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Exact Symbol Error Rate of MQAM with MRC over Flat Fading Nakagami-m and Rician Channels

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Abstract:

This paper presents new, simple, and exact expressions for the average symbol error rate (ASER) of MQAM using antenna diversity with maximum ratio combining (MRC), transmitted over flat fading channels. Two fading distributions are considered: Nakagami-m and Rician. The analysis is based on representing the Gaussian Q function in the form of a sum of exponential functions that, in the limit, approaches the exact value. Our results are compared with exact values for various signal alphabets, fading parameters and diversity orders.

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I. Introduction

Demands for faster data rates on wireless and cellular communication systems, such as 3.5G, have led to much current interest in the use of M-ary Quadrature Amplitude Modulation (MQAM) signaling formats due to its high spectral efficiency. However, in wireless communication, it is well known that the fading phenomenon, which inherently exists in most radio links, constitute one of the boundary conditions of radio communications design. A widely recognized practice for combating fading in digital communications over such a time-varying channel is to employ space diversity combining techniques. The most commonly used diversity combining techniques are maximal ratio combining (MRC), equal gain combining (EGC) and selection combining (SC). A thorough treatment of these diversity combining techniques can be found in [1]. MRC is the optimal linear combining technique and provides the maximum possible improvement that a diversity system can attain through a fading channel [2].

Two fading distributions will be considered here: Nakagami-*m* and Rician distributions. Nakagami-*m* distribution is the best fit for digital signals received in urban multipath fading channels and Rician is the best fit for digital signals received in line-of-sight (LOS) communication links. Both are two parameter distributions that provide more flexibility and accuracy in matching the observed signal statistics [2]-[4].

The exact average symbol error rate (ASER) of MQAM with MRC over independent and identical Nakagami-*m* and Rician fading channels have been previously reported. Zhang *et al* [5] presented a closed form infinite series expression. For the same problem Seo *et al* [6] derived an expression for the ASER using hypergeometric functions. Manjeet *et al* [7]-[8] came up with a closed form expression in the form of single integral for both Nakagami-*m* and Rician channels. Falujah *et al* [9] proposed an expression in the form of hypergeometric functions for MQAM with MRC over Nakagami-*m*. The same problem was solved by Annamalai *et al* [10] with expression in the form of hypergeometric functions.

The rest of the paper is organized as follows: System model is presented in section II. Section III analyzes the ASER of MQAM with MRC over both Rician and Nakagami-*m* channels. Numerical results and discussion are shown in section IV. Finally, some conclusions are given in section V.

II. System Model

Two types of fading channels are considered: Nakagami-*m* and Rician fading channels. Both will be analyzed when space diversity and MRC is employed. We assume that, for both fading models Nakagami-*m* and Rician, there are L diversity channels carrying the same information signal. Each channel is modeled as frequency non-selective slowly fading channel corrupted by additive white Gaussian noise (AWGN) process. The fading processes and the noise processes among the L diversity branches are assumed to be mutually statistically independent. Also the channel state information is assumed to be perfect at the receiver with MRC.

For Nakagami-*m* frequency-nonselective and slowly varying channel the probability density function (*pdf*) of the instantaneous MRC output SNR γ can be written as [11]

$$p(\gamma) = \left(\frac{mL}{\bar{\gamma}}\right)^{mL} \frac{\gamma^{mL-1}\exp\left(-mL\gamma/\bar{\gamma}\right)}{\Gamma(mL)} \quad (1)$$

Where, L is the number of MRC branches, m is the Nakagami-*m* fading parameter, $\Gamma(.)$ is the gamma function and $\bar{\gamma}$ is the average signal-to-noise ratio (SNR) associated with each symbol and it is related to the average SNR per channel ($\bar{\gamma}_l$) as $\bar{\gamma} = \sum_{k=1}^{L} \bar{\gamma}_l$.

For Rician fading channel the pdf of the instantaneous SNR γ at the output of the MRC can be written as [12]

$$p(\gamma) = \frac{L+K}{\bar{\gamma}}\left(\frac{(L+K)\gamma}{K\bar{\gamma}}\right)^{\frac{L-1}{2}} e^{-K} \exp\left[\frac{-\gamma(L+K)}{\bar{\gamma}}\right] I_{L-1}\left(2\sqrt{\frac{\gamma K(L+K)}{\bar{\gamma}}}\right) \quad (2)$$

Where, L is the number of branches, $\bar{\gamma}$ is the expected value of γ (SNR), K is the Rician parameter and $I_{L-1}(.)$ is the modified Bessel function of L-1 order.

III. Performance Analyses

A. Gaussian channel

In MQAM, a symbol is generated according to $\log_2 M$ data bits, and each symbol in a quadrant has different Symbol Error Rate (SER). For square MQAM constellation $\log_2 M$ need to be even and the SER of MQAM in the AWGN channel is given by [2]

$$P_{S,AWGN}(e|\gamma) = 4aQ\left(\sqrt{b\gamma}\right) - 4a^2Q^2\left(\sqrt{b\gamma}\right) \tag{3}$$

Where, $a = \left(1 - 1/\sqrt{M}\right)$, $b = 3\log_2 M/(M-1)$, $\gamma = E_b/N_0$ is the received SNR per bit. By using the alternative form of Q(x) we can write the SER as [4]

$$P_{S,AWGN}(e|\gamma) = \frac{4a}{\pi}\int_0^{\pi/2}\exp\left(\frac{-b\gamma}{2\sin^2\theta}\right) - \frac{4a^2}{\pi}\int_0^{\pi/4}\exp\left(\frac{-b\gamma}{2\sin^2\theta}\right) \tag{4}$$

The main idea of this work is representing Q(x) in exponential form [13], which can be achieved by applying trapezoidal rule on (4) to get (Appendix 1)

$$P_{S,AWGN}(e|\gamma) = \frac{a}{n}\left\{\frac{e^{-b\gamma/2}}{2} - \frac{ae^{-b\gamma}}{2} + (1-a)\sum_{i=1}^{n-1} e^{-b\gamma/s} + \sum_{i=n}^{2n-1} e^{-b\gamma/s}\right\} \tag{5}$$

Where, $s = 2\sin^2\theta_i$, $\theta_i = \frac{i\pi}{4n}$

The average SER can be derived by averaging the conditional SER over the pdf of γ and can be written as [2]

$$ASER = \int_0^{\infty} P_S(e/\gamma).p(\gamma)d\gamma \tag{6}$$

B. Nakagami-*m*

By substituting (5) and (1) into (6), the ASER over Nakagami-m can be calculated with the help of following relationship [14]

$$\int_0^\infty x^{k-1}\exp(-ux)dx = \Gamma(k)u^{-k} \tag{7}$$

The ASER of MQAM with MRC over identical and independent (iid) flat Nakagami-m fading channel is

$$ASER_{Nak} = \frac{a}{n}\left\{\frac{1}{2}\left(1+\frac{b\overline{\gamma}_l}{2m}\right)^{-mL} - \frac{a}{2}\left(1+\frac{b\overline{\gamma}_l}{m}\right)^{-mL} + (1-a)\sum_{i=1}^{n-1}\left(1+\frac{b\overline{\gamma}_l}{ms_i}\right)^{-mL} + \sum_{i=n}^{2n-1}\left(1+\frac{b\overline{\gamma}_l}{ms_i}\right)^{-mL}\right\} \tag{8}$$

For special case, when we use $n = 1$ in (8) we get a simple approximation of ASER with two terms i.e.,

$$ASER_{Nak} \approx \frac{a}{2}\left\{\left(1+\frac{b\overline{\gamma}_l}{2m}\right)^{-mL} + (2-a)\left(1+\frac{b\overline{\gamma}_l}{m}\right)^{-mL}\right\} \tag{9}$$

And for $L = 1$, (8) reduces to the case of MQAM over single Nakagami-m and for $m = 1$ it reduces to the case of MQAM over Rayleigh i.e.,

$$ASER_{Ray} = \frac{a}{n}\left\{\frac{1}{2}\left(\frac{2}{b\overline{\gamma}_l+2}\right)^{L} - \frac{a}{2}\left(\frac{1}{b\overline{\gamma}_l+1}\right)^{L} + (1-a)\sum_{i=1}^{n-1}\left(\frac{s_i}{b\overline{\gamma}_l+s_i}\right)^{L} + \sum_{i=n}^{2n-1}\left(\frac{s_i}{b\overline{\gamma}_l+s_i}\right)^{L}\right\} \tag{10}$$

C. Rician

By substituting (5) and (2) into (6), the ASER over Rician can be calculated with help of the following relationship [14]

$$\int_0^\infty x^{v} e^{-\alpha x} I_{2v}(2\beta\sqrt{x})dx = \alpha^{-(2v+1)}\beta^{2v}\exp\left(\frac{\beta^2}{\alpha}\right) \tag{11}$$

The ASER of MQAM with MRC over identical and independent (iid) flat Rician fading channel is

$$ASER_{Ri} = \frac{a}{n}\left\{\frac{1}{2}\left(\frac{b}{2r}+1\right)^{-L}\exp\left[\frac{-Kb}{b+2r}\right] - \frac{a}{2}\left(\frac{b}{r}+1\right)^{-L}\exp\left[\frac{-Kb}{b+r}\right]\right.$$

$$+(1-a)\sum_{i=1}^{n-1}\left(\frac{b}{rs_i}+1\right)^{-L}\exp\left[\frac{-Kb}{b+rs_i}\right]+\sum_{i=n}^{2n-1}\left(\frac{b}{rs_i}+1\right)^{-L}\exp\left[\frac{-Kb}{b+rs_i}\right]\Bigg\} \qquad (12)$$

Where $r=(L+K)/L\overline{\gamma}_l$

For particular cases of L = 1 (12) reduces to the case of MQAM over single Rician and for K = 0 it reduces to the case of MQAM over Rayleigh i.e. (7). And for n = 1, (12) reduces to

$$ASER_{Ric}\approx\frac{a}{2}\left(\frac{2r}{b+2r}\right)^{L}\exp\left[\frac{-Kb}{b+2r}\right]+\left(a-\frac{a^2}{2}\right)\left(\frac{r}{b+r}\right)^{L}\exp\left[\frac{-Kb}{b+r}\right] \qquad (13)$$

IV. Numerical results and discussion

The ASER versus average SNR per bit per branch (i.e. $\bar{\gamma}/L$) for MQAM with MRC diversity over iid Nakagami-*m* fading channels is plotted for various combinations of the number of branches L, the fading parameter m, and the constellation size M in Fig. 1, 2 & 3. In the same way, Fig. 4 and 5 shows the ASER over iid Rician channels for MQAM with MRC. In (8) it is clear that the system performance is related directly with the product of m and L. In other words, by using MRC we can maintain the link quality to specific value. The figures also show that the larger the product of fading parameter (m or K) and L, the larger the performance improvements obtained. Most of the potential diversity gains are obtained with a small number of channels. The accuracy of the approximation is shown in Fig. 1-5 where the curves are plotted using (8) for Nakagami-*m* and (12) for Rician with n = 500 for the exact and n = 1 for the approximate (i.e. (9) & (13)) except Fig. 3 where n = 2 for the approximate and it shows that the exact and approximate are overlapping. Table 1 shows the difference (error) in dB between the exact and approximate for n = 1, 2, & 3 where the largest values are selected in the table for different combinations of M, L, and m. The maximum error is 0.47 dB for the case when the m and L product is 0.5 and when the product value increase the error decreases. By increasing n to10 the maximum error is less than 0.0005 dB which can be considered as an exact.

V. Conclusion

In this paper, we obtained simple closed form expressions to determine the performance of MQAM with MRC transmitted over slow, flat, identically independently distributed (iid) fading channels and using space diversity in terms of ASER. Two types of fading channels are considered: Nakagami-*m* and Rician. These simple and efficient ASER formulas make it possible for the first time to study, analyze and discuss the parameters of various constellations of square MQAM, diversity order and fading parameters precisely and easily in one simple formula that is in closed form series expressions. Theoretically, to get the exact solution the series must be infinite. But because the series converge rapidly, 10 terms are enough to get an error less than 0.0005 dB in the worst case which is at high order of modulation index (M) and low product value of diversity order and fading parameter (K for Rician or m for Nakagami-m).

Appendix I

Applying trapezoidal rule on (4) we get

$$P_{S,AWGN}\left(e|\gamma\right)=\frac{a}{m}\left[\exp\left(\frac{-b\gamma}{2}\right)+2\sum_{i=1}^{m-1}\exp\left(\frac{-b\gamma}{2\sin^2\phi_i}\right)\right]-\frac{a^2}{2n}\left[\exp\left(-b\gamma\right)+2\sum_{i=1}^{n-1}\exp\left(\frac{-b\gamma}{2\sin^2\theta_i}\right)\right]$$

Where $\phi_i=\dfrac{i\pi}{2m}$ and $\theta_i=\dfrac{i\pi}{4n}$

Let m=2n to get

$$P_{S,AWGN}\left(e|\gamma\right)=\frac{a}{2n}\left[\exp\left(\frac{-b\gamma}{2}\right)+2\sum_{i=1}^{2n-1}\exp\left(\frac{-b\gamma}{2\sin^2\theta_i}\right)\right]-\frac{a^2}{2n}\left[\exp\left(-b\gamma\right)+2\sum_{i=1}^{n-1}\exp\left(\frac{-b\gamma}{2\sin^2\theta_i}\right)\right]$$

$$P_{S,AWGN}\left(e\,|\gamma\right)=\frac{a}{2n}\left[\exp\left(\frac{-b\gamma}{2}\right)+2\sum_{i=1}^{n-1}\exp\left(\frac{-b\gamma}{2\sin^2\theta_i}\right)+\sum_{i=n}^{2n-1}\exp\left(\frac{-b\gamma}{2\sin^2\theta_i}\right)\right]$$

$$-\frac{a^2}{2n}\left[\exp\left(-b\gamma\right)+2\sum_{i=1}^{n-1}\exp\left(\frac{-b\gamma}{2\sin^2\theta_i}\right)\right]$$

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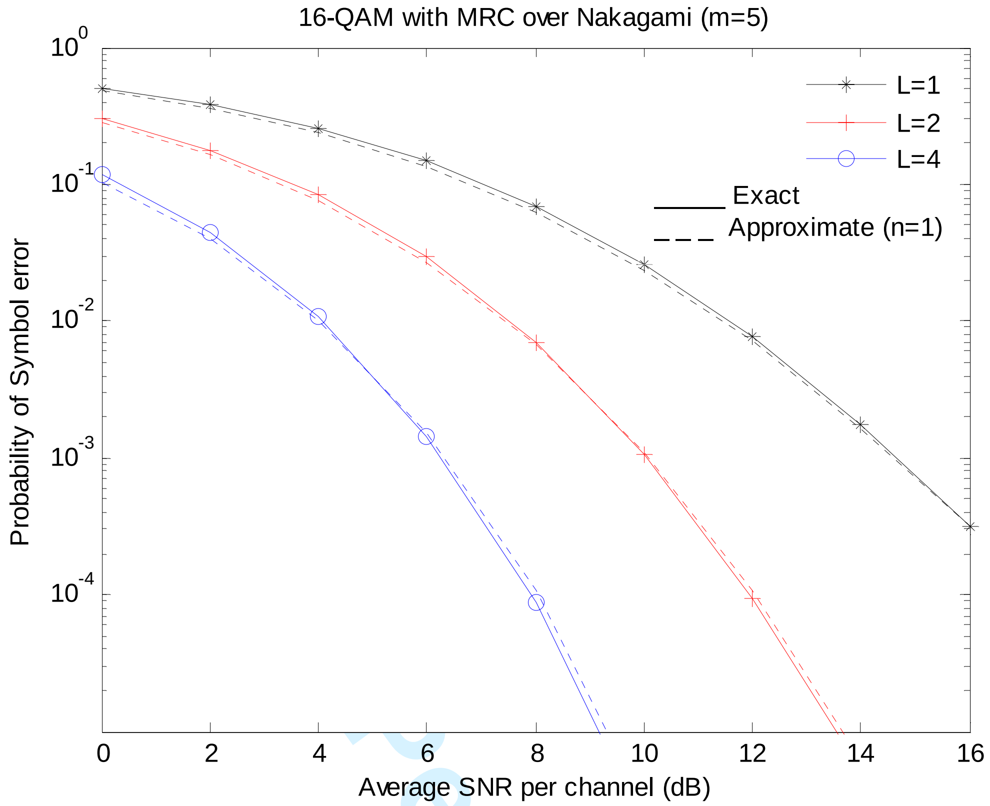

Fig. 1. ASER of 16-QAM with MRC over Nakagami-*m* (m=5) using (8) for both n=100 & 1.

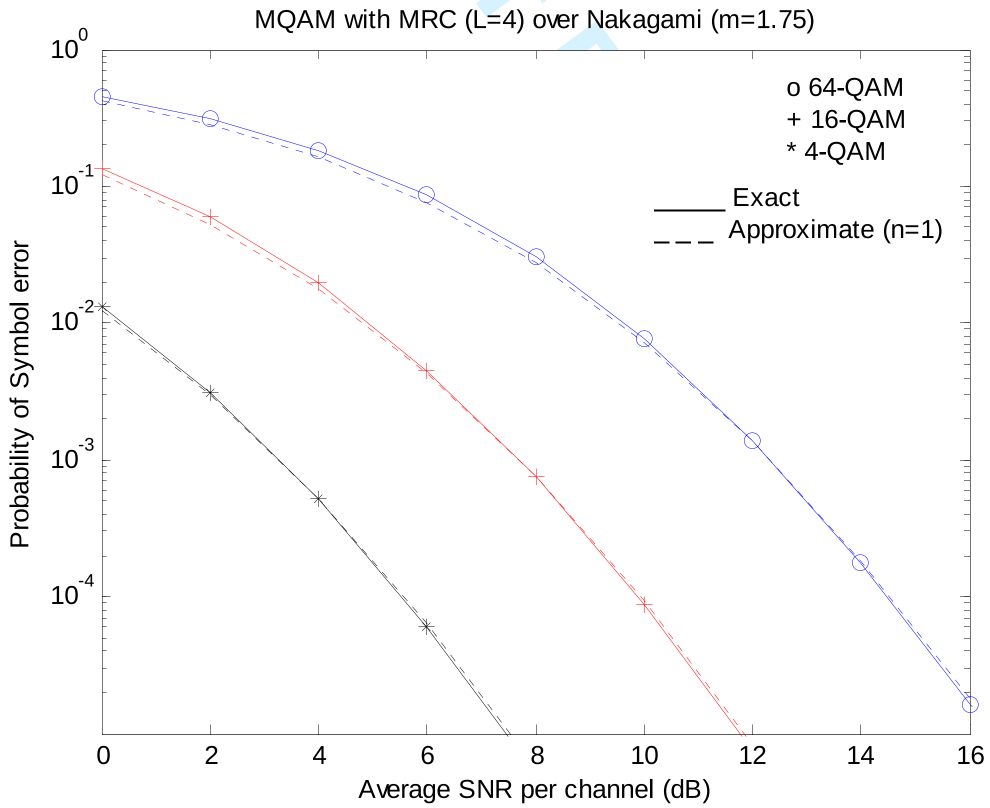

Fig. 2: ASER of MQAM with MRC (L = 4) over Nakagami-*m* with fading figure *m* = 1.75 by using (8) for both n=100 and 1.

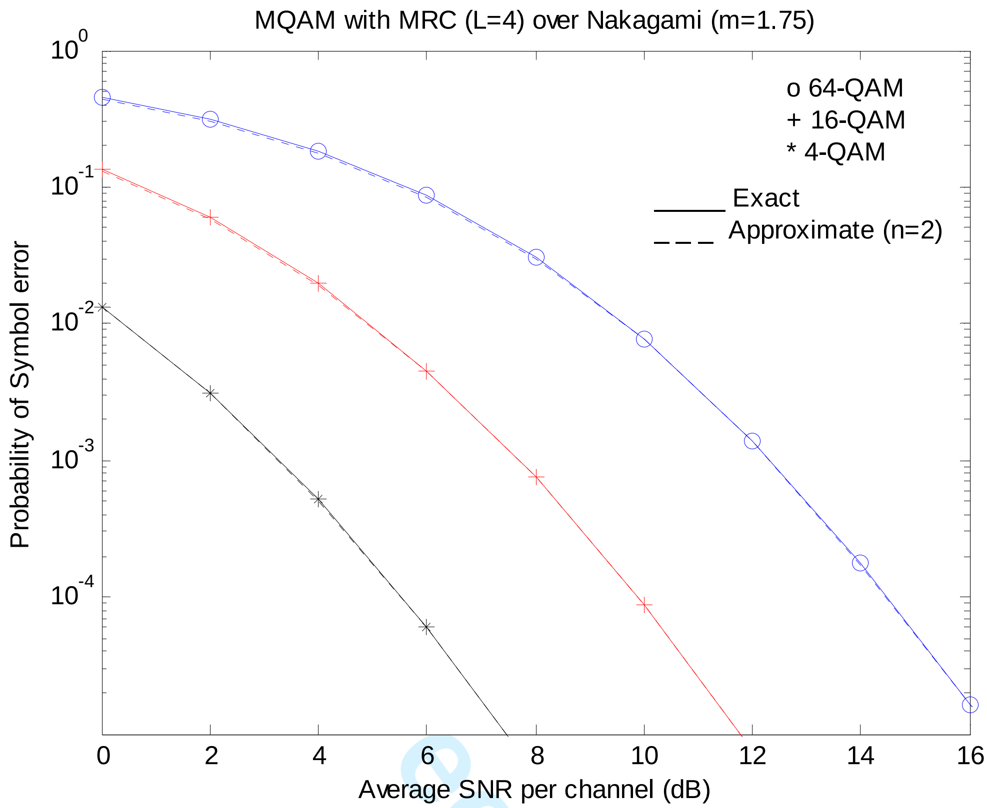

Fig. 3: ASER of MQAM with MRC (L = 4) over Nakagami-*m* with fading figure *m* = 1.75 by using (8) for both n=100 and 2.

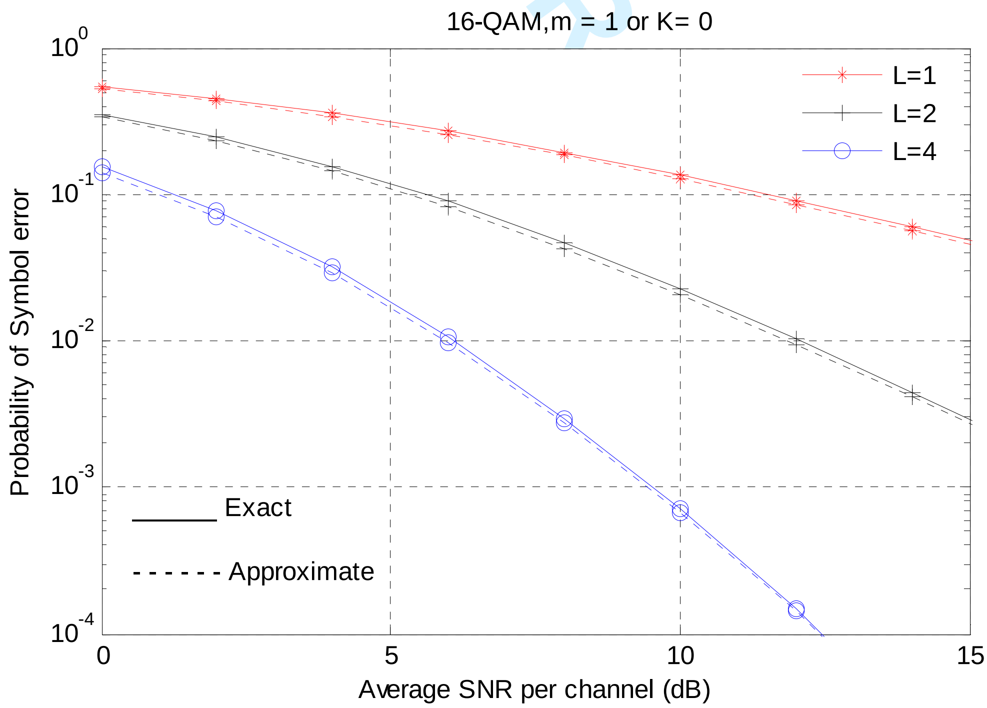

Fig. 4: ASER of 16-QAM with MRC over Rayleigh fading channel by using (12) for both n=100 and 1.

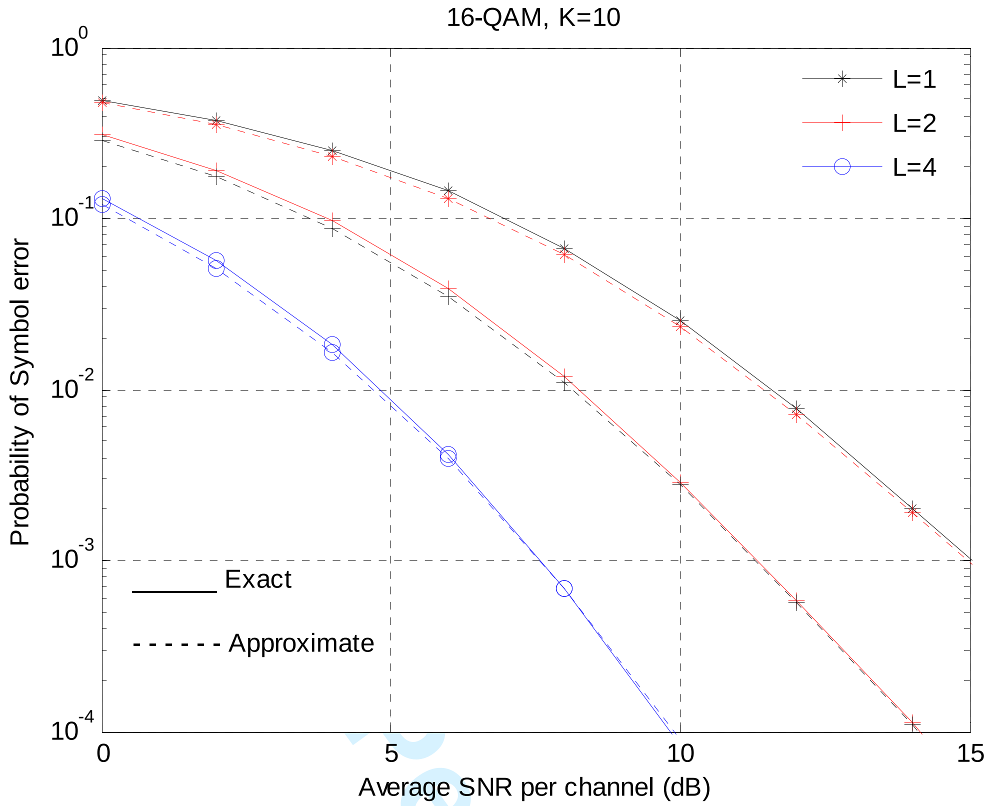

Fig. 5: ASER of MQAM with MRC over Rician with fading figure $K = 10$ by using (12) for both n=100 and 1.

Table1: Difference in dB between exact and approximate for M-QAM with MRC over Nakagami-*m* fading channel.

M	*m*	*L*	The difference in dB *n* = 1	*n* = 2	*n* = 3
16	0.5	1	0.47	0.11	0.05
		2	0.29	0.07	0.03
		4	0.26	0.06	0.03
	1	1	0.29	0.07	0.03
		2	0.26	0.06	0.03
		4	0.27	0.06	0.03
64	0.5	1	0.47	0.11	0.05
		2	0.35	0.08	0.04
		4	0.32	0.07	0.03
	1	1	0.35	0.08	0.04
		2	0.32	0.07	0.03
		4	0.31	0.07	0.03
	2	1	0.32	0.07	0.03
		2	0.31	0.07	0.03
		4	0.30	0.06	0.03
	5	1	0.30	0.06	0.03
		2	0.30	0.06	0.03
		4	0.30	0.06	0.02