lies below s_i.

Linear Recursion

Example – 6 (continued): Binary Search

- Implementation:

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
     else{                          // Recursive part
           int i = (lo+hi/2);
           if(a[i] == x) return i;
           else if(a[i] < x)
                      return search (a, i+1, hi, x)
           else
                      return search (a, lo,i-1, x);
     }
}
```