EQUIVALENT CAPACITY GAINS IN SIGNAL-TO-NOISE RATIO DUE TO CORRELATION AND RICIAN FACTORS IN MIMO SYSTEMS

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I. INTRODUCTION

This paper considers the capacity of single user frequency flat doubly (double-sided) spatially correlated symmetric ($m \times m$) MIMO Rician channels. Double-sided correlation occurs when the antennas are insufficiently spaced at both ends, or when there are a limited number of scatterers surrounding both the transmit and receive terminals. Throughout the paper we assume that the channel is unknown at the transmitter and perfectly known at the receiver. We present the view point that the effect of K factor and correlated fading parameter (r) on the capacity of $m \times m$ MIMO systems can be considered as an equivalent increase or decrease in the SNR (dB). The results are expressed in mathematical forms.

Although, recently some capacity bounds for semi-correlated Rician fading channels are presented in the literature, but to the best of our knowledge, no exact analytical results are available for semi or doubly correlated Rician fading channels. The only exact analytical result available in the literature so far is for the uncorrelated Rician fading channels due to Kang & Alouini. So, we used Monte Carlo simulation to simulate doubly correlated Rician fading channels.

II. SIMULATION RESULTS

First, we validate our Monte Carlo simulations for uncorrelated MIMO Rician channels with the exact analytical results of Kang & Alouini (figure excluded due to space limitation). After validating the Monte Carlo simulations, we fixed the capacity at any arbitrary value and calculated the effect of Rician factor (K) with or without correlated fading parameter(r) in terms of the equivalent gain in the SNR required to achieve this capacity. Negative gain implies loss in the performance. Here, we fixed the capacity at 10 bits/s/Hz. The results are presented in a tabular form. All the data presented in the tables are in dB. The gains in tables 3–7 are calculated with respect to table 1 (i.i.d Rayleigh fading channel, K = 0, and r = 0). Table 1 represents the SNR required to achieve capacity of 10 bits/s/Hz by $m \times m$ systems. Table 3 presents the gains only due to correlation. The second column in tables 4–7 presents gains only due to K factor. The combined gains due to correlation and K factor are presented in tables 4–7 from columns 3–8.

From the tables 1–7, we can deduce the following:

- The relative gain (RG) in SNR (dB) due to correlated Rician fading decreases with m (see Table 2 as an example). It is defined as follows:

$$\text{RG}=\frac{(\text{SNRat } K=0, r=0)-(\text{SNRat } K=K, r=r)}{(\text{SNRat } K=0, r=0)} \quad (1)$$

- For all values of m ($m \neq 1$), the combined gain in the SNR in dB due to K factor and correlated fading parameter (r) in terms of their individual gains in the SNR in dB at any given fixed capacity can be written in the following mathematical forms:

(i) Over a practical range of K and r, i,e. for practical cases, the combined gain can be approximated as the sum of the two individual gains as:

$$L_{total} \approx L_{corr} + L_K, \qquad 0 \le K \le 10, 0 \le r \le 0.95 \quad (2)$$

where, "$\approx$" represent approximation, and

- L_{total} : Total gain in the SNR in dB due to correlation parameter r and Rician factor K
- L_{corr} : Gain in the SNR in dB due to correlation parameter r
- L_K : Gain in the SNR in dB due to Rician factor K

(ii) At high values of K i.e, for AWGN, gain in the SNR in dB due to correlation parameter r becomes negligible i.e.,

$$L_{corr} \approx 0, \qquad 0 \le r \le 1 \quad (3)$$

Thus,

$$L_{total} \approx L_K, \qquad 0 \le r \le 1 \quad (4)$$

(iii) When the correlation parameter r is equal to 1, eq. 1 does not hold because for r equal to 1, the MIMO system reduces to SISO system and at this value of r, the increase in K factor will improve the performance of the system.

$$L_{total} \approx L_{corr} + L_{K(m=1)}, \quad 0 \le K \le 10, \quad (5)$$

where,

- $L_{K(m=1)}$: Gain in the SNR in dB due to Rician factor K for $m = 1$

III. CONCLUSIONS

For $m \times m$ MIMO systems, we found that the relative gain (RG) in SNR due to correlated fading decreases with m. All mathematical expressions give a good estimate of the combined gains in the SNR in dB to within 1 dB. For practical scenarios (eq. 1), the combined gain in the SNR in dB due to K factor and correlated fading parameter (r)

in terms of their individual gains in the SNR in dB is simply the addition of the two gains.

Table 1: Required SNR (dB) at C = 10 bits/s/Hz,

m	SNR (dB) $K = 0$, $r = 0$
1	32.53
2	17.91
4	9.01
6	5.16
10	1.19

Table 2: Relative gain (RG) at C = 10 bits/s/Hz,

m	SNR (dB) $K = 2$, $r = 0.7$	RG $K = 2$, $r = 0.7$
1	0.95	0.95/32.53 = 0.029
2	-4.74	-4.74/17.91 = -0.264
4	-6.13	-6.13/9.01 = -0.68
6	-6.12	-6.12/5.16 = -1.19
10	-5.70	-5.70/1.19 = -4.79

Table 3: Equivalent gain in SNR (dB) at C = 10 bits/s/Hz, $K = 0$

m	$r = 0$	$r = 0.3$	$r = 0.5$	$r = 0.7$	$r = 0.9$	$r = 0.95$	$r = 1$
1	0	0	0	0	0	0	0
2	0	-0.36	-1.18	-2.69	-6.12	-8.13	-11.65
4	0	-0.43	-1.33	-3.13	-7.61	-10.37	-17.50
6	0	-0.39	-1.24	-2.95	-7.49	-10.48	-19.66
10	0	-0.33	-1.02	-2.51	-6.79	-9.85	-21.39

Table 4: Equivalent gain in SNR (dB) at C = 10 bits/s/Hz, $K = 2$

m	$r = 0$	$r = 0.3$	$r = 0.5$	$r = 0.7$	$r = 0.9$	$r = 0.95$	$r = 1$
1	0.95	0.95	0.95	0.95	0.95	0.95	0.95
2	-0.74	-1.98	-3.08	-4.74	-7.94	-9.45	-10.68
4	-1.77	-2.83	-4.07	-6.13	-10.52	-12.89	-16.52
6	-2.11	-2.98	-4.12	-6.12	-10.70	-13.44	-18.73
10	-2.44	-3.07	-3.97	-5.70	-10.16	-13.07	-20.44

Table 5: Equivalent gain in SNR (dB) at C = 10 bits/s/Hz, $K = 4$

m	$r = 0$	$r = 0.3$	$r = 0.5$	$r = 0.7$	$r = 0.9$	$r = 0.95$	$r = 1$
1	1.49	1.49	1.49	1.49	1.49	1.49	1.49
2	-1.51	-2.75	-3.89	-5.57	-8.36	-9.37	-10.17
4	-2.95	-4.04	-5.33	-7.32	-11.52	-13.62	-16.02
6	-3.47	-4.35	-5.48	-7.43	-11.89	-14.36	-18.13
10	-3.92	-4.55	-5.43	-7.13	-11.48	-14.23	-19.82

Table 6: Equivalent gain in SNR (dB) at C = 10 bits/s/Hz, $K = 10$

m	$r = 0$	$r = 0.3$	$r = 0.5$	$r = 0.7$	$r = 0.9$	$r = 0.95$	$r = 1$
1	2.05	2.05	2.05	2.05	2.05	2.05	2.05
2	-2.76	-3.95	-5.07	-6.55	-8.71	-9.28	-9.58
4	-4.87	-5.94	-7.11	-9.07	-12.69	-14.24	-15.48
6	-5.61	-6.47	-7.53	-9.39	-13.39	-15.43	-17.59
10	-6.24	-6.85	-7.67	-9.36	-13.29	-15.69	-19.30

Table 7: Equivalent gain in SNR (dB) at C = 10 bits/s/Hz, AWGN

m	$r = 0$	$r = 0.3$	$r = 0.5$	$r = 0.7$	$r = 0.9$	$r = 0.95$	$r = 1$
1	2.43	2.43	2.43	2.43	2.43	2.43	2.43
2	-9.13	-9.13	-9.15	-9.17	-9.17	-9.17	-9.17
4	-15.05	-15.06	-15.06	-15.06	-15.06	-15.07	-15.07
6	-17.14	-17.14	-17.15	-17.15	-17.15	-17.15	-17.15
10	-18.88	-18.88	-18.88	-18.88	-18.88	-18.90	-18.90